Unit 6: Power Series & Function Approximations

Content Area: **Math**
Course(s): **Calculus AP**
Time Period:
Length: **20 Days**
Status: **Published**

Title Section

Department of Curriculum and Instruction

Belleville Public Schools

Curriculum Guide

Calculus AP, Unit 6

Power Series & Function Approximations

Belleville Board of Education

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Board Approved:September 23, 2019

Unit Overview

- Determine whether a series converges or diverges
- Determine or estimate the sum of a series
- Construct and use Taylor Polynomials
- Write a power series representing a given function

Students will be able to independently use their learning to understand and explain the deep relationship between transcendental functions and algebraic functions and to extend their knowledge of function approximation to develop better approximations of functions which nature does not directly reveal to us.

Enduring Understanding

Students will understand that:

- The sum of an infinite number of real numbers may converge
- A function can be represented by an associated power series over the interval of convergence for the power series

Essential Questions

Essential Questions are:

- How have sequences played a crucial role in the progress of civilization in the last century?
- What is Discrete Mathematics and why do we need it?
- How are Discrete and Continuous mathematical concepts related?
- How are algebraic and transcendental functions related?

Exit Skills

By the end of Unit 6 students will know:

- The nth partial sum is defined as the sum of the first n terms of a sequence
- An infinite series of numbers converges to a real number S, if and only if the limit of its sequence of partial sums exists and equals S
- Common series of numbers include geometric series, the harmonic series, and p-series
- A series may be absolutely convergent, conditionally convergent, or divergent
- If a series converges absolutely, then it converges
- In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of numbers converges or diverges are the nth term tests, the comparison test, the limit comparison test, the integral test, the ratio test, and the alternating series test
- If a is the first term of a geometric series and r is the multiplier, then the convergence of the series depends upon the magnitude of the multiplier. The sum of a convergent geometric series is given by a formula.
- If an alternating series converges by the alternating series test (AST), then the alternating series error bound can be used to estimate how close a partial sum is to the value of the infinite series
- If a series converges absolutely, then any series obtained from it by regrouping or rearranging the terms has the same value
- The coefficient of the nth-degree Taylor polynomial centered at a is the nth derivative of the function evaluated at a, divided by n!

- Taylor polynomials for a function f centered at a can be used to approximate function values of f near a
- In many cases, as the degree of a Taylor polynomial increases, the nth degree polynomial will converge to the original function over some interval
- The Lagrange error bound can be used to bound the error of a Taylor polynomial approximation to a function
- In some situations where the signs of a Taylor polynomial are alternating, the alternating series error bound can be used to bound the error of a Taylor polynomial approximation to the function
- A power series is a series in which the terms include successive powers of the difference between x and a number which represented the center of the series
- The Maclaurin series for sin(x), cos(x), and e^x provide the foundation for constructing the Maclaurin series for other functions
- The Maclaurin series for 1/(1-x) is a geometric series
- A Taylor polynomial for f(x) is a partial sum of the Taylor series for f(x)

New Jersey Student Learning Standards (NJSLS)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.F-BF.A.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.

MA.F-BF.A.1b	Combine standard function types using arithmetic operations.
MA.F-BF.A.1c	Compose functions.
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
MA.F-BF.B.4a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
MA.F-BF.B.4b	Verify by composition that one function is the inverse of another.
MA.F-BF.B.4c	Read values of an inverse function from a graph or a table, given that the function has an inverse.
MA.F-BF.B.4d	Produce an invertible function from a non-invertible function by restricting the domain.
MA.F-BF.B.5	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
MA.F-LE.A.1a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
MA.F-LE.A.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MA.F-LE.A.1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MA.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MA.F-LE.A.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
MA.F-LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to ab to the ct power = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
MA.F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.

