Unit 5: Sequences, Series & Statistics

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Title Section

Department of Curriculum and Instruction

Belleville Public Schools

Curriculum Guide

PRECALCULUS / TRIGONOMETRY HONORS, GRADES 10-12

UNIT 5: SEQUENCES, SERIES & STATISTICS

Belleville Board of Education

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Unit Overview

In this Unit...

- students will learn to use sequences and series to model and solve real-world situations. They will begin by identifying different types of sequences and calculating the sums of series. Then, they will learn how to find the *n*th term of an arithmetic or geometric sequence or series and how to calculate the sum of an infinite geometric series. They will learn about mathematical induction and how it can be applied to prove summation formulas and other properties. Then, they will learn how to use Pascal's Triangle and the Binomial Theorem to write binomial expansions and find specified coefficients. They will investigate power series and their usefulness in representing rational functions and approximating values.
- students will learn the different types of probability distributions and how they can be used. First, they will identify shapes of different distributions and select appropriate statistics based on this information. Then, they will construct probability distributions, including binomial distributions, and calculate their summary statistics. They will also find area and probabilities of normal distributions and find specific values from these probabilities. Finally, they will learn how to apply the Central Limit Theorem to find probabilities.

Enduring Understanding

- Use sequences and series to represent real-world relationships.
- Determine the difference between arithmetic and geometric sequences.
- Find the *n*th term of a sequence or series.
- Calculate the sum of an infinite geometric series.
- Understand the concept of mathematical induction and be able to apply it to prove properties and formulas.
- Use Pascal's Triangle and the Binomial Theorem to write binomial expansions and identify specified coefficients.
- Use power series to represent rational functions and approximate values.
- Identify and construct different types of probability distributions.

- Understand and apply the Central Limit Theorem.

Essential Questions

- What do sequences and series represent?
- What is the difference between an arithmetic and a geometric sequence?
- How can you tell whether a certain relationship can be modeled by an arithmetic or a geometric sequence?
- How can you find the *n*th term of a sequence or series?
- How can you find the sum of an infinite geometric series?
- What is mathematical induction and how is it useful?
- What is a binomial expansion and what is the purpose of Pascal's Triangle?
- How can you use the Binomial Theorem to find specified coefficients in binomial expansions?
- What is a power series and when can it be used?
- What are the different types of probability distributions and how can you identify them?
- How do you find probabilities for normal distributions?
- How can you use probabilities to find different data values?
- What is the Central Limit Theorem and how can it be applied?

Exit Skills

By the end of Unit 5, Precalculus / Trigonometry students should know:

- How to use sigma notation to represent and calculate sums of series.
- How to find *n*th terms of arithmetic sequences and arithmetic series.
- How to find *n*th terms of geometric sequences and geometric series. Find sums of infinite geometric series.
- How to use mathematical induction to prove summation formulas and properties of divisibility involving a positive integer *n*.
- How to use Pascal's Triangle or the Binomial Theorem to write binomial expansions.
- How to use the Binomial Theorem to find the coefficients of specified terms in binomial expansions.
- How to use a power series to represent a rational function.
- How to use power series representations to approximate values of transcendental functions.
- How to identify shapes of distributions.
- How to construct probability distributions, including binomial distributions.
- How to find probabilities for normal distributions and data values given probabilities.
- How to understand and apply the Central Limit Theorem.

New Jersey Student Learning Standards (NJSLS)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.

MA.S-ID.A.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.S-ID.A.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
MA.S-ID.A.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.7	Look for and make use of structure.
MA.A-APR.C.5	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
MA.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.S-MD.A.2	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
MA.S-MD.A.3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
MA.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interdisciplinary Connections

9-12.HS-ETS1-4.4.1	Models (e.g., physical, mathematical, computer models) can be used to simulate systems and interactions— including energy, matter, and information flows— within and between systems at different scales.
9-12.HS-ETS1-4.5	Using Mathematics and Computational Thinking
9-12.HS-ETS1-4.5.1	Use mathematical models and/or computer simulations to predict the effects of a design solution on systems and/or the interactions between systems.
9-12.HS-PS1-8.2.1	Develop a model based on evidence to illustrate the relationships between systems or between components of a system.

Learning Objectives

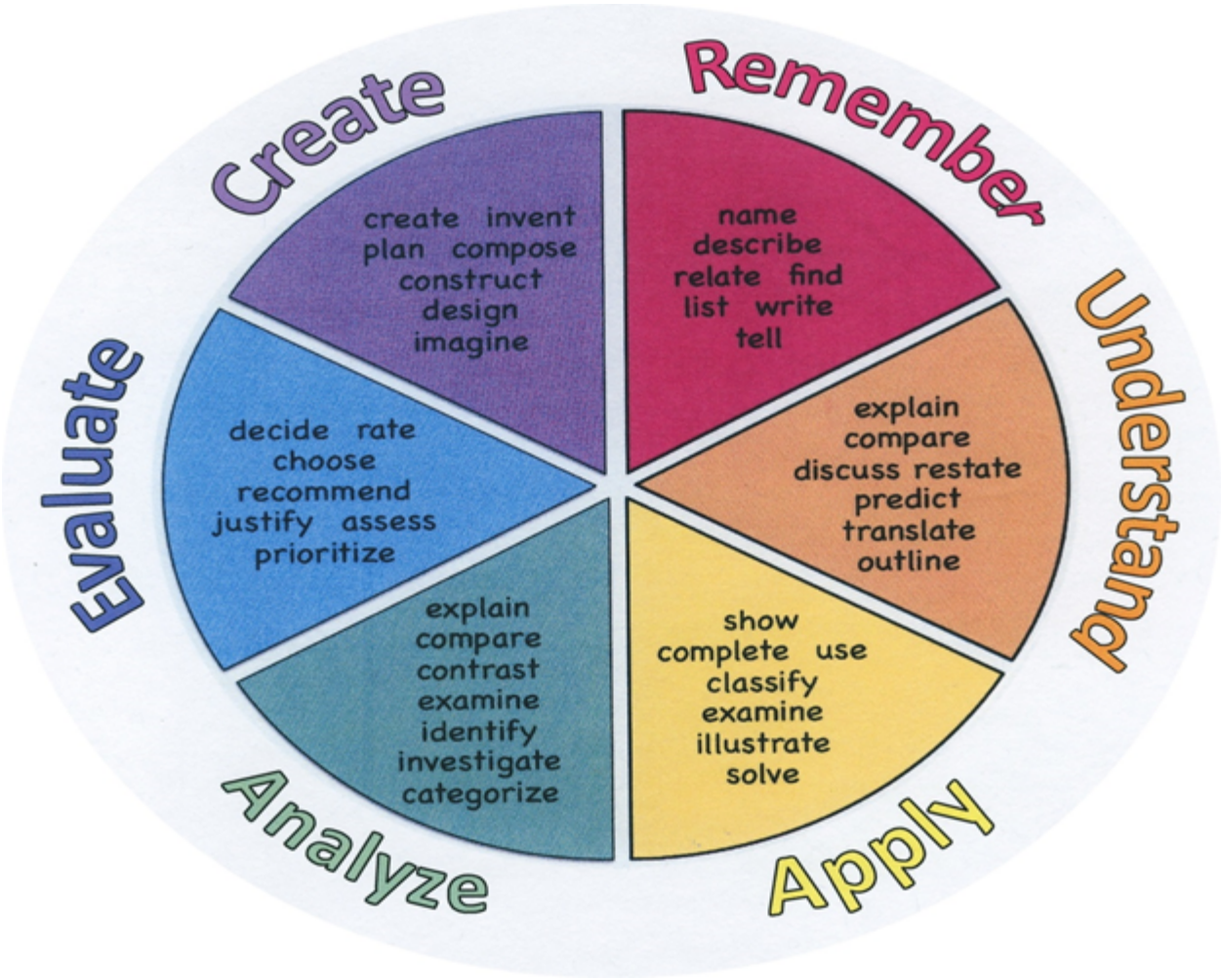
Students will be able to:

- Investigate several different types of sequences.
- Use sigma notation to represent and calculate sums of series.
- Find *n*th terms and arithmetic means of arithmetic sequences.
- Find sums of *n* terms of arithmetic series.

- Find *n*th terms and geometric means of geometric sequences.
- Find sums of *n* terms of geometric series and the sums of infinite geometric series.
- Use mathematical induction to prove summation formulas and properties of divisibility involving a positive integer *n*.
- Use extended mathematical induction.
- Use Pascal's triangle to write binomial expansions.
- Use the Binomial Theorem to write and find the coefficients of specified terms in binomial expansions.
- Use a power series to represent a rational function.
- Use power series representations to approximate values of transcendental functions.
- Identify the shapes of distributions in order to select more appropriate statistics.
- Use measures of position to compare two sets of data.
- Construct a probability distribution, and calculate its summary statistics.
- Construct and use a binomial distribution, and calculate its summary statistics.
- Find area under normal distribution curves.
- Find probabilities for normal distributions, and find data values given probabilities.
- Use the Central Limit Theorem to find probabilities.
- Find normal approximations of binomial distributions.

Action Verbs: Below are examples of action verbs associated with each level of the Revised Bloom's Taxonomy.

Remember	Understand	Apply	Analyze	Evaluate	Create
Choose	Classify	Choose	Categorize	Appraise	Combine
Describe	Defend	Dramatize	Classify	Judge	Compose
Define	Demonstrate	Explain	Compare	Criticize	Construct
Label	Distinguish	Generalize	Differentiate	Defend	Design
List	Explain	Judge	Distinguish	Compare	Develop
Locate	Express	Organize	Identify	Assess	Formulate
Match	Extend	Paint	Infer	Conclude	Hypothesize
Memorize	Give Examples	Prepare	Point out	Contrast	Invent
Name	Illustrate	Produce	Select	Critique	Make
Omit	Indicate	Select	Subdivide	Determine	Originate
Recite	Interrelate	Show	Survey	Grade	Organize
Select	Interpret	Sketch	Arrange	Justify	Plan
State	Infer	Solve	Breakdown	Measure	Produce
Count	Match	Use	Combine	Rank	Role Play
Draw	Paraphrase	Add	Detect	Rate	Drive
Outline	Represent	Calculate	Diagram	Support	Devise
Point	Restate	Change	Discriminate	Test	Generate
Quote	Rewrite	Classify	Illustrate		Integrate
Recall	Select	Complete	Outline		Prescribe
Recognize	Show	Compute	Point out		Propose
Repeat	Summarize	Discover	Separate		Reconstruct
Reproduce	Tell	Divide			Revise
	Translate	Examine			Rewrite
	Associate	Graph			Transform
	Compute	Interpolate			
	Convert	Manipulate			
	Discuss	Modify			
	Estimate	Operate			
	Extrapolate	Subtract			
	Generalize				
	Predict				



Suggested Activities & Best Practices

- Online textbook practice problems, study guides, and worksheets
- Desmos Classroom Activities, such as "Alligator Investigation" activity
- Higher-order thinking tasks, such as Illustrative Mathematics task "Simulating to Decide"
- Practice activities from ALEKS, KUTA Software, Khan Academy, etc., such as "Finding a Specified Term in an Arithmetic Sequence"

Assessment Evidence - Checking for Understanding (CFU)

- Regular Exit Tickets to assess individual learning objectives (Formative)
- Quizzes to assess groups of learning objectives - at least one quiz for each chapter (Chapters 10 and 11) (Summative)
- Chapter Tests given at least once per chapter - at least 2 tests in this unit (Chapters 10 and 11) (Summative)

- Common Quarterly/Benchmark Exams - Quarter 4 Exam for this unit (Benchmark)
- Web-Based Assessments (using Google Forms, ALEKS, Edulastic, Khan Academy, etc.) (Formative/Summative)

- Admit Tickets
- Anticipation Guide
- Common Benchmarks
- Compare & Contrast
- Create a Multimedia Poster
- Define
- Describe
- Evaluate
- Evaluation rubrics
- Exit Tickets
- Explaining
- Fist- to-Five or Thumb-Ometer
- Illustration
- KWL Chart
- Learning Center Activities
- Newspaper Headline
- Outline
- Question Stems
- Quickwrite
- Quizzes
- Red Light, Green Light
- Self- assessments
- Socratic Seminar
- Study Guide
- Surveys
- Teacher Observation Checklist
- Think, Pair, Share
- Think, Write, Pair, Share
- Top 10 List
- Unit review/Test prep
- Unit tests
- Web-Based Assessments

Primary Resources & Materials

- Glencoe McGraw-Hill Precalculus 2014
- Practice Glencoe Precalculus
- Study Guide Glencoe Precalculus

- connected.mcgraw-hill.com

Ancillary Resources

- Glencoe McGraw-Hill Algebra 2 2014
- ALEKS
- Kuta Software

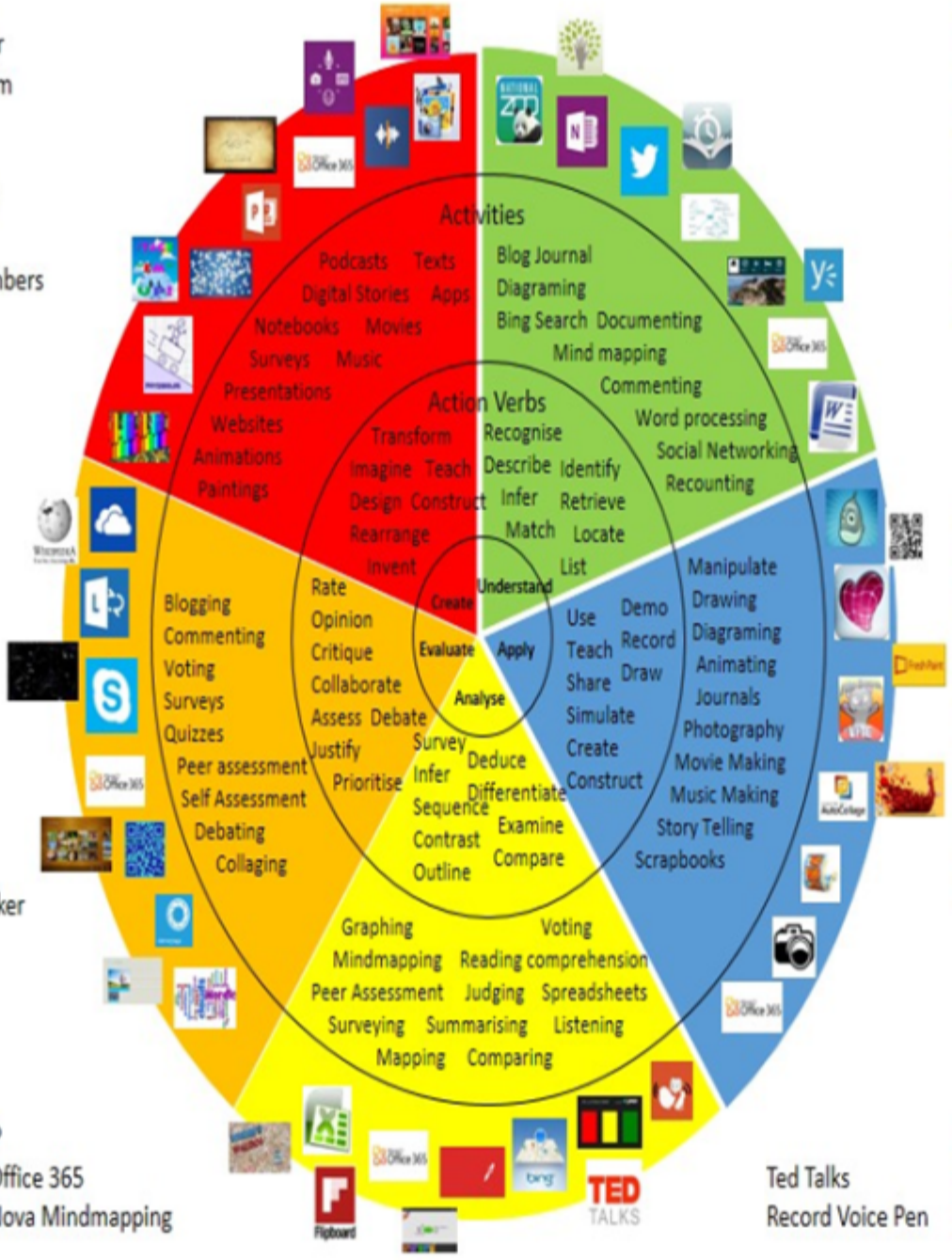
Technology Infusion

- Smart TV - Display and interact with lessons and activities
- Chromebooks - students access activities, slides, and practice problems
- Google Classroom - Slides, Forms, Drive, etc.
- ALEKS - Students practice individual learning objectives such as "Writing an explicit rule for a geometric series"
- Desmos - Students interact with classroom activities or use graphing software to graph and analyze functions
- YouTube - Students watch videos to deepen understanding of specific concepts throughout the unit
- Khan Academy - Students practice individual learning objectives, such as "Graphically add and subtract vectors"
- Calculator/Graphing calculator - Students perform calculations or graph and analyze functions
- Edulastic - Students complete assessments and checks for understanding
- KUTA Software - Teacher generates a variety of assessments and practice problems for individual learning objectives or groups of learning objectives
- Pear Deck - Teacher presents information through an interactive slide show presentation

Win 8.1 Apps/Tools Pedagogy Wheel

Podcasts
Photostory 3
Kid Story Builder
Music Maker Jam
Paint A Story
Office 365
MS PowerPoint
Stack 'Em Up
NqSquared Numbers
Physamajig
Xylophone 8

Wikipedia
Skydrive
Lync
SkyMap
Skype
Office 365
Puzzle Touch
Easy QR
Memorylage
Life Moments
Word Cloud Maker



Where's Waldo?
MS Excel Office 365
Flipboard Nova Mindmapping

Ted Talks
Record Voice Pen

Originally taken from http://www.coetail.com/vzimmer/files/2013/02/iPadagogy-Wheel.001.jpg
And adapted for Windows 8.1 devices by Charlotte Beckhurst @CharBeckhurst

Alignment to 21st Century Skills & Technology

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP2.1	Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP4.1	Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others' time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP6.1	Career-ready individuals regularly think of ideas that solve problems in new and different ways, and they contribute those ideas in a useful and productive manner to improve their organization. They can consider unconventional ideas and suggestions as solutions to issues, tasks or problems, and they discern which ideas and suggestions will add greatest value. They seek new methods, practices, and ideas from a variety of sources and seek to apply those ideas to their own workplace. They take action on their ideas and understand how to bring innovation to an organization.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP8.1	Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP11.1	Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.
TECH.8.1.12	Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

TECH.8.1.12.A	Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems and operations.
TECH.8.1.12.A.3	Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.
TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.A.CS2	Select and use applications effectively and productively.

21st Century Skills/Interdisciplinary Themes

- Communication and Collaboration
- Creativity and Innovation
- Critical thinking and Problem Solving
- ICT (Information, Communications and Technology) Literacy
- Information Literacy
- Life and Career Skills
- Media Literacy

21st Century Skills

- Civic Literacy
- Financial, Economic, Business and Entrepreneurial Literacy
- Global Awareness

Differentiation

- Small group instruction - Teacher utilizes small groups to remediate or enrich specific topics with different groups of students, as necessary.
- Study guides - Teacher provides students with study guides prior to quizzes and tests.
- Problem-based learning - Teacher introduces topics to students as part of a project, such as creating a real-world statistical experiment.
- Open-ended activities - Students complete activities with multiple entry points and more than one possible solution.

Differentiations:

- Small group instruction
- Small group assignments
- Extra time to complete assignments
- Pairing oral instruction with visuals

- Repeat directions
- Use manipulatives
- Center-based instruction
- Study guides
- Scheduled breaks
- Rephrase written directions
- Multisensory approaches
- Additional time
- Preview vocabulary
- Preview content & concepts
- Behavior management plan
- Highlight text
- Student(s) work with assigned partner
- Visual presentation
- Assistive technology
- Auditory presentations
- Small group setting

Hi-Prep Differentiations:

- Alternative formative and summative assessments
- Choice boards
- Games and tournaments
- Group investigations
- Learning contracts
- Multiple intelligence options
- Personal agendas
- Project-based learning
- Problem-based learning
- Stations/centers
- Tiered activities/assignments
- Tiered products
- Varying organizers for instructions

Lo-Prep Differentiations

- Exploration by interest
- Flexible grouping
- Goal setting with students
- Jigsaw
- Mini workshops to re-teach or extend skills
- Open-ended activities
- Think-Pair-Share
- Varied supplemental materials

Special Education Learning (IEP's & 504's)

- Provide modifications as dictated in student's IEP/504 - Teacher modifies tests/assesments as necessary.
- Additional time for skill mastery - Teacher allows students additional time to master particular learning objectives.
- Center-Based Instruction - Teacher utilizes different sets of stations/centers in order to differentiate and provide students with varied learning settings.
- Modify assignments/tests - Teacher modifies tests/assesments by, for example, writing multi-part answers for questions that require students to complete multiple steps.
- Utilize computers or electronic devices - Teacher uses chromebooks and smart TV to provide students with visualizations of graphs/models and allow students to interact with them them.
- Extended time on tests/quizzes - Teacher allows students to have extended time on tests/quizzes as dictated by their IEP/504.
- Use of calculator on tests/quizzes - Students are allowed to use calculators on tests/quizzes.
- Use of study guide, reference sheets, or notes on tests/quizzes - Teacher allows students to use reference sheets or study guides on tests/quizes that contain information such as formulas for arithmetic and geometric series.

- printed copy of board work/notes provided
- additional time for skill mastery
- assistive technology
- behavior management plan
- Center-Based Instruction
- check work frequently for understanding
- computer or electronic device utilizes
- extended time on tests/ quizzes
- have student repeat directions to check for understanding
- highlighted text visual presentation
- modified assignment format
- modified test content
- modified test format
- modified test length
- multi-sensory presentation
- multiple test sessions
- preferential seating
- preview of content, concepts, and vocabulary
- Provide modifications as dictated in the student's IEP/504 plan
- secure attention before giving instruction/directions
- shortened assignments
- student working with an assigned partner
- teacher initiated weekly assignment sheet
- Use open book, study guides, test prototypes

English Language Learning (ELL)

- Using videos, illustrations, pictures and drawings to explain or clarify - Teacher provides tools such as visualizations of graphs that students can interact with.
- Eliminate nonessential information - Teacher explains concepts using only the vocabulary that is essential to understand a concept.
- Tutoring by peers - Teacher allows peers to explain concepts to ELL students.
- Allow students to correct errors - Teacher allows students to gain back points by correcting their errors on a test/quiz.
- Modify assignments/tests - Teacher modifies tests/assesments by, for example, writing multi-part answers for questions that require students to complete multiple steps.
- Use of study guide, reference sheets, or notes on tests/quizzes - Teacher allows students to use reference sheets or study guides on tests/quizes that contain information such as formulas for arithmetic and geometric series.

- teaching key aspects of a topic. Eliminate nonessential information
- using videos, illustrations, pictures, and drawings to explain or clarif
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning;
- allowing students to correct errors (looking for understanding)
- allowing the use of note cards or open-book during testing
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- modifying tests to reflect selected objectives
- providing study guides
- reducing the number of answer choices on a multiple choice test
- tutoring by peers

At Risk

- Decrease the amount of work presented or required - Teacher allows students to submit less work, for example, only complete 3 out of the 5 practice problems for a specific learning objective.
- Using videos, illustrations, pictures, and drawings to explain or clarify - Teacher provides tools such as visualizations of graphs that students can interact with.
- Tutoring by peers - Teacher allows peers to explain concepts to at risk students.
- Providing study guides - Teacher provides students with study guides prior to quizzes and tests. For example, a variety of practice problems related to the topics being assessed.
- Allowing students to correct errors - Teacher allows students to gain back points by correcting their errors on a test/quiz.
- Allowing students to select from given choices - Teacher gives students a choice of activities to complete, such as draw a graph, create an equation, or write a sentence to model a situation.
- Allowing the use of notes, study guides, or reference sheets on tests/quizzes - Teacher allows students to use reference sheets or study guides on tests/quizes that contain information such as formulas for arithmetic and geometric series.

- allowing students to correct errors (looking for understanding)
- teaching key aspects of a topic. Eliminate nonessential information
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning
- allowing students to select from given choices
- allowing the use of note cards or open-book during testing
- collaborating (general education teacher and specialist) to modify vocabulary, omit or modify items to

reflect objectives for the student, eliminate sections of the test, and determine how the grade will be determined prior to giving the test.

- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- marking students' correct and acceptable work, not the mistakes
- modifying tests to reflect selected objectives
- providing study guides
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using authentic assessments with real-life problem-solving
- using videos, illustrations, pictures, and drawings to explain or clarify

Talented and Gifted Learning (T&G)

- Create a plan to solve an issue presented in the class - Teacher allows students to use their understanding of functions to model a specific problem of their choosing.
- Complete activities alligned with above grade level standards - Students learn concepts that are more complex, such as finding limits of infinite geometric series.
- Utilize problem-based learning for greater depth of knowledge - Teacher introduces topics to students as part of a project, such as creating creating real-world statistical studies.
- Allow students to work at a faster pace - Teacher provides resources for students to move ahead if they are able to demonstrate mastery of learning objectives at a faster pace.

- Above grade level placement option for qualified students
- Advanced problem-solving
- Allow students to work at a faster pace
- Cluster grouping
- Complete activities aligned with above grade level text using Benchmark results
- Create a plan to solve an issue presented in the class or in a text
- Flexible skill grouping within a class or across grade level for rigor
- Higher order, critical & creative thinking skills, and discovery
- Multi-disciplinary unit and/or project
- Teacher-selected instructional strategies that are focused to provide challenge, engagement, and growth opportunities
- Utilize exploratory connections to higher-grade concepts
- Utilize project-based learning for greater depth of knowledge

Sample Lesson

Using the template below, please develop a **Sample Lesson** for the first unit only.

Unit Name:

NJSLS:

Interdisciplinary Connection:

Statement of Objective:

Anticipatory Set/Do Now:

Learning Activity:

Student Assessment/CFU's:

Materials:

21st Century Themes and Skills:

Differentiation/Modifications:

Integration of Technology: