Unit 4: The Integral

Content Area: **Math**
Course(s): **Calculus AP**
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Status: **Published**

Title Section

Department of Curriculum and Instruction

Belleville Public Schools

Curriculum Guide

Calculus AP, Unit 4

The Integral

Belleville Board of Education

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Unit Overview

- Recognize antiderivatives of basic functions
- Interpret the definite integral as the limit of a Riemann sum
- Express the limit of a Riemann sum in integral notation
- Approximate a definite integral
- Calculate a definite integral using areas and properties of definite integrals
- Evaluate an improper integral or show that an improper integral diverges (BC only)
- Analyze functions defined by an integral
- Calculate antiderivatives
- Evaluate definite integrals

Students will be able to independently use their learning to understand, explain, and solve problems involving real-world phenomena where rate processes are known.

Enduring Understanding

Students will understand that:

- Antidifferentiation is the inverse process of differentiation
- The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies
- The Fundamental Theorem of Calculus (FTOC), which has two distinct formulations, connects differentiations and integration

Essential Questions

Essential Questions are:

- How can we count the uncountable?
- Why is counting important?
- How does the integral extend geometry?

Exit Skills

By the end of UNIT 4 students will know:

- An antiderivative of a function f is a function g whose derivative f
- Differentiation rules provide the foundation for finding antiderivatives
- A Riemann sum, which requires a partition of an interval I, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition
- The definite integral of a continuous function f over a closed interval, and denoted by definite integral

notation is the limit of Riemann sums as the widths of the subintervals approach 0.

- The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral
- Definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, and verbally
- Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions
- In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area
- Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals
- An improper integral is an integral that has one or both limits infinite or has an integrand that is unbounded in the interval of integration (BC only)
- Improper integrals can be determined using limits of definite integrals (BC only)
- The definite integral can be used to define new functions such as accumulation functions
- If f is a continuous function on a closed interval, then the derivative of an accumulation function of f with respect to the upper limit of integrals equals f itself
- Graphical, numerical, analytical, and verbal representations of a function f provide information about an accumulation function of f
- The function defined by an accumulation function of f is an antiderivative of f
- If f is continuous on a closed interval and F is the antiderivative of f, then the Fundamental Theorem of Calculus can be used to evaluate a definite integral analytically
- Indefinite integral notation can be used to describe indefinite integrals
- Many functions do not have closed form antiderivatives
- Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables, integration by parts (BC only), and nonrepeating linear partial fractions (BC only)

New Jersey Student Learning Standards (NJSLS)

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Please list only the content-level and cross-curricular **New Jersey Student Learning Standards** applicable to the unit. **Do not list**

standards that are not used in the unit.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.K-12.5	Use appropriate tools strategically.
MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.K-12.6	Attend to precision.
MA.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
MA.K-12.7	Look for and make use of structure.
MA.F-IF.C	Analyze functions using different representations
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
MA.F-LE.B	Interpret expressions for functions in terms of the situation they model
MA.G-MG.A	Apply geometric concepts in modeling situations

Interdisciplinary Connections

section above.

LA.RI.11-12.1	Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.
LA.W.11-12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
9-12.HS-ETS1-4	Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem.
9-12.HS-ETS1-3	Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics, as well as possible social, cultural, and environmental impacts.
9-12.HS-ETS1-1	Analyze a major global challenge to specify qualitative and quantitative criteria and constraints for solutions that account for societal needs and wants.
9-12.HS-ETS1-2	Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

Learning Objectives

Students Will Be Able To:

1: Apply basic integration rules to evaluate indefinite integrals

2: Find the constant of integration given an initial condition

3: Find the net area between a curves and the x-axis using a definite integrals, graphically, numerically, or analytically

4: Apply properties of definite integrals to compute them graphically or analytically

5: Approximate the net area between a curve and the x-axis using a left, right, or midpoint Riemann sum, or a Trapezoid Sum

6: Construct an accumulation function to represent the net area between a curve and the x-axis

7: Use the Fundamental Theorem of Calculus to compute the net area between a curve and the x-axis

8: Use the 2nd Fundamental Theorem of Calculus to differentiate accumulation functions