Interdisciplinary Connections

LA.RI.11-12.1	Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.
LA.W.11-12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
9-12.HS-ETS1-4	Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem.
9-12.HS-ETS1-3	Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics, as well as possible social, cultural, and environmental impacts.
9-12.HS-ETS1-1	Analyze a major global challenge to specify qualitative and quantitative criteria and constraints for solutions that account for societal needs and wants.
9-12.HS-ETS1-2	Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Learning Objectives

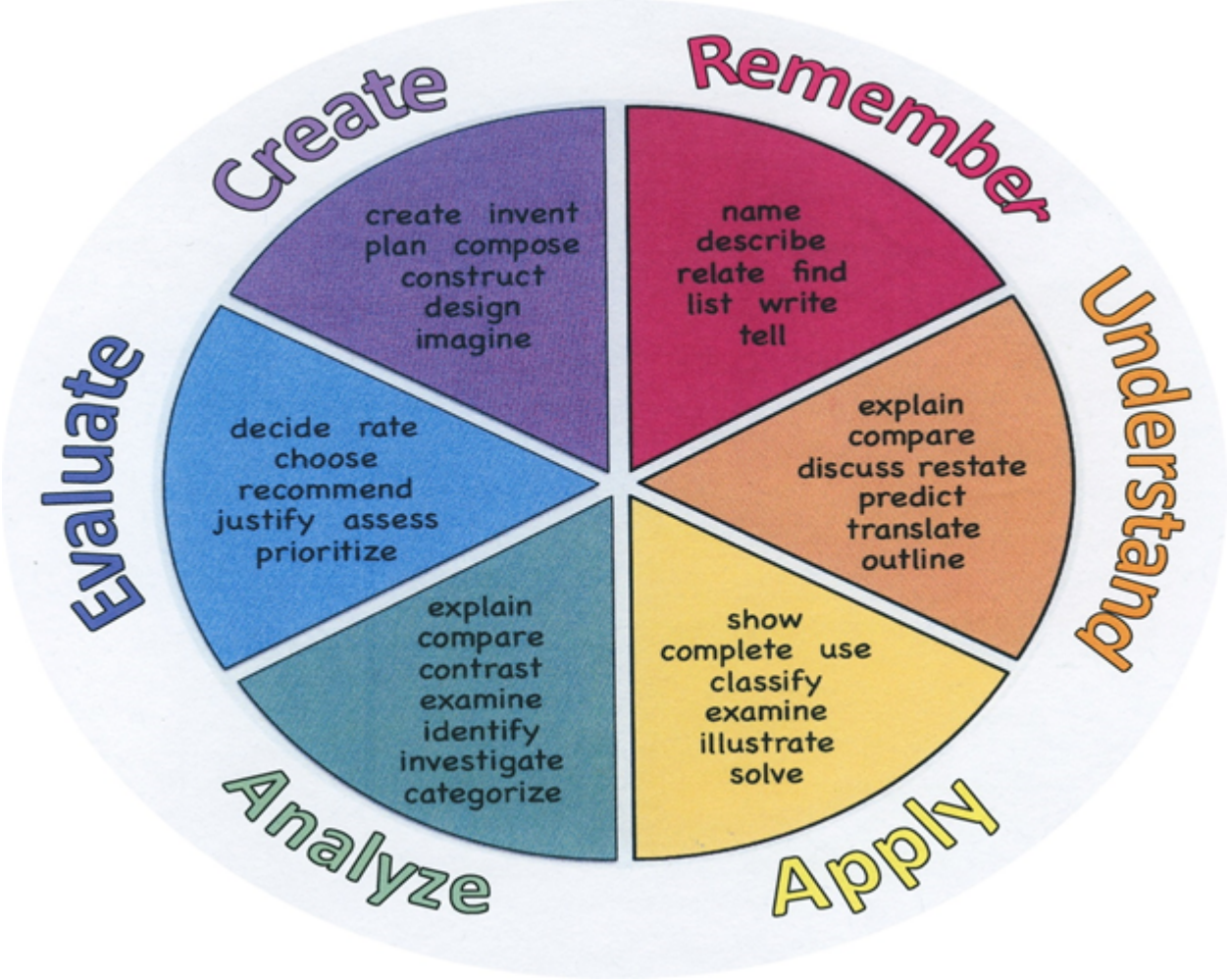
Students will be able to:

1. Determine the convergence/divergence of sequences
2. Determine the convergence/divergence and sum of geometric series
3. Apply the Nth Term Test to determine the divergence of a series
4. Apply the Direct and Limit Comparison Tests to determine the convergence/divergence of a series
5. Apply the Integral Test or p-series to determine the convergence/divergence of a series
6. Apply the Ratio Test to determine the convergence/divergence of a series
7. Apply the Alternating Series Test to determine the convergence of an alternating series
8. Approximate an alternating series and find the approximation error bound using the Alternating Series Remainder
9. Derive Power Series from geometric series and Maclaurin series for sine, cosine and exponential functions by using arithmetic operations, substitution, differentiation or integration
10. Construct Taylor Polynomials for a function at a point
11. Approximate the value of a function using a Taylor Polynomial and find the approximation error bound using either the Alternating Series Remainder or Lagrange Remainder

Action Verbs: Below are examples of action verbs associated with each level of the Revised Bloom's Taxonomy.

Remember	Understand	Apply	Analyze	Evaluate	Create
Choose	Classify	Choose	Categorize	Appraise	Combine
Describe	Defend	Dramatize	Classify	Judge	Compose
Define	Demonstrate	Explain	Compare	Criticize	Construct
Label	Distinguish	Generalize	Differentiate	Defend	Design
List	Explain	Judge	Distinguish	Compare	Develop
Locate	Express	Organize	Identify	Assess	Formulate
Match	Extend	Paint	Infer	Conclude	Hypothesize
Memorize	Give Examples	Prepare	Point out	Contrast	Invent
Name	Illustrate	Produce	Select	Critique	Make
Omit	Indicate	Select	Subdivide	Determine	Originate
Recite	Interrelate	Show	Survey	Grade	Organize
Select	Interpret	Sketch	Arrange	Justify	Plan
State	Infer	Solve	Breakdown	Measure	Produce
Count	Match	Use	Combine	Rank	Role Play
Draw	Paraphrase	Add	Detect	Rate	Drive

Outline Point Quote Recall Recognize Repeat Reproduce	Represent Restate Rewrite Select Show Summarize Tell Translate Associate Compute Convert Discuss Estimate Extrapolate Generalize Predict	Calculate Change Classify Complete Compute Discover Divide Examine Graph Interpolate Manipulate Modify Operate Subtract	Diagram Discriminate Illustrate Outline Point out Separate	Support Test	Devise Generate Integrate Prescribe Propose Reconstruct Revise Rewrite Transform



Suggested Activities & Best Practices

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not

needed or used.

Guidelines for Suggested Activities:

- Includes activities **appropriate & specific** to the development of the Unit;
- Is comprised of the variety of learning activities that will be referenced in lesson plans, constructed/developed and instructionally delivered in the classroom;
- Are authentic;
- Recognizes the learning styles of the students;
- Integrates problem- or project-based learning.

Assessment Evidence - Checking for Understanding (CFU)

- Provide open-ended problems that mirror AP Exam questions to measure comprehension (Formative)
- Peer/Self Evaluation Rubrics to measure progress (Formative)
- At the end of each chapter in the unit, summative assessments will be administered(Summative)
- Benchmark assessemnts will be administered during each quarter (Benchmark)

- Admit Tickets
- Anticipation Guide
- Common Benchmarks
- Compare & Contrast
- Create a Multimedia Poster
- DBQ's
- Define
- Describe
- Evaluate
- Evaluation rubrics
- Exit Tickets
- Explaining

- Fist- to-Five or Thumb-Ometer
- Illustration
- Journals
- KWL Chart
- Learning Center Activities
- Multimedia Reports
- Newspaper Headline
- Outline
- Question Stems
- Quickwrite
- Quizzes
- Red Light, Green Light
- Self- assessments
- Socratic Seminar
- Study Guide
- Surveys
- Teacher Observation Checklist
- Think, Pair, Share
- Think, Write, Pair, Share
- Top 10 List
- Unit review/Test prep
- Unit tests
- Web-Based Assessments
- Written Reports

Primary Resources & Materials

Textbook: Larson, R., & Edwards, B. H. (2014). *AP Calculus 10e* (10th ed.). Independence, KY: Cengage Learning.

Texas Instruments TI-84 graphing calculator

Ancillary Resources

KHAN Academy

https://www.khanacademy.org/math/calculus-home

Massachusetts Institute of Technology (MIT) Open Courseware for High School

https://ocw.mit.edu/high-school/mathematics/

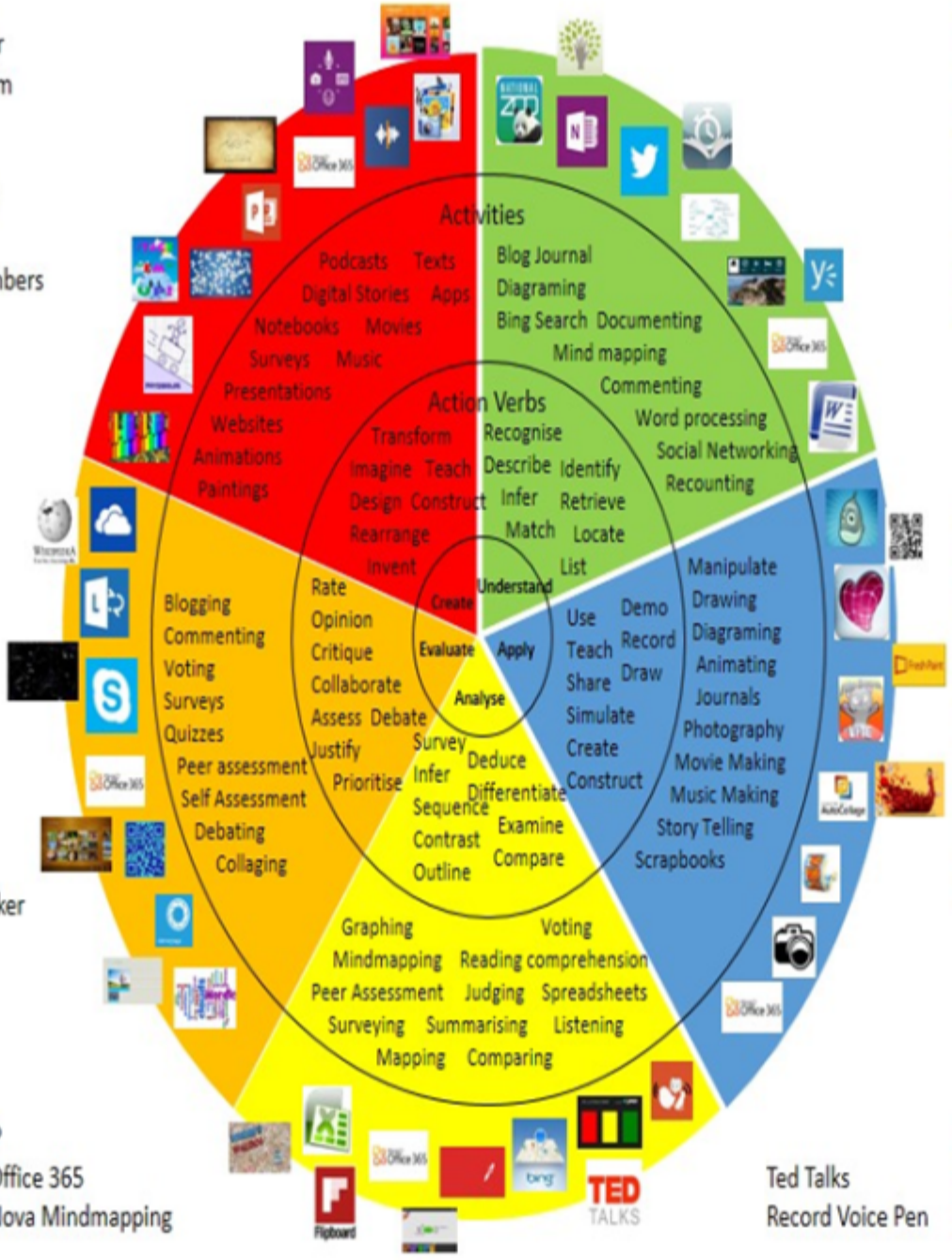
Technology Infusion

1. Textbook online resources at my.hrw.com (assessments, learning tools)
● iPad (for above, and YouTube math videos, as appropriate)

Win 8.1 Apps/Tools Pedagogy Wheel

Podcasts
Photostory 3
Kid Story Builder
Music Maker Jam
Paint A Story
Office 365
MS PowerPoint
Stack 'Em Up
NqSquared Numbers
Physamajig
Xylophone 8

Wikipedia
Skydrive
Lync
SkyMap
Skype
Office 365
Puzzle Touch
Easy QR
Memorylage
Life Moments
Word Cloud Maker



Where's Waldo?
MS Excel Office 365
Flipboard Nova Mindmapping

Ted Talks
Record Voice Pen

Originally taken from http://www.coetail.com/vzimmer/files/2013/02/iPadagogy-Wheel.001.jpg
And adapted for Windows 8.1 devices by Charlotte Beckhurst @CharBeckhurst

Alignment to 21st Century Skills & Technology

Mastery and infusion of **21st Century Skills & Technology** and their Alignment to the core content areas is essential to student learning. The core content areas include:

- English Language Arts;
- Mathematics;
- Science and Scientific Inquiry (Next Generation);
- Social Studies, including American History, World History, Geography, Government and Civics, and Economics;
- World languages;
- Technology;
- Visual and Performing Arts.

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.1	Review career goals and determine steps necessary for attainment.
CAEP.9.2.12.C.3	Identify transferable career skills and design alternate career plans.
CAEP.9.2.12.C.4	Analyze how economic conditions and societal changes influence employment trends and future education.
CAEP.9.2.12.C.5	Research career opportunities in the United States and abroad that require knowledge of world languages and diverse cultures.
TECH.8.1.12.A.3	Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.
TECH.8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
TECH.8.1.12.D.1	Demonstrate appropriate application of copyright, fair use and/or Creative Commons to an original work.
TECH.8.1.12.D.2	Evaluate consequences of unauthorized electronic access (e.g., hacking) and disclosure, and on dissemination of personal information.
TECH.8.1.12.F.1	Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.
TECH.8.2.12.E.1	Demonstrate an understanding of the problem-solving capacity of computers in our world.

21st Century Skills/Interdisciplinary Themes

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Please list only the **21st Century/Interdisciplinary Themes** that will be incorporated into this unit.

- Communication and Collaboration
- Creativity and Innovation
- Critical thinking and Problem Solving
- ICT (Information, Communications and Technology) Literacy
- Information Literacy
- Life and Career Skills
- Media Literacy

21st Century Skills

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Please list only the **21st Century Skills** that will be incorporated into this unit.

- Civic Literacy
- Environmental Literacy
- Financial, Economic, Business and Entrepreneurial Literacy
- Global Awareness
- Health Literacy

Differentiation

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Please remember: Effective educational **Differentiation** in a lesson lies within content, process, and/or product.

Please identify the ones that will be employed in this unit.

Differentiations:

- Small group instruction
- Small group assignments
- Extra time to complete assignments
- Pairing oral instruction with visuals
- Repeat directions
- Use manipulatives
- Center-based instruction
- Token economy
- Study guides
- Teacher reads assessments allowed
- Scheduled breaks
- Rephrase written directions
- Multisensory approaches
- Additional time
- Preview vocabulary
- Preview content & concepts
- Story guides
- Behavior management plan
- Highlight text
- Student(s) work with assigned partner
- Visual presentation
- Assistive technology
- Auditory presentations
- Large print edition
- Dictation to scribe
- Small group setting

Hi-Prep Differentiations:

- Alternative formative and summative assessments
- Choice boards
- Games and tournaments
- Group investigations
- Guided Reading
- Independent research and projects
- Interest groups
- Learning contracts
- Leveled rubrics
- Literature circles
- Multiple intelligence options
- Multiple texts
- Personal agendas
- Project-based learning
- Problem-based learning
- Stations/centers
- Think-Tac-Toes
- Tiered activities/assignments

- Tiered products
- Varying organizers for instructions

Lo-Prep Differentiations

- Choice of books or activities
- Cubing activities
- Exploration by interest
- Flexible grouping
- Goal setting with students
- Jigsaw
- Mini workshops to re-teach or extend skills
- Open-ended activities
- Think-Pair-Share
- Reading buddies
- Varied journal prompts
- Varied supplemental materials

Special Education Learning (IEP's & 504's)

