Unit 1: Limits and Continuity

Content Area: **Math**
Course(s): **Calculus AP**
Time Period:
Length: **30 Days**
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Title Section

Department of Curriculum and Instruction

Belleville Public Schools

Curriculum Guide

Calculus AP, Unit 1

Limits and Continuity

Belleville Board of Education

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Unit Overview

- Express limits symbolically using correct notation
- Interpret limits expressed symbolically
- Estimate limits of functions
- Determine limits of functions
- Deduce and interpret behavior of functions using limits
- Analyze functions for intervals of continuity or points of discontinuity
- Determine the applicability of important calculus theorems using continuity

Students will be able to independently use their learning to understand and explain the limiting process as a tool for solving problems and a foundational element of the Derivative and the Definite Integral

Enduring Understanding

Students will understand that:

- The concept of a limit can be used to understand the behavior of functions
- Continuity is a key property of functions that is defined using limits

Essential Questions

Essential Questions are:

- How can we describe the aesthetic properties real-world phenomena?
- Why are functions that "behave well" or "behave badly" important to describing the world we live in?
- How can we distinguish among infinite quantities or infinitesimal quantities?
- How do limits extend and expand the power of geometry and algebra?

Exit Skills

By the end of Unit I Students will know:

- The meaning of a limit: Given a function f, the limit of f(x) as x approaches c is a read number R if f(x) can be made arbitrarily close to R be taking x sufficiently close to c (but not equal to c).
- The relationship between meaning and notation of the limit of a function as x approaches c
- The concept of a limit can be extended to include one-sided limits, limits at infinity and infinite limits
- A limit might not exist for some functions at particular values of x. Some ways that the limit might not exist are if the function is unbounded (blowup), if the function is oscillating near this value (infinite oscillation), or if the limit from the left does not equal the limit from the right (jump)
- Numerical and graphical information can be used to estimate limits
- Limits of sums, differences, products, quotients and composite functions can be found using the basic theorems of limits and algebraic rules
- The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem
- Limits of the indeterminate forms 0/0 and Inf/Inf may be evaluated using L'Hospital's Rule
- Asymptotic and unbounded behavior of functions can be explained and described using limits
- Relative magnitudes of functions and their rates of change can be compared using limits

- A function f is continuous at c if the limit at c exists and the limit equals the function value
- Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains
- Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes
- Continuity is an essential condition for theorems such as te Intermediate Value Theorem (IVT), Extreme Value Theorem (EVT), and the Mean Value Theorem (MVT)

New Jersey Student Learning Standards (NJSLS)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.N-RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MA.A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MA.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of

	exponents.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MA.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
MA.K-12.4	Model with mathematics.
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.K-12.5	Use appropriate tools strategically.
MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines.
MA.K-12.6	Attend to precision.
MA.A-SSE.B.3b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
MA.A-SSE.B.3c	Use the properties of exponents to transform expressions for exponential functions.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.K-12.7	Look for and make use of structure.
MA.A-SSE.B.4	Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.F-IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MA.F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MA.F-IF.C.7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.A-APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

MA.F-IF.C.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MA.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MA.F-IF.C.8b	Use the properties of exponents to interpret expressions for exponential functions.
MA.A-APR.C.4	Prove polynomial identities and use them to describe numerical relationships.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.A-APR.C.5	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
MA.A-APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.G-GPE.B.4	Use coordinates to prove simple geometric theorems algebraically.
MA.F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
MA.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
MA.F-TF.A.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
MA.F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
MA.F-TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
MA.F-TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
MA.F-TF.C.8	Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ given $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle.
MA.F-TF.C.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Interdisciplinary Connections

LA.RI.11-12.1	Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.
LA.W.11-12.1.B	Develop claim(s) and counterclaims avoiding common logical fallacies and using sound reasoning and thoroughly, supplying the most relevant evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level, concerns, values, and possible biases.
SOC.6.2.12.C.6.b	Compare and contrast demographic trends in industrialized and developing nations, and evaluate the potential impact of these trends on the economy, political stability, and use of resources.
SOC.6.3.12.D.2	Analyze a current foreign policy issue by considering current and historical perspectives, examining strategies, and presenting possible actions.

