Unit 3: Applications of the Derivative

Content Area: **Math**
Course(s): **Calculus AP**
Time Period:
Length: **30 Days**
Status: **Published**

Title Section

Department of Curriculum and Instruction

Belleville Public Schools

Curriculum Guide

Calculus AP, Unit 3

Applications of the Derivative

Belleville Board of Education

102 Passaic Avenue

Belleville, NJ 07109

Prepared by: INSERT YOUR TITLE, FIRST NAME, LAST NAME HERE

Dr. Richard Tomko, Ph.D., M.J., Superintendent of Schools

Ms. LucyAnn Demikoff, Director of Curriculum and Instruction K-12

Ms. Nicole Shanklin, Director of Elementary Education K-8, ESL Coordinator K-12

Mr. George Droste, Director of Secondary Education

Board Approved:September 23, 2019

Unit Overview

- Use derivatives to analyze properties of a function.
- Interpret the meaning of a derivative within a problem
- Solve problems involving the slope of a tangent line
- Solve problems involving related rates, optimization, rectilinear motion and planar motion
- Solve problems involving rates of change in applied contexts
- Apply the Mean Value Theorem (MVT) to describe the behavior of a function over an interval

Students will be able to independently use their learning to solve, understand and explain new classes of problems involving real-world phenomena, including optimization, function approximation, and establishing relationships between various rate processes within a system.

Enduring Understanding

Students will understand:

- A function's derivative which is itself a function, can be used to understand the behavior of the function
- The derivative has multiple interpretations and applications including those that involve instantaneous rates of change
- The Mean Value Theorem (MVT) connects the behavior of a differentiable function over an interval to the behavior of the derivative of that function at a particular point in the interval

Essential Questions

Essential Questions are:

- How can the derivative be used to describe the properties of real-world phenomena?
- How can the derivative be used to make our lives better?
- How can the derivative be used to save time and money in solving problems?
- How can the derivative be used to make geometry more valuable as a problem-solving tool?
- How much should we be willing to spend on the precision of knowledge?

Exit Skills

By the end of UNIT 3 students will know:

- First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection
- Key features of functions and their derivatives can be identified and related to their graphical,

numerical, and analytical representations

- Key features of the graphs of f, f’ and f’’ are related to one another
- The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration
- The unit for f’(x) is the unit of f divided by the unit for x
- The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable
- The derivative at a point is the slope of the line tangent to the graph at that point on the graph
- The tangent line is the graph of a locally linear approximation of the function near the point of tangency
- The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to the other quantities whose rates of change are known
- The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval
- The derivative can be used to express information about rates of change in applied contexts

New Jersey Student Learning Standards (NJSLS)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.K-12.5	Use appropriate tools strategically.

MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.F-IF.C	Analyze functions using different representations
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
MA.F-LE.B	Interpret expressions for functions in terms of the situation they model
MA.G-GMD.A	Explain volume formulas and use them to solve problems
MA.G-GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
MA.G-MG.A	Apply geometric concepts in modeling situations
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MA.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
MA.G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Interdisciplinary Connections

LA.RI.11-12.1	Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.
LA.W.11-12.1.B	Develop claim(s) and counterclaims avoiding common logical fallacies and using sound reasoning and thoroughly, supplying the most relevant evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level, concerns, values, and possible biases.
9-12.HS-ETS1-4	Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem.
9-12.HS-ETS1-2	Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
9-12.HS-ETS1-1	Analyze a major global challenge to specify qualitative and quantitative criteria and constraints for solutions that account for societal needs and wants.
9-12.HS-ETS1-3	Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and

aesthetics, as well as possible social, cultural, and environmental impacts.

Learning Objectives

Students will be able to:

1: Determine the intervals on which a function is increasing or decreasing

2: Find relative extrema using the 1st Derivative Test

3: Find critical number of a function

4: Find absolute extrema of a function on a closed interval

5: Determine intervals on which a function is concave up or concave down, graphically and analytically

6: Find inflection points of a function graphically and analytically

7: Find relative extrema using the 2nd Derivative Test

8: Approximate a function value using a tangent line approximation

9: Analyze the motion of an object along an axis

10: Solve related rates problems

11: Use the derivative to understand rates of change in an applied context

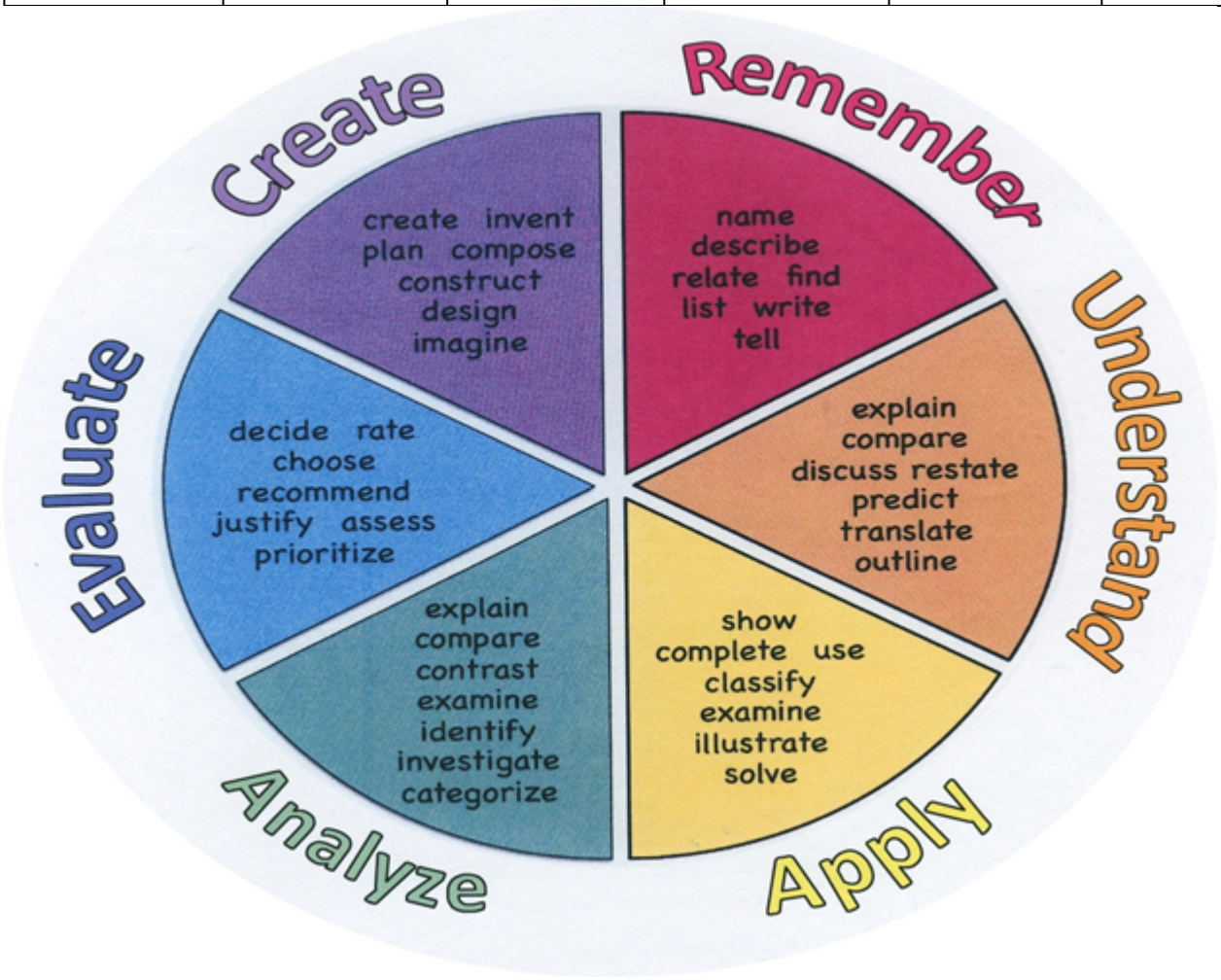
12: Use the Mean Value Theorem to relate average rate of change to instantaneous rate of change

13: Use L'Hospital's Rule to evaluate limits analytically

Action Verbs: Below are examples of action verbs associated with each level of the Revised Bloom's Taxonomy.

Remember	Understand	Apply	Analyze	Evaluate	Create
Choose	Classify	Choose	Categorize	Appraise	Combine
Describe	Defend	Dramatize	Classify	Judge	Compose
Define	Demonstrate	Explain	Compare	Criticize	Construct
Label	Distinguish	Generalize	Differentiate	Defend	Design
List	Explain	Judge	Distinguish	Compare	Develop
Locate	Express	Organize	Identify	Assess	Formulate
Match	Extend	Paint	Infer	Conclude	Hypothesize
Memorize	Give Examples	Prepare	Point out	Contrast	Invent
Name	Illustrate	Produce	Select	Critique	Make
Omit	Indicate	Select	Subdivide	Determine	Originate
Recite	Interrelate	Show	Survey	Grade	Organize
Select	Interpret	Sketch	Arrange	Justify	Plan
State	Infer	Solve	Breakdown	Measure	Produce
Count	Match	Use	Combine	Rank	Role Play
Draw	Paraphrase	Add	Detect	Rate	Drive
Outline	Represent	Calculate	Diagram	Support	Devise

Point Quote Recall Recognize Repeat Reproduce	Restate Rewrite Select Show Summarize Tell Translate Associate Compute Convert Discuss Estimate Extrapolate Generalize Predict	Change Classify Complete Compute Discover Divide Examine Graph Interpolate Manipulate Modify Operate Subtract	Discriminate Illustrate Outline Point out Separate	Test	Generate Integrate Prescribe Propose Reconstruct Revise Rewrite Transform



Suggested Activities & Best Practices

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not

needed or used.

Guidelines for Suggested Activities:

- Includes activities **appropriate & specific** to the development of the Unit;
- Is comprised of the variety of learning activities that will be referenced in lesson plans, constructed/developed and instructionally delivered in the classroom;
- Are authentic;
- Recognizes the learning styles of the students;
- Integrates problem- or project-based learning.

Assessment Evidence - Checking for Understanding (CFU)

- Provide open-ended problems that mirror AP Exam questions to measure comprehension (Formative)
- Peer/Self Evaluation Rubrics to measure progress (Formative)
- At the end of each chapter in the unit, summative assessments will be administered (Summative)
- Benchmark assessemnts will be administered during each quarter (Benchmark)

- Admit Tickets
- Anticipation Guide
- Common Benchmarks
- Compare & Contrast
- Create a Multimedia Poster
- DBQ's
- Define
- Describe
- Evaluate
- Evaluation rubrics
- Exit Tickets
- Explaining

- Fist- to-Five or Thumb-Ometer
- Illustration
- Journals
- KWL Chart
- Learning Center Activities
- Multimedia Reports
- Newspaper Headline
- Outline
- Question Stems
- Quickwrite
- Quizzes
- Red Light, Green Light
- Self- assessments
- Socratic Seminar
- Study Guide
- Surveys
- Teacher Observation Checklist
- Think, Pair, Share
- Think, Write, Pair, Share
- Top 10 List
- Unit review/Test prep
- Unit tests
- Web-Based Assessments
- Written Reports

Primary Resources & Materials

Textbook: Larson, R., & Edwards, B. H. (2014). *AP Calculus 10e* (10th ed.). Independence, KY: Cengage Learning.

Texas Instruments TI-84 graphing calculator

Ancillary Resources

KHAN Academy

https://www.khanacademy.org/math/calculus-home

Massachusetts Institute of Technology (MIT) Open Courseware for High School

https://ocw.mit.edu/high-school/mathematics/

Technology Infusion

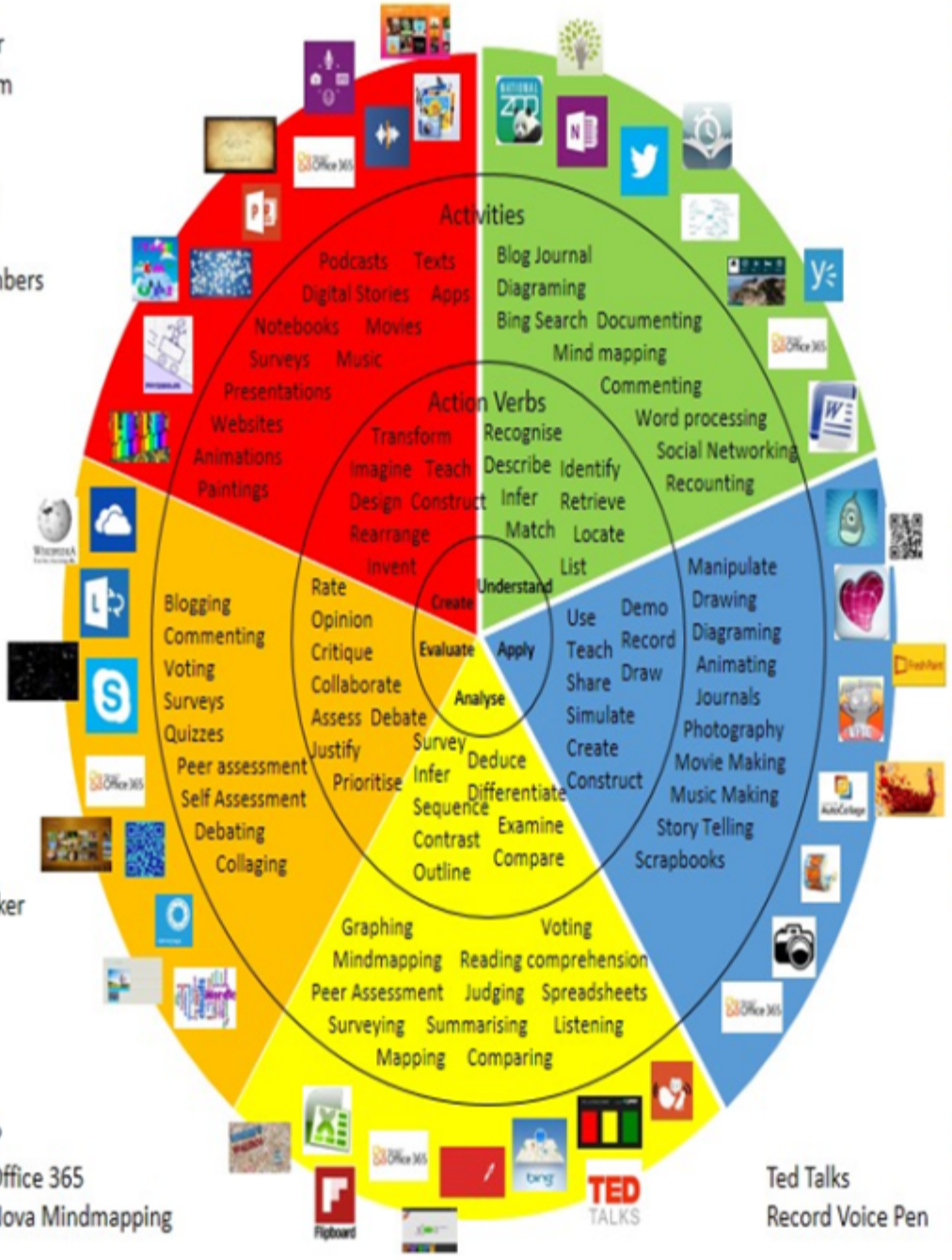
Upon completion of this sections, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

What **Technology Infusion** and/or strategies are integrated into this unit to enhance learning? Please list all hardware, software and strategies. Please find a technology pedagogy wheel for assistance while completing this section.

Win 8.1 Apps/Tools Pedagogy Wheel

Podcasts
Photostory 3
Kid Story Builder
Music Maker Jam
Paint A Story
Office 365
MS PowerPoint
Stack 'Em Up
NqSquared Numbers
Physamajig
Xylophone 8

Wikipedia
Skydrive
Lync
SkyMap
Skype
Office 365
Puzzle Touch
Easy QR
Memorylage
Life Moments
Word Cloud Maker



Where's Waldo?
MS Excel Office 365
Flipboard Nova Mindmapping

Ted Talks
Record Voice Pen

Originally taken from http://www.coetail.com/vzimmer/files/2013/02/iPadagogy-Wheel.001.jpg
And adapted for Windows 8.1 devices by Charlotte Beckhurst @CharBeckhurst

Alignment to 21st Century Skills & Technology

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Mastery and infusion of **21st Century Skills & Technology** and their Alignment to the core content areas is essential to student learning. The core content areas include:

- English Language Arts;
- Mathematics;
- Science and Scientific Inquiry (Next Generation);
- Social Studies, including American History, World History, Geography, Government and Civics, and Economics;
- World languages;
- Technology;
- Visual and Performing Arts.

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.1	Review career goals and determine steps necessary for attainment.
CAEP.9.2.12.C.3	Identify transferable career skills and design alternate career plans.
CAEP.9.2.12.C.4	Analyze how economic conditions and societal changes influence employment trends and future education.
TECH.8.1.12.A.4	Construct a spreadsheet workbook with multiple worksheets, rename tabs to reflect the data on the worksheet, and use mathematical or logical functions, charts and data from all worksheets to convey the results.
TECH.8.1.12.A.5	Create a report from a relational database consisting of at least two tables and describe the process, and explain the report results.
TECH.8.2.12.E.CS1	Computational thinking and computer programming as tools used in design and engineering.

21st Century Skills/Interdisciplinary Themes

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Please list only the **21st Century/Interdisciplinary Themes** that will be incorporated into this unit.

- Communication and Collaboration
- Creativity and Innovation
- Critical thinking and Problem Solving
- ICT (Information, Communications and Technology) Literacy
- Information Literacy
- Life and Career Skills
- Media Literacy

21st Century Skills

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Please list only the **21st Century Skills** that will be incorporated into this unit.

- Civic Literacy
- Environmental Literacy
- Financial, Economic, Business and Entrepreneurial Literacy
- Global Awareness
- Health Literacy

Differentiation

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Please remember: Effective educational **Differentiation** in a lesson lies within content, process, and/or product.

Please identify the ones that will be employed in this unit.

Differentiations:

- Small group instruction
- Small group assignments

- Extra time to complete assignments
- Pairing oral instruction with visuals
- Repeat directions
- Use manipulatives
- Center-based instruction
- Token economy
- Study guides
- Teacher reads assessments allowed
- Scheduled breaks
- Rephrase written directions
- Multisensory approaches
- Additional time
- Preview vocabulary
- Preview content & concepts
- Story guides
- Behavior management plan
- Highlight text
- Student(s) work with assigned partner
- Visual presentation
- Assistive technology
- Auditory presentations
- Large print edition
- Dictation to scribe
- Small group setting

Hi-Prep Differentiations:

- Alternative formative and summative assessments
- Choice boards
- Games and tournaments
- Group investigations
- Guided Reading
- Independent research and projects
- Interest groups
- Learning contracts
- Leveled rubrics
- Literature circles
- Multiple intelligence options
- Multiple texts
- Personal agendas
- Project-based learning
- Problem-based learning
- Stations/centers
- Think-Tac-Toes
- Tiered activities/assignments
- Tiered products
- Varying organizers for instructions

Lo-Prep Differentiations

- Choice of books or activities
- Cubing activities
- Exploration by interest

- Flexible grouping
- Goal setting with students
- Jigsaw
- Mini workshops to re-teach or extend skills
- Open-ended activities
- Think-Pair-Share
- Reading buddies
- Varied journal prompts
- Varied supplemental materials

Special Education Learning (IEP's & 504's)

Please identify the **Special Education Learning** adaptations that will be employed in the unit, using the ones identified below.

● Allow students to use notes
● Provide modifications and accommodations as listed in the student's IEP/504 plan
● Position student near helping peer or have quick access to teacher
● Modify or reduce assignments/texts by creating partial graphs when graphing limits
● Reduce length of assignment for different mode of delivery
● Increase one-to-one time
● Utilize working contract between you and student at risk
● Prioritize tasks
● Provide manipulatives, graphic organizers and math journals to keep track of formulas and techniques of differentiation (i.e. product rule, quotient rule, behavior of limits)
● Use online resources for skill building and allow students to see notes of processes of differentiation and graphing)
● Use collaborative grouping strategies such as small groups to reinforce the process of differentiation
● NJDOE resources

- printed copy of board work/notes provided
- additional time for skill mastery
- assistive technology
- behavior management plan
- Center-Based Instruction
- check work frequently for understanding
- computer or electronic device utilizes
- extended time on tests/ quizzes
- have student repeat directions to check for understanding
- highlighted text visual presentation

- modified assignment format
- modified test content
- modified test format
- modified test length
- multi-sensory presentation
- multiple test sessions
- preferential seating
- preview of content, concepts, and vocabulary
- Provide modifications as dictated in the student's IEP/504 plan
- reduced/shortened reading assignments
- Reduced/shortened written assignments
- secure attention before giving instruction/directions
- shortened assignments
- student working with an assigned partner
- teacher initiated weekly assignment sheet
- Use open book, study guides, test prototypes

English Language Learning (ELL)

- Place student next to same-language speaker, if possible
- Vocabulary aides: limits, derivatives
- Provide use of translation dictionary or software for differentiation and optimization
- Implement strategy groups
- Continually have progress update with students and teachers on a weekly basis
- Provide graphic organizers
- Allow students to use notes and examples
- Modification plan
- NJDOE resources

- teaching key aspects of a topic. Eliminate nonessential information
- using videos, illustrations, pictures, and drawings to explain or clarif
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning;
- allowing students to correct errors (looking for understanding)
- allowing the use of note cards or open-book during testing
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments

- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using computer word processing spell check and grammar check features
- using true/false, matching, or fill in the blank tests in lieu of essay tests

At Risk

● NJDOE resources
● Create weekly check-ins outside class
● Utilize online resources such as http://www.tenmarks.com or www.khanacademy.org
● Provide students with opportunities to create projects such as ones that illustrate what the first and second derivative tell us about the graph of a function

- allowing students to correct errors (looking for understanding)
- teaching key aspects of a topic. Eliminate nonessential information
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning
- allowing students to select from given choices
- allowing the use of note cards or open-book during testing
- collaborating (general education teacher and specialist) to modify vocabulary, omit or modify items to reflect objectives for the student, eliminate sections of the test, and determine how the grade will be determined prior to giving the test.
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- marking students' correct and acceptable work, not the mistakes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using authentic assessments with real-life problem-solving
- using true/false, matching, or fill in the blank tests in lieu of essay tests
- using videos, illustrations, pictures, and drawings to explain or clarify

Talented and Gifted Learning (T&G)

● Process should be modified:higher-order-thinking skills, open-ended thinking, discovery
● Utilize project-based learning for greater depth of knowledge and assign students to aid struggling students
● Utilize exploratory connections to higher grade concepts
● Contents should be modified: abstraction, complexity, variety, organization
● Learning environments should be modified: student-centered learning, independence, openness, complexity, groups varied
● Use of web-based resources such as http://www.tenmarks.com, www.khanacademy.org, geogebra.org

● NJDOE resources
● Allows students to explore topics on their own related to this unit.

- Above grade level placement option for qualified students
- Advanced problem-solving
- Allow students to work at a faster pace
- Cluster grouping
- Complete activities aligned with above grade level text using Benchmark results
- Create a blog or social media page about their unit
- Create a plan to solve an issue presented in the class or in a text
- Debate issues with research to support arguments
- Flexible skill grouping within a class or across grade level for rigor
- Higher order, critical & creative thinking skills, and discovery
- Multi-disciplinary unit and/or project
- Teacher-selected instructional strategies that are focused to provide challenge, engagement, and growth opportunities
- Utilize exploratory connections to higher-grade concepts
- Utilize project-based learning for greater depth of knowledge

Sample Lesson