● Provide modifications and accommodations as listed in the student's IEP/504 plan
● Position student near helping peer or have quick access to teacher
● Modify or reduce assignments/texts
● Reduce length of assignment fordifferent mode of delivery
● Increase one-to-one time
● Utilize working contract between you and student at risk
● Prioritize tasks
● Provide manipulatives
● Use graphic organizers
● Use interactive math journals
● Use online resources for skill building
● Provide teacher notes outlining how to set up and solve word problems
● Use collaborative grouping strategies to form a plan to relate to real world situations
● Use online resources to find similar problems that can be applied to problem solving with Calculus
● Break down word problems intosmaller tasks that can be verified by the teacher before moving on
● NJDOE resources

- printed copy of board work/notes provided
- additional time for skill mastery
- assistive technology
- behavior management plan
- Center-Based Instruction
- check work frequently for understanding

- computer or electronic device utilizes
- extended time on tests/ quizzes
- have student repeat directions to check for understanding
- highlighted text visual presentation
- modified assignment format
- modified test content
- modified test format
- modified test length
- multi-sensory presentation
- multiple test sessions
- preferential seating
- preview of content, concepts, and vocabulary
- Provide modifications as dictated in the student's IEP/504 plan
- reduced/shortened reading assignments
- Reduced/shortened written assignments
- secure attention before giving instruction/directions
- shortened assignments
- student working with an assigned partner
- teacher initiated weekly assignment sheet
- Use open book, study guides, test prototypes

English Language Learning (ELL)

- Place student next to same-language speaker, if possible
- Provide text to speech for long word problems and provide diagrams
- Use of translation dictionary or software
- Implement strategy groups
- Confer frequently
- Provide graphic organizers
- Modification plan
- Assign a partner to explain words that may not exist in a different language
- NJDOE resources

- teaching key aspects of a topic. Eliminate nonessential information
- using videos, illustrations, pictures, and drawings to explain or clarif
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning;
- allowing students to correct errors (looking for understanding)
- allowing the use of note cards or open-book during testing
- decreasing the amount of workpresented or required

- having peers take notes or providing a copy of the teacher's notes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using computer word processing spell check and grammar check features
- using true/false, matching, or fill in the blank tests in lieu of essay tests

At Risk

- NJDOE resources
- Create weekly check-ins outside class
- Utilize online resources such as http://www.tenmarks.com or www.khanacademy.org
- Have students keep a journal of their progress and which questions need to be addressed in and out of class

- allowing students to correct errors (looking for understanding)
- teaching key aspects of a topic. Eliminate nonessential information
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning
- allowing students to select from given choices
- allowing the use of note cards or open-book during testing
- collaborating (general education teacher and specialist) to modify vocabulary, omit or modify items to reflect objectives for the student, eliminate sections of the test, and determine how the grade will be determined prior to giving the test.
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- marking students' correct and acceptable work, not the mistakes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using authentic assessments with real-life problem-solving
- using true/false, matching, or fill in the blank tests in lieu of essay tests
- using videos, illustrations, pictures, and drawings to explain or clarify

Talented and Gifted Learning (T&G)

- Process should be modified: higher-order-thinking skills, open-ended thinking, discovery
- Utilize project-based learning for greater depth of knowledge
- Utilize exploratory connections to higher grade concepts

● Contents should be modified: abstraction, complexity, variety, organization
● Products should be modified:real-world problems, audiences, deadlines, evaluation, transformations
● Learning environments should be modified: student-centered learning, independence, openness, complexity, groups varied
● Use of web-based resources such as http://www.tenmarks.com, www.khanacademy.org, geogebra.org desmos.com
● NJDOE resources
● Students can be assigned as tutors to help other students with difficult topic areas

- Above grade level placement option for qualified students
- Advanced problem-solving
- Allow students to work at a faster pace
- Cluster grouping
- Complete activities aligned with above grade level text using Benchmark results
- Create a blog or social media page about their unit
- Create a plan to solve an issue presented in the class or in a text
- Debate issues with research to support arguments
- Flexible skill grouping within a class or across grade level for rigor
- Higher order, critical & creative thinking skills, and discovery
- Multi-disciplinary unit and/or project
- Teacher-selected instructional strategies that are focused to provide challenge, engagement, and growth opportunities
- Utilize exploratory connections to higher-grade concepts
- Utilize project-based learning for greater depth of knowledge

Sample Lesson