9: Use accumulation functions to write the solutions to initial value problems

10: Evaluate definite integrals of piecewise functions analytically

11: Evaluate improper integrals (BC only)

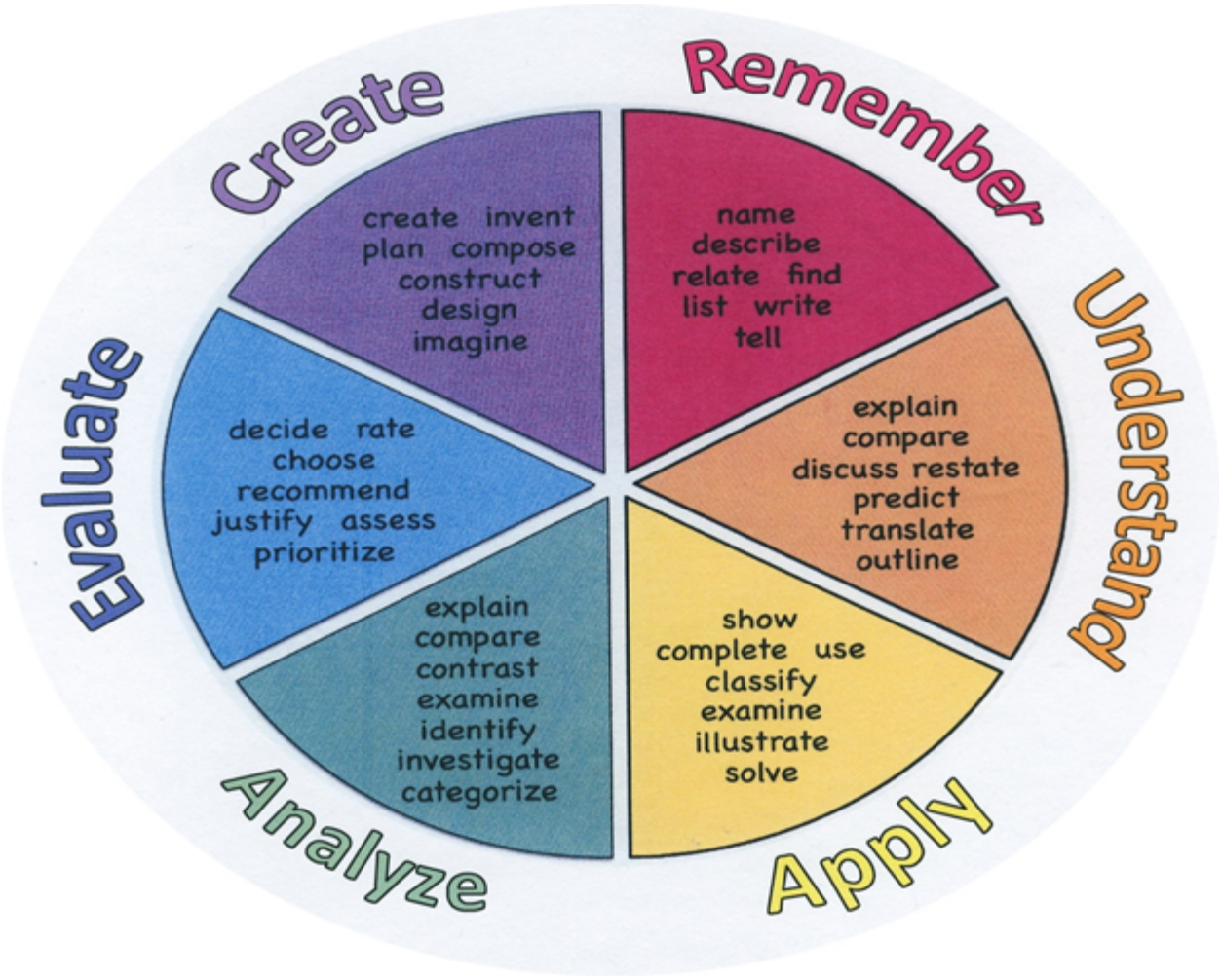
12: Evaluate indefinite and definite integrals using u-substitution

13: Evaluate indefinite and definite integrals using integration by parts (BC only)

14: Evaluate indefinite and definite integrals using partial fractions expansion (BC only)

Action Verbs: Below are examples of action verbs associated with each level of the Revised Bloom's Taxonomy.

Remember	Understand	Apply	Analyze	Evaluate	Create
Choose	Classify	Choose	Categorize	Appraise	Combine
Describe	Defend	Dramatize	Classify	Judge	Compose
Define	Demonstrate	Explain	Compare	Criticize	Construct
Label	Distinguish	Generalize	Differentiate	Defend	Design
List	Explain	Judge	Distinguish	Compare	Develop
Locate	Express	Organize	Identify	Assess	Formulate
Match	Extend	Paint	Infer	Conclude	Hypothesize
Memorize	Give Examples	Prepare	Point out	Contrast	Invent
Name	Illustrate	Produce	Select	Critique	Make
Omit	Indicate	Select	Subdivide	Determine	Originate
Recite	Interrelate	Show	Survey	Grade	Organize
Select	Interpret	Sketch	Arrange	Justify	Plan
State	Infer	Solve	Breakdown	Measure	Produce
Count	Match	Use	Combine	Rank	Role Play
Draw	Paraphrase	Add	Detect	Rate	Drive
Outline	Represent	Calculate	Diagram	Support	Devise
Point	Restate	Change	Discriminate	Test	Generate
Quote	Rewrite	Classify	Illustrate		Integrate
Recall	Select	Complete	Outline		Prescribe
Recognize	Show	Compute	Point out		Propose
Repeat	Summarize	Discover	Separate		Reconstruct
Reproduce	Tell	Divide			Revise
	Translate	Examine			Rewrite
	Associate	Graph			Transform
	Compute	Interpolate			
	Convert	Manipulate			
	Discuss	Modify			
	Estimate	Operate			
	Extrapolate	Subtract			
	Generalize				
	Predict				



Suggested Activities & Best Practices

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Guidelines for Suggested Activities:

- Includes activities **appropriate & specific** to the development of the Unit;
- Is comprised of the variety of learning activities that will be referenced in lesson plans, constructed/developed and instructionally delivered in the classroom;
- Are authentic;
- Recognizes the learning styles of the students;
- Integrates problem- or project-based learning.

Assessment Evidence - Checking for Understanding (CFU)

- Provide open-ended problems that mirror AP Exam questions to measure comprehension (Formative)
- Peer/Self Evaluation Rubrics to measure progress (Formative)
- At the end of each chapter in the unit, summative assessments will be administered (Summative)
- Benchmark assessemnts will be administered during each quarter (Benchmark)

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- Admit Tickets
- Anticipation Guide
- Common Benchmarks
- Compare & Contrast
- Create a Multimedia Poster
- DBQ's
- Define
- Describe
- Evaluate
- Evaluation rubrics
- Exit Tickets
- Explaining
- Fist- to-Five or Thumb-Ometer
- Illustration
- Journals
- KWL Chart
- Learning Center Activities
- Multimedia Reports
- Newspaper Headline
- Outline

- Question Stems
- Quickwrite
- Quizzes
- Red Light, Green Light
- Self- assessments
- Socratic Seminar
- Study Guide
- Surveys
- Teacher Observation Checklist
- Think, Pair, Share
- Think, Write, Pair, Share
- Top 10 List
- Unit review/Test prep
- Unit tests
- Web-Based Assessments
- Written Reports

Primary Resources & Materials

Textbook: Larson, R., & Edwards, B. H. (2014). *AP Calculus 10e* (10th ed.). Independence, KY: Cengage Learning.

Texas Instruments TI-84 graphing calculator

Ancillary Resources

KHAN Academy

https://www.khanacademy.org/math/calculus-home

Massachusetts Institute of Technology (MIT) Open Courseware for High School

https://ocw.mit.edu/high-school/mathematics/

Technology Infusion

1. Use khanacademy.org to find instructional videos on processes of integration and finding area between curves
- iPad (for above, and YouTube math videos, as appropriate)
- Use desmos.com to find the intersection of two graphs

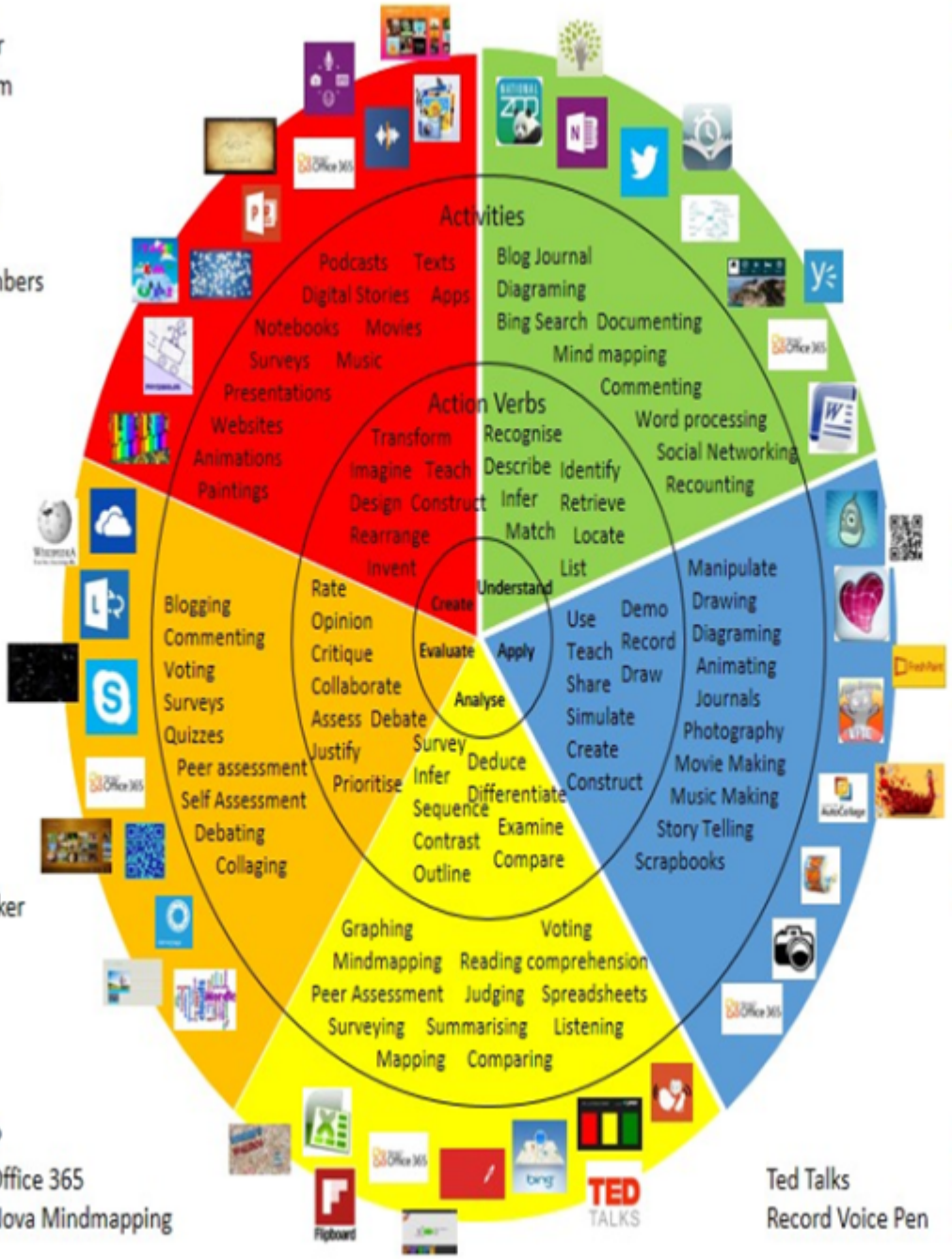
Win 8.1 Apps/Tools Pedagogy Wheel

Podcasts
Photostory 3
Kid Story Builder
Music Maker Jam
Paint A Story
Office 365
MS PowerPoint
Stack 'Em Up
NqSquared Numbers
Physamajig
Xylophone 8

Wikipedia
Skydrive
Lync
SkyMap
Skype
Office 365
Puzzle Touch
Easy QR
Memorylage
Life Moments
Word Cloud Maker

Where's Waldo?
MS Excel
Flipboard
Office 365
Nova Mindmapping

Ted Talks
Record Voice Pen



Originally taken from http://www.coetail.com/vzimmer/files/2013/02/iPadagogy-Wheel.001.jpg
And adapted for Windows 8.1 devices by Charlotte Beckhurst @CharBeckhurst

Alignment to 21st Century Skills & Technology

Mastery and infusion of **21st Century Skills & Technology** and their Alignment to the core content areas is essential to student learning. The core content areas include:

- English Language Arts;
- Mathematics;
- Science and Scientific Inquiry (Next Generation);
- Social Studies, including American History, World History, Geography, Government and Civics, and Economics;
- World languages;
- Technology;
- Visual and Performing Arts.

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.1	Review career goals and determine steps necessary for attainment.
CAEP.9.2.12.C.3	Identify transferable career skills and design alternate career plans.
CAEP.9.2.12.C.4	Analyze how economic conditions and societal changes influence employment trends and future education.
CAEP.9.2.12.C.5	Research career opportunities in the United States and abroad that require knowledge of world languages and diverse cultures.
TECH.8.1.12.A.3	Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.
TECH.8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
TECH.8.1.12.D.1	Demonstrate appropriate application of copyright, fair use and/or Creative Commons to an original work.
TECH.8.1.12.D.2	Evaluate consequences of unauthorized electronic access (e.g., hacking) and disclosure, and on dissemination of personal information.
TECH.8.1.12.F.1	Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.

TECH.8.2.12.E.1 Demonstrate an understanding of the problem-solving capacity of computers in our world.

21st Century Skills/Interdisciplinary Themes

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Please list only the **21st Century/Interdisciplinary Themes** that will be incorporated into this unit.

- Communication and Collaboration
- Creativity and Innovation
- Critical thinking and Problem Solving
- ICT (Information, Communications and Technology) Literacy
- Information Literacy
- Life and Career Skills
- Media Literacy

21st Century Skills

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Please list only the **21st Century Skills** that will be incorporated into this unit.

- Civic Literacy
- Environmental Literacy
- Financial, Economic, Business and Entrepreneurial Literacy
- Global Awareness
- Health Literacy

Differentiation

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Please remember: Effective educational **Differentiation** in a lesson lies within content, process, and/or product.

Please identify the ones that will be employed in this unit.

Differentiations:

- Small group instruction
- Small group assignments
- Extra time to complete assignments
- Pairing oral instruction with visuals
- Repeat directions
- Use manipulatives
- Center-based instruction
- Token economy
- Study guides
- Teacher reads assessments allowed
- Scheduled breaks
- Rephrase written directions
- Multisensory approaches
- Additional time
- Preview vocabulary
- Preview content & concepts
- Story guides
- Behavior management plan
- Highlight text
- Student(s) work with assigned partner
- Visual presentation
- Assistive technology
- Auditory presentations
- Large print edition
- Dictation to scribe
- Small group setting

Hi-Prep Differentiations:

- Alternative formative and summative assessments
- Choice boards
- Games and tournaments
- Group investigations
- Guided Reading
- Independent research and projects
- Interest groups
- Learning contracts
- Leveled rubrics
- Literature circles
- Multiple intelligence options
- Multiple texts
- Personal agendas
- Project-based learning
- Problem-based learning
- Stations/centers
- Think-Tac-Toes

- Tiered activities/assignments
- Tiered products
- Varying organizers for instructions

Lo-Prep Differentiations

- Choice of books or activities
- Cubing activities
- Exploration by interest
- Flexible grouping
- Goal setting with students
- Jigsaw
- Mini workshops to re-teach or extend skills
- Open-ended activities
- Think-Pair-Share
- Reading buddies
- Varied journal prompts
- Varied supplemental materials

Special Education Learning (IEP's & 504's)

● Provide modifications and accommodations as listed in the student's IEP/504 plan
● Position student near helping peer or have quick access to teacher
● Modify or reduce assignments/texts
● Reduce length of assignment for different mode of delivery
● Break down tasks of integration to separate questions to allow students to complete one task before moving onto another
● Utilize working contract between you and student at risk
● Prioritize tasks
● Provide manipulatives
● Use graphic organizers to show various methods of integration that still arrive at the same core answer
● Use interactive math journals
● Use online resources for skill building
● Provide teacher notes on google classroom that students can print out and use in class and at home
● Use collaborative grouping strategies such as small groups
● Use online resources
● NJDOE resources

- printed copy of board work/notes provided
- additional time for skill mastery
- assistive technology
- behavior management plan

- Center-Based Instruction
- check work frequently for understanding
- computer or electronic device utilizes
- extended time on tests/ quizzes
- have student repeat directions to check for understanding
- highlighted text visual presentation
- modified assignment format
- modified test content
- modified test format
- modified test length
- multi-sensory presentation
- multiple test sessions
- preferential seating
- preview of content, concepts, and vocabulary
- Provide modifications as dictated in the student's IEP/504 plan
- reduced/shortened reading assignments
- Reduced/shortened written assignments
- secure attention before giving instruction/directions
- shortened assignments
- student working with an assigned partner
- teacher initiated weekly assignment sheet
- Use open book, study guides, test prototypes

English Language Learning (ELL)

- Place student next to same-language speaker, if possible
- Vocabulary aides; integrals
- Use of translation dictionary or software to help explain finding area between curves
- Implement strategy groups
- Confer frequently
- Provide graphic organizers
- Modification plan
- NJDOE resources

- teaching key aspects of a topic. Eliminate nonessential information
- using videos, illustrations, pictures, and drawings to explain or clarif
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards,

charts, graphs, slide shows, videos, etc.) to demonstrate student's learning;

- allowing students to correct errors (looking for understanding)
- allowing the use of note cards or open-book during testing
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using computer word processing spell check and grammar check features
- using true/false, matching, or fill in the blank tests in lieu of essay tests

At Risk

● NJDOE resources
● Create weekly check-ins outside class
● Utilize online resources such as http://www.tenmarks.com or www.khanacademy.org
● Keep in contact with parents of students and track their progress, noting each communication log. Involve guidance and other teachers as necessary

- allowing students to correct errors (looking for understanding)
- teaching key aspects of a topic. Eliminate nonessential information
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning
- allowing students to select from given choices
- allowing the use of note cards or open-book during testing
- collaborating (general education teacher and specialist) to modify vocabulary, omit or modify items to reflect objectives for the student, eliminate sections of the test, and determine how the grade will be determined prior to giving the test.
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- marking students' correct and acceptable work, not the mistakes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using authentic assessments with real-life problem-solving
- using true/false, matching, or fill in the blank tests in lieu of essay tests
- using videos, illustrations, pictures, and drawings to explain or clarify

Talented and Gifted Learning (T&G)

● Process should be modified: higher-order-thinking skills, open-ended thinking, discovery
● Utilize project-based learning for greater depth of knowledge
● Utilize exploratory connections to higher grade concepts
● Contents should be modified: abstraction, complexity, variety, organization
● Learning environments should be modified: student-centered learning, independence, openness, complexity, groups varied
● Use of web based resources such as http://www.tenmarks.com, www.khanacademy.org geogebra.org, http://www.wolframalpha.com/calculators/integral-calculator/
● NJDOE resources
● Assign projects that demonstrate real-world uses for integration to find area

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- Above grade level placement option for qualified students
- Advanced problem-solving
- Allow students to work at a faster pace
- Cluster grouping
- Complete activities aligned with above grade level text using Benchmark results
- Create a blog or social media page about their unit
- Create a plan to solve an issue presented in the class or in a text
- Debate issues with research to support arguments
- Flexible skill grouping within a class or across grade level for rigor
- Higher order, critical & creative thinking skills, and discovery
- Multi-disciplinary unit and/or project
- Teacher-selected instructional strategies that are focused to provide challenge, engagement, and growth opportunities
- Utilize exploratory connections to higher-grade concepts
- Utilize project-based learning for greater depth of knowledge

Sample Lesson