Unit 4: Vectors, Polar Coordinates & Complex Numbers

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Title Section

Department of Curriculum and Instruction

Belleville Public Schools

Curriculum Guide

PRECALCULUS / TRIGONOMETRY, GRADES 10-12

UNIT 4: VECTORS, POLAR COORDINATES & COMPLEX NUMBERS

Belleville Board of Education

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Unit Overview

In this Unit...

- students will learn to use vectors to model and solve real-world situations. They will begin by learning the many different ways that vectors can be expressed and graphed. Then, they will perform operations on vectors and find dot products, projections and angles between them. They will learn how to graph vectors in three-dimensional space and use them to find area and volume of shapes.
- students will learn the relationship between rectangular and polar coordinates. They will begin by learning how to convert between the two types of coordinates and equations. Then, they will use polar coordinates and equations to represent different problems, including conic sections. They will also learn how to represent complex numbers with polar coordinates and how to perform a variety of operations in this form.

Enduring Understanding

- Use vectors to represent and solve real-world problems.
- Express and graph vectors in different forms.
- Find dot products, projections, and angles between vectors.
- Graph vectors in three-dimensional space.
- Use vectors to find area and volume of shapes.
- Understand the relationship between rectangular and polar coordinates.
- Convert between polar and rectangular coordinates and equations.

- Identify the polar equations of conic sections.
- Perform operations on complex numbers using polar coordinates.

Essential Questions

- What is a vector and what does it represent?
- What are the different ways that vectors can be written and graphed?
- How do you find the dot product and the angle between two vectors?
- What is the projection of a vector onto another vector?
- How can vectors be graphed in three-dimensional space?
- How can you use vectors to find area and volume?
- What are polar coordinates and what is their relationship to rectangular coordinates?
- How can you convert between polar and rectangular coordinates and equations?
- What do the polar equations of conic sections look like?
- How can you express and perform operations on complex numbers using polar coordinates?

Exit Skills

By the end of Unit 4, Precalculus / Trigonometry students should know:

- How to represent and operate with vectors both geometrically and algebraically.
- How to resolve vectors into their rectangular components.
- How to write a vector as the linear combination of unit vectors.
- How to find the dot product of two vectors and use the dot product to find the angle between them.
- How to find the projection of one vector onto another.
- How to graph and operate with vectors in space.
- How to find the dot and cross products of and angles between vectors in space.
- How to find areas of parallelograms and volumes of parallelepipeds in space.
- How to graph points with polar coordinates.
- How to graph polar equations.
- How to identify and graph classic polar curves.
- How to convert between polar and rectangular coordinates.
- How to convert between polar and rectangular equations.
- How to identify polar equations of conics.
- How to write and graph the polar equation of a conic given its eccentricity and the equation of its directrix.
- How to convert complex numbers from rectangular to polar form and vice versa.
- How to find products, quotients, powers, and roots of complex numbers in polar form.

New Jersey Student Learning Standards (NJSLS)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.N-CN.A.1	Know there is a complex number i such that i^2 = -1, and every complex number has the form $a + bi$ with a and b real.
MA.N-CN.A.2	Use the relation i^2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
MA.N-CN.A.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
MA.N-CN.B.4	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
MA.N-CN.B.5	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.
MA.N-CN.B.6	Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
MA.N-VM.A.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lvert\lvert\boldsymbol{v}\rvert\rvert$, $\boldsymbol{v}$).
MA.N-VM.A.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
MA.N-VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
MA.N-VM.B.4	Add and subtract vectors.
MA.N-VM.B.4b	Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
MA.N-VM.B.4c	Understand vector subtraction $\boldsymbol{v} - w$ as $\boldsymbol{v} + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
MA.N-VM.B.5	Multiply a vector by a scalar.
MA.N-VM.B.5a	Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(\boldsymbol{v}_x, \boldsymbol{v}$ subscript $y) = (c\boldsymbol{v}_x, c\boldsymbol{v}$ subscript $y)$.
MA.N-VM.B.5b	Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\lvert\lvert c\boldsymbol{v}\rvert\rvert = \lvert c\rvert\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\lvert c\rvert\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).
MA.A-REI.C.8	Represent a system of linear equations as a single matrix equation in a vector variable.

Interdisciplinary Connections

9-12.HS-ETS1-4.4.1	Models (e.g., physical, mathematical, computer models) can be used to simulate systems and interactions— including energy, matter, and information flows— within and between systems at different scales.
9-12.HS-ETS1-4.5	Using Mathematics and Computational Thinking
9-12.HS-ETS1-4.5.1	Use mathematical models and/or computer simulations to predict the effects of a design solution on systems and/or the interactions between systems.
9-12.HS-PS1-8.2.1	Develop a model based on evidence to illustrate the relationships between systems or between components of a system.

Learning Objectives

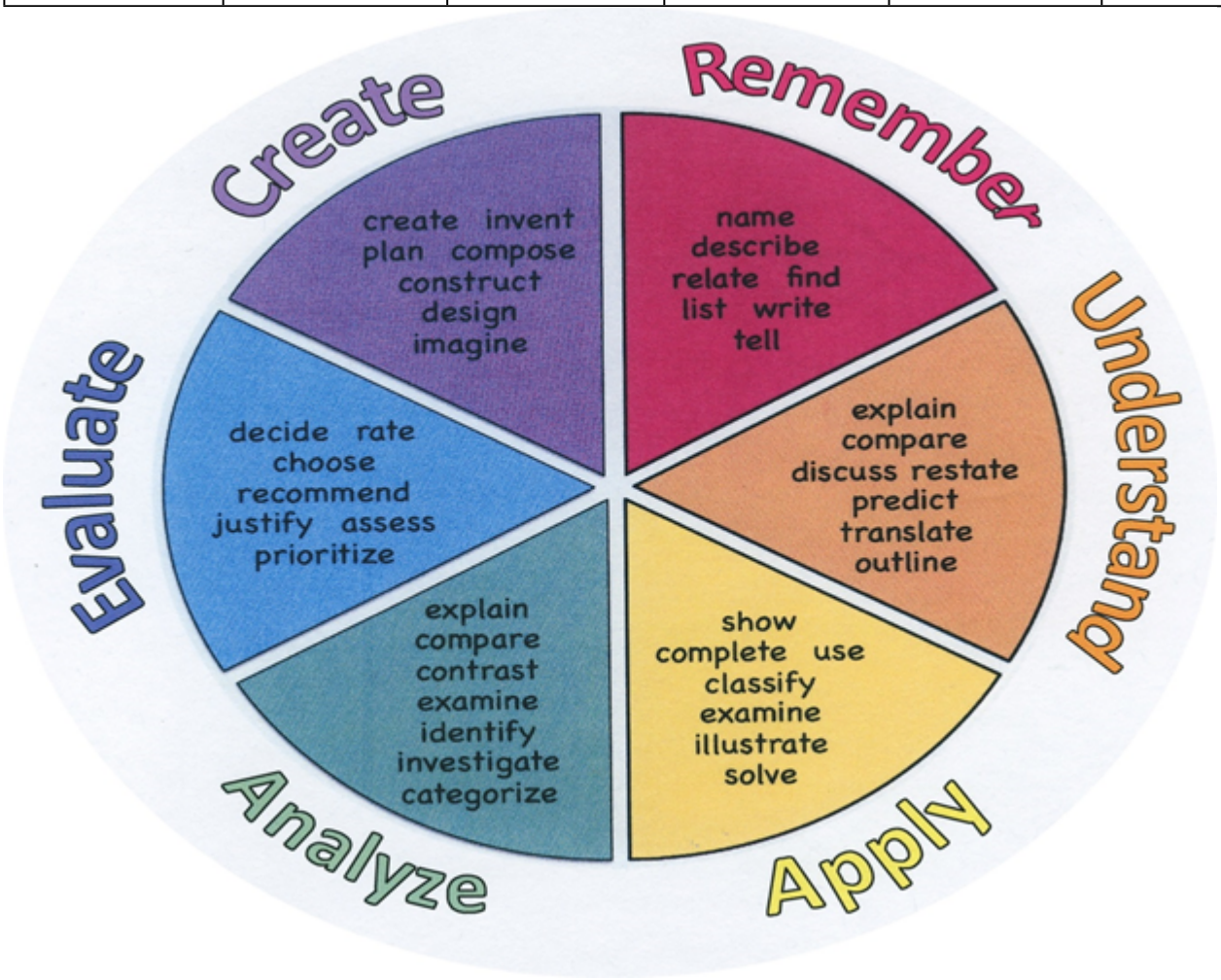
Students will be able to:

- Represent and operate with vectors geometrically.
- Solve vector problems, and resolve vectors into their rectangular components.
- Represent and operate with vectors in the coordinate plane.
- Write a vector as a linear combination of unit vectors.
- Find the dot product of two vectors, and use the dot product to find the angle between them.
- Find the projection of one vector onto another.
- Plot points and vectors in the three-dimensional coordinate system.
- Express algebraically and operate with vectors in space.
- Find dot products of and angles between vectors in space.
- Find cross products of vectors in space, and use cross products to find area and volume.
- Graph points with polar coordinates.
- Graph simple polar equations.
- Graph polar equations.
- Identify and graph classical curves.
- Convert between polar and rectangular coordinates.
- Convert between polar and rectangular equations.
- Identify polar equations of conics.
- Write and graph the polar equation of a conic given its eccentricity and the equation of its directrix.
- Convert complex numbers from rectangular to polar form and vice versa.
- Find products, quotients, powers, and roots of complex numbers in polar form.

Action Verbs: Below are examples of action verbs associated with each level of the Revised Bloom's Taxonomy.

Remember	Understand	Apply	Analyze	Evaluate	Create
Choose	Classify	Choose	Categorize	Appraise	Combine
Describe	Defend	Dramatize	Classify	Judge	Compose
Define	Demonstrate	Explain	Compare	Criticize	Construct
Label	Distinguish	Generalize	Differentiate	Defend	Design
List	Explain	Judge	Distinguish	Compare	Develop
Locate	Express	Organize	Identify	Assess	Formulate
Match	Extend	Paint	Infer	Conclude	Hypothesize
Memorize	Give Examples	Prepare	Point out	Contrast	Invent
Name	Illustrate	Produce	Select	Critique	Make
Omit	Indicate	Select	Subdivide	Determine	Originate
Recite	Interrelate	Show	Survey	Grade	Organize
Select	Interpret	Sketch	Arrange	Justify	Plan
State	Infer	Solve	Breakdown	Measure	Produce
Count	Match	Use	Combine	Rank	Role Play

Draw Outline Point Quote Recall Recognize Repeat Reproduce	Paraphrase Represent Restate Rewrite Select Show Summarize Tell Translate Associate Compute Convert Discuss Estimate Extrapolate Generalize Predict	Add Calculate Change Classify Complete Compute Discover Divide Examine Graph Interpolate Manipulate Modify Operate Subtract	Detect Diagram Discriminate Illustrate Outline Point out Separate	Rate Support Test	Drive Devise Generate Integrate Prescribe Propose Reconstruct Revise Rewrite Transform



Suggested Activities & Best Practices

- Online textbook practice problems, study guides, and worksheets
- Higher-order thinking tasks, such as Illustrative Mathematics task "Complex Distance"
- Practice activities from ALEKS, KUTA Software, Khan Academy, etc., such as "Vector Addition and Scalar Multiplication"

Assessment Evidence - Checking for Understanding (CFU)