Learning Objectives

Students will be able to:

1: Compute limits numerically

2: Compute limits graphically

3: Compute one-sided limits

4: Determine continuity of a function at a point

5: Use properties of limits to compute limits analytically

6: Determine the interval(s) on which a function is continuous

7: Apply the Intermediate Value Theorem to identify characteristics of a function

8: Identify and handle determinate and indeterminate forms

9: Compute limits analytically using algebraic and trigonometric techniques

10: Identify the zeroes and poles of rational functions and determine sketch them accordingly

11: Find limits of piecewise functions analytically

12: Identify and characterize discontinuities graphically and analytically

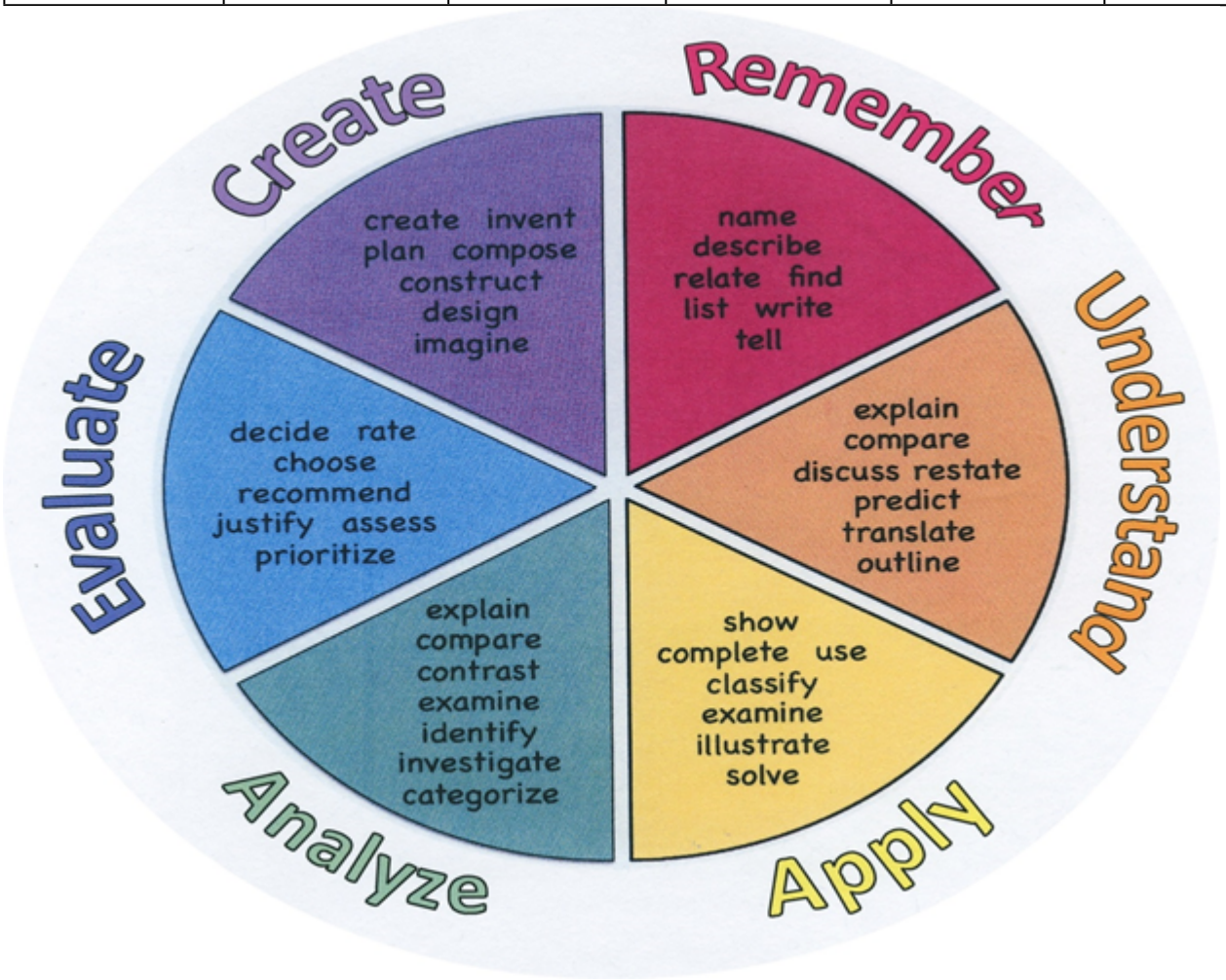
13: Compute infinite limits graphically

14: Compute infinite limits analytically across all function families

Action Verbs: Below are examples of action verbs associated with each level of the Revised Bloom's Taxonomy.

Remember	Understand	Apply	Analyze	Evaluate	Create
Choose	Classify	Choose	Categorize	Appraise	Combine
Describe	Defend	Dramatize	Classify	Judge	Compose
Define	Demonstrate	Explain	Compare	Criticize	Construct
Label	Distinguish	Generalize	Differentiate	Defend	Design
List	Explain	Judge	Distinguish	Compare	Develop

Locate	Express	Organize	Identify	Assess	Formulate
Match	Extend	Paint	Infer	Conclude	Hypothesize
Memorize	Give Examples	Prepare	Point out	Contrast	Invent
Name	Illustrate	Produce	Select	Critique	Make
Omit	Indicate	Select	Subdivide	Determine	Originate
Recite	Interrelate	Show	Survey	Grade	Organize
Select	Interpret	Sketch	Arrange	Justify	Plan
State	Infer	Solve	Breakdown	Measure	Produce
Count	Match	Use	Combine	Rank	Role Play
Draw	Paraphrase	Add	Detect	Rate	Drive
Outline	Represent	Calculate	Diagram	Support	Devise
Point	Restate	Change	Discriminate	Test	Generate
Quote	Rewrite	Classify	Illustrate		Integrate
Recall	Select	Complete	Outline		Prescribe
Recognize	Show	Compute	Point out		Propose
Repeat	Summarize	Discover	Separate		Reconstruct
Reproduce	Tell	Divide			Revise
	Translate	Examine			Rewrite
	Associate	Graph			Transform
	Compute	Interpolate			
	Convert	Manipulate			
	Discuss	Modify			
	Estimate	Operate			
	Extrapolate	Subtract			
	Generalize				
	Predict				



Suggested Activities & Best Practices

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Guidelines for Suggested Activities:

- Includes activities **appropriate & specific** to the development of the Unit;
- Is comprised of the variety of learning activities that will be referenced in lesson plans, constructed/developed and instructionally delivered in the classroom;
- Are authentic;
- Recognizes the learning styles of the students;
- Integrates problem- or project-based learning.

Assessment Evidence - Checking for Understanding (CFU)

- Provide open-ended problems that mirror AP Exam questions to measure comprehension (Formative)
- Peer/Self Evaluation Rubrics to measure progress (Formative)
- At the end of each chapter in the unit, summative assessments will be administered (Summative)
- Benchmark assessemnts will be administered during each quarter (Benchmark)

- Admit Tickets
- Anticipation Guide
- Common Benchmarks
- Compare & Contrast
- Create a Multimedia Poster
- DBQ's

- Define
- Describe
- Evaluate
- Evaluation rubrics
- Exit Tickets
- Explaining
- Fist- to-Five or Thumb-Ometer
- Illustration
- Journals
- KWL Chart
- Learning Center Activities
- Multimedia Reports
- Newspaper Headline
- Outline
- Question Stems
- Quickwrite
- Quizzes
- Red Light, Green Light
- Self- assessments
- Socratic Seminar
- Study Guide
- Surveys
- Teacher Observation Checklist
- Think, Pair, Share
- Think, Write, Pair, Share
- Top 10 List
- Unit review/Test prep
- Unit tests
- Web-Based Assessments
- Written Reports

Primary Resources & Materials

Textbook: Larson, R., & Edwards, B. H. (2014). *AP Calculus 10e* (10th ed.). Independence, KY: Cengage Learning.

Texas Instruments TI-84 graphing calculator

Ancillary Resources

KHAN Academy

https://www.khanacademy.org/math/calculus-home

Massachusetts Institute of Technology (MIT) Open Courseware for High School

https://ocw.mit.edu/high-school/mathematics/

Technology Infusion

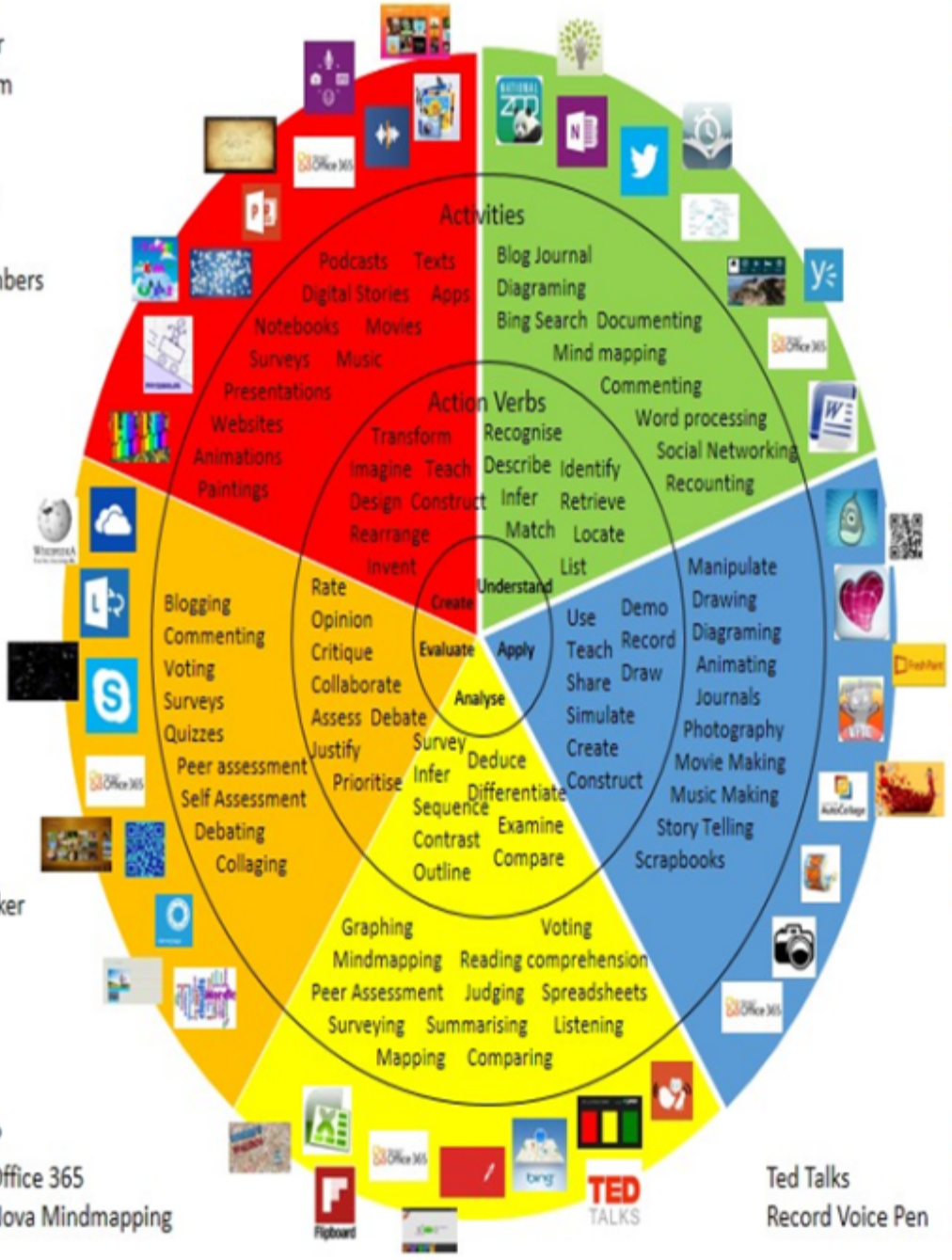
Upon completion of this sections, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

What **Technology Infusion** and/or strategies are integrated into this unit to enhance learning? Please list all hardware, software and strategies. Please find a technology pedagogy wheel for assistance while completing this section.

Win 8.1 Apps/Tools Pedagogy Wheel

Podcasts
Photostory 3
Kid Story Builder
Music Maker Jam
Paint A Story
Office 365
MS PowerPoint
Stack 'Em Up
NqSquared Numbers
Physamajig
Xylophone 8

Wikipedia
Skydrive
Lync
SkyMap
Skype
Office 365
Puzzle Touch
Easy QR
Memorylage
Life Moments
Word Cloud Maker



Where's Waldo?
MS Excel Office 365
Flipboard Nova Mindmapping

Ted Talks
Record Voice Pen

Originally taken from http://www.coetail.com/vzimmer/files/2013/02/iPadagogy-Wheel.001.jpg
And adapted for Windows 8.1 devices by Charlotte Beckhurst @CharBeckhurst

Alignment to 21st Century Skills & Technology

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Mastery and infusion of **21st Century Skills & Technology** and their Alignment to the core content areas is essential to student learning. The core content areas include:

- English Language Arts;
- Mathematics;
- Science and Scientific Inquiry (Next Generation);
- Social Studies, including American History, World History, Geography, Government and Civics, and Economics;
- World languages;
- Technology;
- Visual and Performing Arts.

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.1	Review career goals and determine steps necessary for attainment.
CAEP.9.2.12.C.3	Identify transferable career skills and design alternate career plans.
CAEP.9.2.12.C.4	Analyze how economic conditions and societal changes influence employment trends and future education.
CAEP.9.2.12.C.5	Research career opportunities in the United States and abroad that require knowledge of world languages and diverse cultures.
TECH.8.1.12.A.3	Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.
TECH.8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
TECH.8.1.12.D.1	Demonstrate appropriate application of copyright, fair use and/or Creative Commons to an original work.
TECH.8.1.12.D.2	Evaluate consequences of unauthorized electronic access (e.g., hacking) and disclosure, and on dissemination of personal information.
TECH.8.1.12.E.1	Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.

TECH.8.1.12.F.1	Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.
TECH.8.2.12.E.1	Demonstrate an understanding of the problem-solving capacity of computers in our world.

21st Century Skills/Interdisciplinary Themes

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Please list only the **21st Century/Interdisciplinary Themes** that will be incorporated into this unit.

- Communication and Collaboration
- Creativity and Innovation
- Critical thinking and Problem Solving
- ICT (Information, Communications and Technology) Literacy
- Information Literacy
- Life and Career Skills
- Media Literacy

21st Century Skills

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Please list only the **21st Century Skills** that will be incorporated into this unit.

- Civic Literacy
- Environmental Literacy
- Financial, Economic, Business and Entrepreneurial Literacy
- Global Awareness
- Health Literacy

Differentiation

Upon completion of this section, please remove all remaining descriptions, notes, outlines, examples and/or illustrations that are not needed or used.

Please remember: Effective educational **Differentiation** in a lesson lies within content, process, and/or product.

Please identify the ones that will be employed in this unit.

Differentiations:

- Small group instruction
- Small group assignments
- Extra time to complete assignments
- Pairing oral instruction with visuals
- Repeat directions
- Use manipulatives
- Center-based instruction
- Token economy
- Study guides
- Teacher reads assessments allowed
- Scheduled breaks
- Rephrase written directions
- Multisensory approaches
- Additional time
- Preview vocabulary
- Preview content & concepts
- Story guides
- Behavior management plan
- Highlight text
- Student(s) work with assigned partner
- Visual presentation
- Assistive technology
- Auditory presentations
- Large print edition
- Dictation to scribe
- Small group setting

Hi-Prep Differentiations:

- Alternative formative and summative assessments
- Choice boards
- Games and tournaments
- Group investigations
- Guided Reading
- Independent research and projects
- Interest groups
- Learning contracts
- Leveled rubrics
- Literature circles
- Multiple intelligence options
- Multiple texts
- Personal agendas
- Project-based learning

- Problem-based learning
- Stations/centers
- Think-Tac-Toes
- Tiered activities/assignments
- Tiered products
- Varying organizers for instructions

Lo-Prep Differentiations

- Choice of books or activities
- Cubing activities
- Exploration by interest
- Flexible grouping
- Goal setting with students
- Jigsaw
- Mini workshops to re-teach or extend skills
- Open-ended activities
- Think-Pair-Share
- Reading buddies
- Varied journal prompts
- Varied supplemental materials

Special Education Learning (IEP's & 504's)

● Allow students to use notes
● Provide modifications and accommodations as listed in the student's IEP/504 plan
● Position student near helping peer or have quick access to teacher
● Modify or reduce assignments/texts by creating partial graphs when graphing limits
● Reduce length of assignment for different mode of delivery
● Increase one-to-one time
● Utilize working contract between you and student at risk
● Prioritize tasks
● Provide manipulatives, graphic organizers and math journals to keep track of formulas and techniques of differentiation (i.e. product rule, quotient rule, behavior of limits)
● Use online resources for skill building and allow students to see notes of processes of differentiation and graphing)
● Use collaborative grouping strategies such as small groups to reinforce the process of differentiation
● NJDOE resources

- printed copy of board work/notes provided
- additional time for skill mastery
- assistive technology
- behavior management plan
- Center-Based Instruction
- check work frequently for understanding
- computer or electronic device utilizes
- extended time on tests/ quizzes
- have student repeat directions to check for understanding
- highlighted text visual presentation
- modified assignment format
- modified test content
- modified test format
- modified test length
- multi-sensory presentation
- multiple test sessions
- preferential seating
- preview of content, concepts, and vocabulary
- Provide modifications as dictated in the student's IEP/504 plan
- reduced/shortened reading assignments
- Reduced/shortened written assignments
- secure attention before giving instruction/directions
- shortened assignments
- student working with an assigned partner
- teacher initiated weekly assignment sheet
- Use open book, study guides, test prototypes

English Language Learning (ELL)

● Place student next to same-language speaker, if possible
● Vocabulary aides: limits, derivatives
● Provide use of translation dictionary or software for differentiation and optimization
● Implement strategy groups
● Continually have progress update with students and teachers on a weekly basis
● Provide graphic organizers
● Allow students to use notes and examples
● Modification plan
● NJDOE resources

- teaching key aspects of a topic. Eliminate nonessential information
- using videos, illustrations, pictures, and drawings to explain or clarif
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning;
- allowing students to correct errors (looking for understanding)
- allowing the use of note cards or open-book during testing
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using computer word processing spell check and grammar check features
- using true/false, matching, or fill in the blank tests in lieu of essay tests

At Risk

● NJDOE resources
● Create weekly check-ins outside class
● Utilize online resources such as http://www.tenmarks.com or www.khanacademy.org
● Provide students with opportunities to create projects such as ones that illustrate what the first and second derivative tell us about the graph of a function

- allowing students to correct errors (looking for understanding)
- teaching key aspects of a topic. Eliminate nonessential information
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning
- allowing students to select from given choices
- allowing the use of note cards or open-book during testing
- collaborating (general education teacher and specialist) to modify vocabulary, omit or modify items to reflect objectives for the student, eliminate sections of the test, and determine how the grade will be determined prior to giving the test.
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- marking students' correct and acceptable work, not the mistakes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test

- tutoring by peers
- using authentic assessments with real-life problem-solving
- using true/false, matching, or fill in the blank tests in lieu of essay tests
- using videos, illustrations, pictures, and drawings to explain or clarify

Talented and Gifted Learning (T&G)

● Process should be modified:higher-order-thinking skills, open-ended thinking, discovery
● Utilize project-based learning for greater depth of knowledge and assign students to aid struggling students
● Utilize exploratory connections to higher grade concepts
● Contents should be modified: abstraction, complexity, variety, organization
● Learning environments should be modified: student-centered learning, independence, openness, complexity, groups varied
● Use of web-based resources such as http://www.tenmarks.com, www.khanacademy.org, geogebra.org
● NJDOE resources
● Allows students to explore topics on their own related to this unit.

- Above grade level placement option for qualified students
- Advanced problem-solving
- Allow students to work at a faster pace
- Cluster grouping
- Complete activities aligned with above grade level text using Benchmark results
- Create a blog or social media page about their unit
- Create a plan to solve an issue presented in the class or in a text
- Debate issues with research to support arguments
- Flexible skill grouping within a class or across grade level for rigor
- Higher order, critical & creative thinking skills, and discovery
- Multi-disciplinary unit and/or project
- Teacher-selected instructional strategies that are focused to provide challenge, engagement, and growth opportunities
- Utilize exploratory connections to higher-grade concepts
- Utilize project-based learning for greater depth of knowledge

Sample Lesson

Unit Name:Limits and Continuity

NJSLS:See Below

Interdisciplinary Connection: See Below

Statement of Objective: Student will be able to define, understand the concept of, and evaluate limits.

Anticipatory Set/Do Now: How can we deduce and interpret behaviors of functions using limits?

Learning Activity: Review HW, "Do Now", Guided/Unguided Instruction, Class led Discussion, Closure and assigning of HW

Student Assessment/CFU's:Obeservation, Verbal Responses, Student Modeling, Q&A

Materials:SmartTV, TI-84 Graphing Calculators, Graph Paper, Notebook, Textbook

21st Century Themes and Skills:Critical thinking and Problem Solving, Creativity and Innovation

Differentiation/Modifications:Modeling, Direct/Guided Instruction, Tiered Questions, Peer Partners, Color-Coded Notes

Integration of Technology:TI-84 Graphic Calculator to Model Examples, SmartTV

MA.A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
MA.A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MA.F-IF.C	Analyze functions using different representations
LA.RI.11-12.1	Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.
MA.A-APR.D	Rewrite rational expressions
LA.W.11-12.1.B	Develop claim(s) and counterclaims avoiding common logical fallacies and using sound reasoning and thoroughly, supplying the most relevant evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level, concerns, values, and possible biases.