

Unit 2: The Derivative

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Department of Curriculum and Instruction



Belleville Public Schools

Curriculum Guide

Calculus AP, Unit 2

The Derivative

Belleville Board of Education

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Unit Overview

- Identify the derivative of a function as the limit of a difference quotient
- Estimate Derivatives
- Calculate Derivatives
- Determine higher order derivatives
- Determine Recognize the connection between differentiability and continuity

Students will be able to independently use their learning to understand and explain rate of change concepts in mathematical models of real-world phenomena and generalize their knowledge of algebraic concepts such as slope through the Derivative.

Enduring Understanding

Students will understand that:

- The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies

- A function's derivative which is itself a function, can be used to understand the behavior of the function
- The derivative has multiple interpretations and applications including those that involve instantaneous rates of change

Essential Questions

Essential Questions are:

- 1: Why are rate of change concepts of value to society?
- 2: How did the derivative extend physics and other sciences?
- 3: How does the derivative extend algebra?

Exit Skills

By the end of Unit 2 Students will know:

- The difference quotient and alternative form the difference quotient express the average rate of change (AROC) of a function on an interval
- The instantaneous rate of change of a function at a point can be expressed as the limit of a difference quotient, provided that the limit exists. The formal and alternative definitions of the derivative can be denoted with $f'(c)$ notation
- The derivative of f is the function whose value (y -coordinate) at x is the limit of the difference quotient, provided that limit exists
- Leibniz, Euler, and hybrid notation can be used to describe the derivative of a function.
- The derivative can be represented graphically, numerically, analytically, and verbally.
- The derivative at a point can be estimated from information given in tables or graphs

- Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential and logarithmic functions
- Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- Sums, differences, products, and quotients of functions can be differentiated using derivative rules
- The chain rule provides a way to differentiate composite functions
- The chain rule is the basis for implicit differentiation
- The chain rule can be used to find the derivative of an inverse function
- Differentiating f' produces the second derivative f'' , provided the derivative of f' exists; repeating this process produces higher order derivatives of f
- Higher order derivatives can be represented with Leibniz, Euler or hybrid notations
- A continuous function may fail to be differentiable at a point in its domain
- If a function is differentiable at a point, then it is continuous at that point

New Jersey Student Learning Standards (NJSL)

MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MA.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.F-IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MA.F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MA.F-IF.C.7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.F-IF.C.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MA.F-IF.C.8b	Use the properties of exponents to interpret expressions for exponential functions.
MA.F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
MA.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.
MA.F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
MA.F-TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
MA.F-TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
MA.F-TF.C.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
MA.F-TF.C.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.

MA.K-12.6	Attend to precision.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.N-RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
MA.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MA.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
MA.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MA.A-APR.C.4	Prove polynomial identities and use them to describe numerical relationships.
MA.A-APR.C.5	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n , where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
MA.A-APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MA.A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
MA.A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MA.A-SSE.B.4	Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
MA.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines.
MA.A-SSE.B.3b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
MA.A-SSE.B.3c	Use the properties of exponents to transform expressions for exponential functions.
MA.G-GPE.B.4	Use coordinates to prove simple geometric theorems algebraically.

Interdisciplinary Connections

LA.W.11-12.1.B	Develop claim(s) and counterclaims avoiding common logical fallacies and using sound reasoning and thoroughly, supplying the most relevant evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level, concerns, values, and possible biases.
LA.RI.11-12.1	Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.
SOC.6.2.12.C.6.b	Compare and contrast demographic trends in industrialized and developing nations, and evaluate the potential impact of these trends on the economy, political stability, and use of resources.

Learning Objectives

Students will be able to:

- 1: Compute the average rate of change of a function on an interval numerically, graphically and analytically
- 2: Estimate the slope of a function at a point graphically or numerically
- 3: Compute the slope of a function at a point analytically
- 4: Determine the differentiability of a function at a point graphically or analytically
- 5: Determine the intervals on which a function is differentiable
- 6: Write the equation of a tangent line to a function at a point
- 7: Sketch the derivative a function given its graph
- 8: Use the power rule to differentiate algebraic functions

9: Use basic differentiation rules to differentiate transcendental functions

10: Use product and quotient rule to differentiate functions

11: Use chain rule to differentiate functions

12: Differentiate implicit functions

13: Compute higher-order derivatives

Action Verbs: Below are examples of action verbs associated with each level of the Revised Bloom's Taxonomy.

Remember	Understand	Apply	Analyze	Evaluate	Create
Choose	Classify	Choose	Categorize	Appraise	Combine
Describe	Defend	Dramatize	Classify	Judge	Compose
Define	Demonstrate	Explain	Compare	Criticize	Construct
Label	Distinguish	Generalize	Differentiate	Defend	Design
List	Explain	Judge	Distinguish	Compare	Develop
Locate	Express	Organize	Identify	Assess	Formulate
Match	Extend	Paint	Infer	Conclude	Hypothesize
Memorize	Give Examples	Prepare	Point out	Contrast	Invent
Name	Illustrate	Produce	Select	Critique	Make
Omit	Indicate	Select	Subdivide	Determine	Originate
Recite	Interrelate	Show	Survey	Grade	Organize
Select	Interpret	Sketch	Arrange	Justify	Plan
State	Infer	Solve	Breakdown	Measure	Produce
Count	Match	Use	Combine	Rank	Role Play
Draw	Paraphrase	Add	Detect	Rate	Drive
Outline	Represent	Calculate	Diagram	Support	Devise
Point	Restate	Change	Discriminate	Test	Generate
Quote	Rewrite	Classify	Illustrate		Integrate
Recall	Select	Complete	Outline		Prescribe
Recognize	Show	Compute	Point out		Propose
Repeat	Summarize	Discover	Separate		Reconstruct
Reproduce	Tell	Divide			Revise
	Translate	Examine			Rewrite
	Associate	Graph			Transform
	Compute	Interpolate			
	Convert	Manipulate			
	Discuss	Modify			
	Estimate	Operate			
	Extrapolate	Subtract			
	Generalize				
	Predict				



Suggested Activities & Best Practices

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Guidelines for Suggested Activities:

- Includes activities **appropriate & specific** to the development of the Unit;
- Is comprised of the variety of learning activities that will be referenced in lesson plans, constructed/developed and instructionally delivered in the classroom;
- Are authentic;
- Recognizes the learning styles of the students;
- Integrates problem- or project-based learning.

Assessment Evidence - Checking for Understanding (CFU)

- Provide open-ended problems that mirror AP Exam questions to measure comprehension (Formative)
- Peer/Self Evaluation Rubrics to measure progress (Formative)
- At the end of each chapter in the unit, summative assessments will be administered (Summative)
- Benchmark assessments will be administered during each quarter (Benchmark)

- Admit Tickets
- Anticipation Guide
- Common Benchmarks
- Compare & Contrast
- Create a Multimedia Poster
- DBQ's
- Define
- Describe
- Evaluate
- Evaluation rubrics
- Exit Tickets
- Explaining
- Fist- to-Five or Thumb-Ometer
- Illustration
- Journals
- KWL Chart
- Learning Center Activities
- Multimedia Reports
- Newspaper Headline
- Outline
- Question Stems
- Quickwrite

- Quizzes
- Red Light, Green Light
- Self- assessments
- Socratic Seminar
- Study Guide
- Surveys
- Teacher Observation Checklist
- Think, Pair, Share
- Think, Write, Pair, Share
- Top 10 List
- Unit review/Test prep
- Unit tests
- Web-Based Assessments
- Written Reports

Primary Resources & Materials

Textbook: Larson, R., & Edwards, B. H. (2014). *AP Calculus 10e* (10th ed.). Independence, KY: Cengage Learning.

Texas Instruments TI-84 graphing calculator

Ancillary Resources

KHAN Academy

<https://www.khanacademy.org/math/calculus-home>

Massachusetts Institute of Technology (MIT) Open Courseware for High School

<https://ocw.mit.edu/high-school/mathematics/>

Technology Infusion

Upon completion of this sections, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

What **Technology Infusion** and/or strategies are integrated into this unit to enhance learning? Please list all hardware, software and strategies. Please find a technology pedagogy wheel for assistance while completing this section.

Win 8.1 Apps/Tools Pedagogy Wheel

Podcasts
 Photostory 3
 Kid Story Builder
 Music Maker Jam
 Paint A Story
 Office 365
 MS PowerPoint
 Stack 'Em Up
 NqSquared Numbers
 Physamajig
 Xylophone 8

Wikipedia
 Skydrive
 Lync
 SkyMap
 Skype
 Office 365
 Puzzle Touch
 Easy QR
 Memorylage
 Life Moments
 Word Cloud Maker

Where's Waldo?
 MS Excel
 Flipboard
 Office 365
 Nova Mindmapping

Ted Talks
 Record Voice Pen



Originally taken from <http://www.coetail.com/vzimmer/files/2013/02/iPadagogy-Wheel.001.jpg>
 And adapted for Windows 8.1 devices by Charlotte Beckhurst @CharBeckhurst

Alignment to 21st Century Skills & Technology

Mastery and infusion of **21st Century Skills & Technology** and their Alignment to the core content areas is essential to student learning. The core content areas include:

- English Language Arts;
- Mathematics;
- Science and Scientific Inquiry (Next Generation);
- Social Studies, including American History, World History, Geography, Government and Civics, and Economics;
- World languages;
- Technology;
- Visual and Performing Arts.

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.1	Review career goals and determine steps necessary for attainment.
CAEP.9.2.12.C.3	Identify transferable career skills and design alternate career plans.
CAEP.9.2.12.C.4	Analyze how economic conditions and societal changes influence employment trends and future education.
TECH.8.1.12.A.3	Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.
TECH.8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
TECH.8.1.12.D.1	Demonstrate appropriate application of copyright, fair use and/or Creative Commons to an original work.
TECH.8.1.12.D.2	Evaluate consequences of unauthorized electronic access (e.g., hacking) and disclosure, and on dissemination of personal information.
TECH.8.1.12.F.1	Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.
TECH.8.2.12.E.1	Demonstrate an understanding of the problem-solving capacity of computers in our world.

21st Century Skills/Interdisciplinary Themes

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Please list only the **21st Century/Interdisciplinary Themes** that will be incorporated into this unit.

- Communication and Collaboration
- Creativity and Innovation
- Critical thinking and Problem Solving
- ICT (Information, Communications and Technology) Literacy
- Information Literacy
- Life and Career Skills
- Media Literacy

21st Century Skills

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Please list only the **21st Century Skills** that will be incorporated into this unit.

- Civic Literacy
- Environmental Literacy
- Financial, Economic, Business and Entrepreneurial Literacy
- Global Awareness
- Health Literacy

Differentiation

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Please remember: Effective educational **Differentiation** in a lesson lies within content, process, and/or product.

Please identify the ones that will be employed in this unit.

Differentiations:

- Small group instruction
- Small group assignments
- Extra time to complete assignments
- Pairing oral instruction with visuals
- Repeat directions
- Use manipulatives
- Center-based instruction
- Token economy
- Study guides
- Teacher reads assessments allowed
- Scheduled breaks
- Rephrase written directions
- Multisensory approaches
- Additional time
- Preview vocabulary
- Preview content & concepts
- Story guides
- Behavior management plan
- Highlight text
- Student(s) work with assigned partner
- Visual presentation
- Assistive technology
- Auditory presentations
- Large print edition
- Dictation to scribe
- Small group setting

Hi-Prep Differentiations:

- Alternative formative and summative assessments
- Choice boards
- Games and tournaments
- Group investigations
- Guided Reading
- Independent research and projects
- Interest groups
- Learning contracts
- Leveled rubrics
- Literature circles
- Multiple intelligence options
- Multiple texts
- Personal agendas
- Project-based learning
- Problem-based learning
- Stations/centers
- Think-Tac-Toes
- Tiered activities/assignments
- Tiered products

- Varying organizers for instructions

Lo-Prep Differentiations

- Choice of books or activities
- Cubing activities
- Exploration by interest
- Flexible grouping
- Goal setting with students
- Jigsaw
- Mini workshops to re-teach or extend skills
- Open-ended activities
- Think-Pair-Share
- Reading buddies
- Varied journal prompts
- Varied supplemental materials

Special Education Learning (IEP's & 504's)

- Allow students to use notes
- Provide modifications and accommodations as listed in the student's IEP/504 plan
- Position student near helping peer or have quick access to teacher
- Modify or reduce assignments/texts by creating partial graphs when graphing limits
- Reduce length of assignment for different mode of delivery
- Increase one-to-one time
- Utilize working contract between you and student at risk
- Prioritize tasks
- Provide manipulatives, graphic organizers and math journals to keep track of formulas and techniques of differentiation (i.e. product rule, quotient rule, behavior of limits)
- Use online resources for skill building and allow students to see notes of processes of differentiation and graphing)
- Use collaborative grouping strategies such as small groups to reinforce the process of differentiation
- NJDOE resources
 - printed copy of board work/notes provided
 - additional time for skill mastery
 - assistive technology
 - behavior management plan
 - Center-Based Instruction
 - check work frequently for understanding
 - computer or electronic device utilizes
 - extended time on tests/ quizzes

- have student repeat directions to check for understanding
- highlighted text visual presentation
- modified assignment format
- modified test content
- modified test format
- modified test length
- multiple test sessions
- multi-sensory presentation
- preferential seating
- preview of content, concepts, and vocabulary
- Provide modifications as dictated in the student's IEP/504 plan
- reduced/shortened reading assignments
- Reduced/shortened written assignments
- secure attention before giving instruction/directions
- shortened assignments
- student working with an assigned partner
- teacher initiated weekly assignment sheet
- Use open book, study guides, test prototypes

English Language Learning (ELL)

- Place student next to same-language speaker, if possible
 - Vocabulary aides: limits, derivatives
 - Provide use of translation dictionary or software for differentiation and optimization
 - Implement strategy groups
 - Continually have progress update with students and teachers on a weekly basis
 - Provide graphic organizers
 - Allow students to use notes and examples
 - Modification plan
 - NJDOE resources
-
- teaching key aspects of a topic. Eliminate nonessential information
 - using videos, illustrations, pictures, and drawings to explain or clarify
 - allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning;
 - allowing students to correct errors (looking for understanding)
 - allowing the use of note cards or open-book during testing
 - decreasing the amount of work presented or required
 - having peers take notes or providing a copy of the teacher's notes
 - modifying tests to reflect selected objectives

- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using computer word processing spell check and grammar check features
- using true/false, matching, or fill in the blank tests in lieu of essay tests

At Risk

- NJDOE resources
- Create weekly check-ins outside class
- Utilize online resources such as <http://www.tenmarks.com> or www.khanacademy.org
- Provide students with opportunities to create projects such as ones that illustrate what the first and second derivative tell us about the graph of a function

- allowing students to correct errors (looking for understanding)
- teaching key aspects of a topic. Eliminate nonessential information
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning
- allowing students to select from given choices
- allowing the use of note cards or open-book during testing
- collaborating (general education teacher and specialist) to modify vocabulary, omit or modify items to reflect objectives for the student, eliminate sections of the test, and determine how the grade will be determined prior to giving the test.
- decreasing the amount of work presented or required
- having peers take notes or providing a copy of the teacher's notes
- marking students' correct and acceptable work, not the mistakes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using authentic assessments with real-life problem-solving
- using true/false, matching, or fill in the blank tests in lieu of essay tests
- using videos, illustrations, pictures, and drawings to explain or clarify

Talented and Gifted Learning (T&G)

- Process should be modified: higher-order-thinking skills, open-ended thinking, discovery
- Utilize project-based learning for greater depth of knowledge and assign students to aid struggling students
- Utilize exploratory connections to higher grade concepts
- Contents should be modified: abstraction, complexity, variety, organization
- Learning environments should be modified: student-centered learning, independence, openness, complexity, groups varied
- Use of web-based resources such as <http://www.tenmarks.com>, www.khanacademy.org, [geogebra.org](http://www.geogebra.org)

- NJDOE resources
- Allows students to explore topics on their own related to this unit.
 - Above grade level placement option for qualified students
 - Advanced problem-solving
 - Allow students to work at a faster pace
 - Cluster grouping
 - Complete activities aligned with above grade level text using Benchmark results
 - Create a blog or social media page about their unit
 - Create a plan to solve an issue presented in the class or in a text
 - Debate issues with research to support arguments
 - Flexible skill grouping within a class or across grade level for rigor
 - Higher order, critical & creative thinking skills, and discovery
 - Multi-disciplinary unit and/or project
 - Teacher-selected instructional strategies that are focused to provide challenge, engagement, and growth opportunities
 - Utilize exploratory connections to higher-grade concepts
 - Utilize project-based learning for greater depth of knowledge

Sample Lesson