- Regular Exit Tickets to assess individual learning objectives (Formative)
- Quizzes to assess groups of learning objectives - at least one quiz for each chapter (Chapters 8 and 9) (Summative)
- Chapter Tests given at least once per chapter - at least 2 tests in this unit (Chapters 8 and 9) (Summative)
- Common Quarterly/Benchmark Exams - Quarter 4 Exam for this unit (Benchmark)
- Web-Based Assessments (using Google Forms, ALEKS, Edulastic, Khan Academy, etc.) (Formative/Summative)

- Admit Tickets
- Anticipation Guide
- Common Benchmarks
- Compare & Contrast
- Create a Multimedia Poster
- Define
- Describe
- Evaluate
- Evaluation rubrics
- Exit Tickets
- Explaining
- Fist- to-Five or Thumb-Ometer
- Illustration
- KWL Chart
- Learning Center Activities
- Newspaper Headline
- Outline
- Question Stems
- Quickwrite
- Quizzes
- Red Light, Green Light
- Self- assessments
- Socratic Seminar
- Study Guide
- Surveys
- Teacher Observation Checklist

- Think, Pair, Share
- Think, Write, Pair, Share
- Top 10 List
- Unit review/Test prep
- Unit tests
- Web-Based Assessments

Primary Resources & Materials

- Glencoe McGraw-Hill Precalculus 2014
- Practice Glencoe Precalculus
- Study Guide Glencoe Precalculus
- connected.mcgraw-hill.com

Ancillary Resources

- Glencoe McGraw-Hill Algebra 2 2014
- ALEKS
- Kuta Software

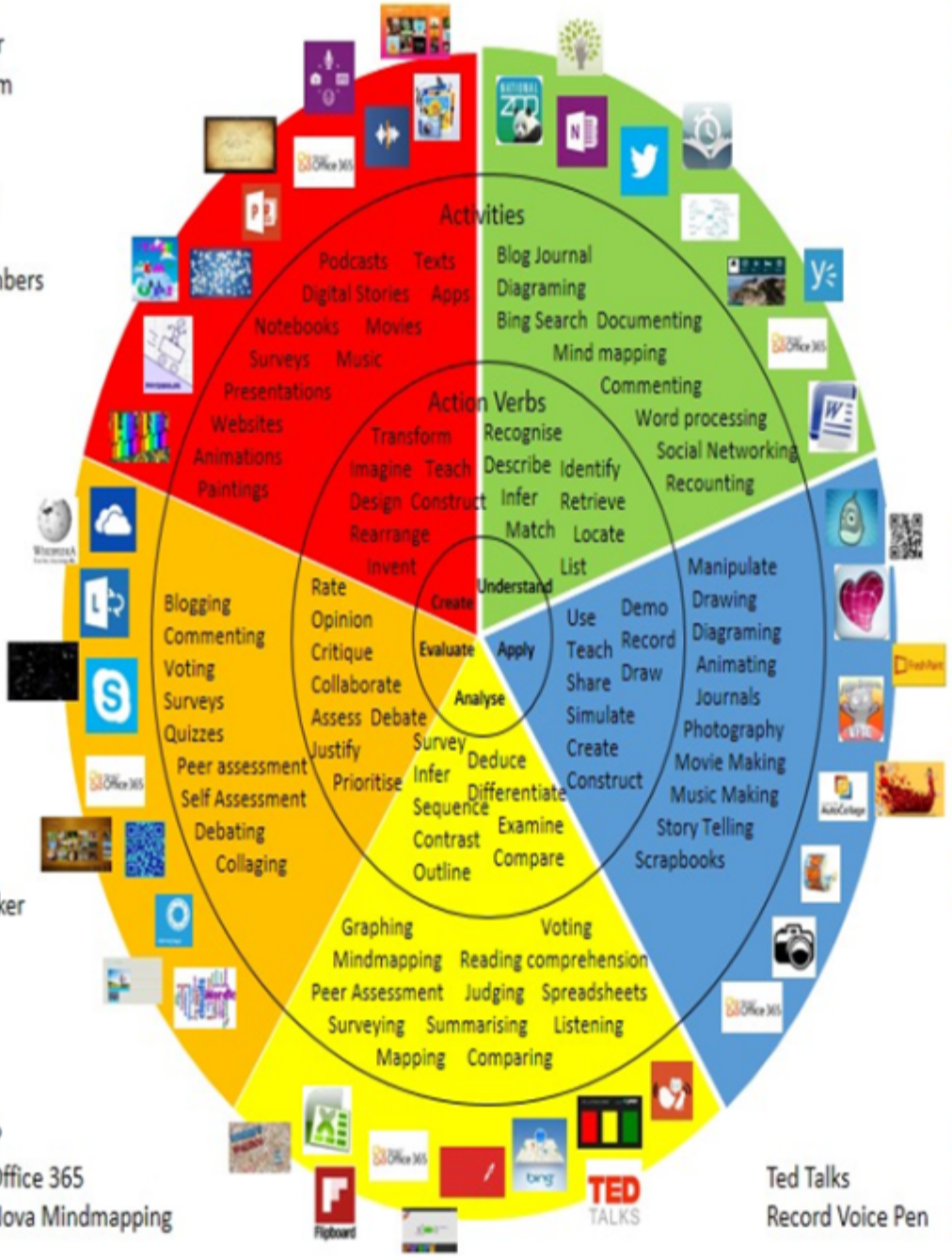
Technology Infusion

- Smart TV - Display and interact with lessons and activities
- Chromebooks - students access activities, slides, and practice problems
- Google Classroom - Slides, Forms, Drive, etc.
- ALEKS - Students practice individual learning objectives such as "Converting rectangular coordinates to polar coordinates"
- Desmos - Students interact with classroom activities or use graphing software to graph and analyze functions
- YouTube - Students watch videos to deepen understanding of specific concepts throughout the unit
- Khan Academy - Students practice individual learning objectives, such as "Combined vector operations"
- Calculator/Graphing calculator - Students perform calculations or graph and analyze functions
- Edulastic - Students complete assessments and checks for understanding
- KUTA Software - Teacher generates a variety of assessments and practice problems for individual learning objectives or groups of learning objectives
- Pear Deck - Teacher presents information through an interactive slide show presentation

Win 8.1 Apps/Tools Pedagogy Wheel

Podcasts
Photostory 3
Kid Story Builder
Music Maker Jam
Paint A Story
Office 365
MS PowerPoint
Stack 'Em Up
NqSquared Numbers
Physamajig
Xylophone 8

Wikipedia
Skydrive
Lync
SkyMap
Skype
Office 365
Puzzle Touch
Easy QR
Memorylage
Life Moments
Word Cloud Maker



Where's Waldo?
MS Excel Office 365
Flipboard Nova Mindmapping

Ted Talks
Record Voice Pen

Originally taken from http://www.coetail.com/vzimmer/files/2013/02/iPadagogy-Wheel.001.jpg
And adapted for Windows 8.1 devices by Charlotte Beckhurst @CharBeckhurst

Alignment to 21st Century Skills & Technology

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP2.1	Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP4.1	Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others' time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP6.1	Career-ready individuals regularly think of ideas that solve problems in new and different ways, and they contribute those ideas in a useful and productive manner to improve their organization. They can consider unconventional ideas and suggestions as solutions to issues, tasks or problems, and they discern which ideas and suggestions will add greatest value. They seek new methods, practices, and ideas from a variety of sources and seek to apply those ideas to their own workplace. They take action on their ideas and understand how to bring innovation to an organization.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP8.1	Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP11.1	Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.
TECH.8.1.12	Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

TECH.8.1.12.A	Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems and operations.
TECH.8.1.12.A.3	Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.
TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.A.CS2	Select and use applications effectively and productively.

21st Century Skills/Interdisciplinary Themes

- Communication and Collaboration
- Creativity and Innovation
- Critical thinking and Problem Solving
- ICT (Information, Communications and Technology) Literacy
- Information Literacy
- Life and Career Skills
- Media Literacy

21st Century Skills

- Civic Literacy
- Financial, Economic, Business and Entrepreneurial Literacy
- Global Awareness

Differentiation

- Small group instruction - Teacher utilizes small groups to remediate or enrich specific topics with different groups of students, as necessary.
- Study guides - Teacher provides students with study guides prior to quizzes and tests.
- Problem-based learning - Teacher introduces topics to students as part of a project, such as using vectors to model velocity.
- Open-ended activities - Students complete activities with multiple entry points and more than one possible solution.

Differentiations:

- Small group instruction
- Small group assignments
- Extra time to complete assignments
- Pairing oral instruction with visuals
- Repeat directions

- Use manipulatives
- Center-based instruction
- Study guides
- Scheduled breaks
- Rephrase written directions
- Multisensory approaches
- Additional time
- Preview vocabulary
- Preview content & concepts
- Behavior management plan
- Highlight text
- Student(s) work with assigned partner
- Visual presentation
- Assistive technology
- Auditory presentations
- Small group setting

Hi-Prep Differentiations:

- Alternative formative and summative assessments
- Choice boards
- Games and tournaments
- Group investigations
- Learning contracts
- Multiple intelligence options
- Personal agendas
- Project-based learning
- Problem-based learning
- Stations/centers
- Tiered activities/assignments
- Tiered products
- Varying organizers for instructions

Lo-Prep Differentiations

- Exploration by interest
- Flexible grouping
- Goal setting with students
- Jigsaw
- Mini workshops to re-teach or extend skills
- Open-ended activities
- Think-Pair-Share
- Varied supplemental materials

Special Education Learning (IEP's & 504's)

- Provide modifications as dictated in student's IEP/504 - Teacher modifies tests/assesments as necessary.
- Additional time for skill mastery - Teacher allows students additional time to master particular learning objectives.
- Center-Based Instruction - Teacher utilizes different sets of stations/centers in order to differentiate and provide students with varied learning settings.
- Modify assignments/tests - Teacher modifies tests/assesments by, for example, writing multi-part answers for questions that require students to complete multiple steps.
- Utilize computers or electronic devices - Teacher uses chromebooks and smart TV to provide students with visualizations of graphs/models and allow students to interact with them them.
- Extended time on tests/quizzes - Teacher allows students to have extended time on tests/quizzes as dictated by their IEP/504.
- Use of calculator on tests/quizzes - Students are allowed to use calculators on tests/quizzes.
- Use of study guide, reference sheets, or notes on tests/quizzes - Teacher allows students to use reference sheets or study guides on tests/quizes that contain information such as rules for converting between polar and rectangular coordinates.

- printed copy of board work/notes provided
- additional time for skill mastery
- assistive technology
- behavior management plan
- Center-Based Instruction
- check work frequently for understanding
- computer or electronic device utilizes
- extended time on tests/ quizzes
- have student repeat directions to check for understanding
- highlighted text visual presentation
- modified assignment format
- modified test content
- modified test format
- modified test length
- multi-sensory presentation
- multiple test sessions
- preferential seating
- preview of content, concepts, and vocabulary
- Provide modifications as dictated in the student's IEP/504 plan
- secure attention before giving instruction/directions
- shortened assignments
- student working with an assigned partner
- teacher initiated weekly assignment sheet
- Use open book, study guides, test prototypes

English Language Learning (ELL)

- Using videos, illustrations, pictures and drawings to explain or clarify - Teacher provides tools such as visualizations of

graphs that students can interact with.

- Eliminate nonessential information - Teacher explains concepts using only the vocabulary that is essential to understand a concept.
- Tutoring by peers - Teacher allows peers to explain concepts to ELL students.
- Allow students to correct errors - Teacher allows students to gain back points by correcting their errors on a test/quiz.
- Modify assignments/tests - Teacher modifies tests/assesments by, for example, writing multi-part answers for questions that require students to complete multiple steps.
- Use of study guide, reference sheets, or notes on tests/quizzes - Teacher allows students to use reference sheets or study guides on tests/quizes that contain information such as rules for converting between rectangular and polar coordinates.

- teaching key aspects of a topic. Eliminate nonessential information
- using videos, illustrations, pictures, and drawings to explain or clarif
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning;
- allowing students to correct errors (looking for understanding)
- allowing the use of note cards or open-book during testing
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- modifying tests to reflect selected objectives
- providing study guides
- reducing the number of answer choices on a multiple choice test
- tutoring by peers

At Risk

- Decrease the amount of work presented or required - Teacher allows students to submit less work, for example, only complete 3 out of the 5 practice problems for a specific learning objective.
- Using videos, illustrations, pictures, and drawings to explain or clarify - Teacher provides tools such as visualizations of graphs that students can interact with.
- Tutoring by peers - Teacher allows peers to explain concepts to at risk students.
- Providing study guides - Teacher provides students with study guides prior to quizzes and tests. For example, a variety of practice problems related to the topics being assessed.
- Allowing students to correct errors - Teacher allows students to gain back points by correcting their errors on a test/quiz.
- Allowing students to select from given choices - Teacher gives students a choice of activities to complete, such as draw a graph, create an equation, or write a sentence to model a situation.
- Allowing the use of notes, study guides, or reference sheets on tests/quizzes - Teacher allows students to use reference sheets or study guides on tests/quizes that contain information such as rules for converting between polar and rectangular coordinates.

- allowing students to correct errors (looking for understanding)
- teaching key aspects of a topic. Eliminate nonessential information
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning
- allowing students to select from given choices
- allowing the use of note cards or open-book during testing
- collaborating (general education teacher and specialist) to modify vocabulary, omit or modify items to

reflect objectives for the student, eliminate sections of the test, and determine how the grade will be determined prior to giving the test.

- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- marking students' correct and acceptable work, not the mistakes
- modifying tests to reflect selected objectives
- providing study guides
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using authentic assessments with real-life problem-solving
- using videos, illustrations, pictures, and drawings to explain or clarify

Talented and Gifted Learning (T&G)

- Create a plan to solve an issue presented in the class - Teacher allows students to use their understanding of functions to model a specific problem of their choosing.
- Complete activities alligned with above grade level standards - Students solve problems that are more complex, such as converting an equation for a conic section into polar form.
- Utilize problem-based learning for greater depth of knowledge - Teacher introduces topics to students as part of a project, such as using vectors to model velocity.
- Allow students to work at a faster pace - Teacher provides resources for students to move ahead if they are able to demonstrate mastery of learning objectives at a faster pace.

- Above grade level placement option for qualified students
- Advanced problem-solving
- Allow students to work at a faster pace
- Cluster grouping
- Complete activities aligned with above grade level text using Benchmark results
- Create a plan to solve an issue presented in the class or in a text
- Flexible skill grouping within a class or across grade level for rigor
- Higher order, critical & creative thinking skills, and discovery
- Multi-disciplinary unit and/or project
- Teacher-selected instructional strategies that are focused to provide challenge, engagement, and growth opportunities
- Utilize exploratory connections to higher-grade concepts
- Utilize project-based learning for greater depth of knowledge

Sample Lesson

Using the template below, please develop a **Sample Lesson** for the first unit only.

Unit Name:

NJSLS:

Interdisciplinary Connection:

Statement of Objective:

Anticipatory Set/Do Now:

Learning Activity:

Student Assessment/CFU's:

Materials:

21st Century Themes and Skills:

Differentiation/Modifications:

Integration of Technology: