Unit 3: Trigonometry

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Department of Curriculum and Instruction

Belleville Public Schools

Curriculum Guide

PRECALCULUS / TRIGONOMETRY, GRADES 10-12

UNIT 3: TRIGONOMETRY

Belleville Board of Education

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Unit Overview

In this Unit...

- students will learn to use the trigonometric functions to model and solve real-world situations. They will begin by using the trigonometric functions to solve for the missing sides and angles of right triangles. Then, they will determine the relationship between degree and radian measure and how to convert between the two. They will use the unit circle, reference angles, and coterminal angles to find values of the trigonometric functions of any angle. They will also learn to graph the trigonometric functions and their inverse functions, as well as transformations of all of these functions. Then they will use properties and laws involving the trigonometric functions to solve oblique triangles and find the area of oblique triangles.
- students will learn the different trigonometric identities. First, they will verify the trigonometric identities. Then, they will use these identities to simplify and solve a variety of trigonometric equations.

Enduring Understanding

- Use trigonometric functions to model real-world situations.
- Find missing sides and angles of right triangles using the trigonometric functions.
- Understand the relationship between degrees and radians.
- Convert degrees to radians and vice versa.
- Use reference angles, coterminal angles, and the unit circle to find values of the trigonometric functions for any angle.
- Identify the important characteristics of the graphs of the trigonometric functions and their inverses.
- Transform the graphs of the trigonometric functions and express these transformations in the equations of the functions.

- Verify the trigonometric identities and use them to simplify and solve equations.

Essential Questions

- What are the trigonometric functions and what do they represent?
- How can you find missing sides and angles of right triangles using the trigonometric functions?
- What is the relationship between degrees and radians?
- How can you convert degrees to radians and vice versa?
- How can the unit circle be used to find trigonometric values of any angle?
- What are coterminal and reference angles?
- In what ways can a trigonometric function be used to represent a real-world situation?
- What are the important characteristics of the graphs of the trigonometric functions?
- How are the graphs of the trigonometric functions affected by transformations?
- How are the graphs of the trigonometric functions related to the graphs of their inverse functions?
- What are some of the different trigonometric identities?
- What are some of the ways that the trigonometric identities can be used to simplify and solve equations?

Exit Skills

By the end of Unit 3, Precalculus / Trigonometry students should know:

- How to solve right triangles using trigonometric and inverse trigonometric functions.
- How to convert between degrees and radians.
- How to find the value of trigonometric functions for any angle on the unit circle.
- How to solve real-world problems using trigonometric functions.
- How to graph trigonometric functions and their inverses.
- How to solve oblique triangles and find their area using various laws and formulas.
- How to identify and use trigonometric identities to find trigonometric values.
- How to use trigonometric identities to simplify and rewrite trigonometric expressions.
- How to verify trigonometric identities.
- How to solve trigonometric equations.
- How to use sum and difference identities to evaluate trigonometric functions.
- How to use double-angle, power-reducing, half-angle, and product-to-sum identities to evaluate trigonometric expressions and solve trigonometric equations.

New Jersey Student Learning Standards (NJSLS)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.

MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.K-12.5	Use appropriate tools strategically.
MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.G-SRT.C.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
MA.G-SRT.C.7	Explain and use the relationship between the sine and cosine of complementary angles.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
MA.G-SRT.D.11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
MA.F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
MA.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π – x, π + x, and 2π – x in terms of their values for x, where x is any real number.
MA.F-TF.A.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
MA.F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
MA.F-TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
MA.F-TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
MA.F-TF.C.8	Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ given $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle.
MA.F-TF.C.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Interdisciplinary Connections

9-12.HS-ETS1-4.4.1	Models (e.g., physical, mathematical, computer models) can be used to simulate systems and interactions— including energy, matter, and information flows— within and between systems at different scales.
9-12.HS-ETS1-4.5	Using Mathematics and Computational Thinking
9-12.HS-ETS1-4.5.1	Use mathematical models and/or computer simulations to predict the effects of a design solution on systems and/or the interactions between systems.
9-12.HS-PS1-8.2.1	Develop a model based on evidence to illustrate the relationships between systems or between components of a system.

Learning Objectives

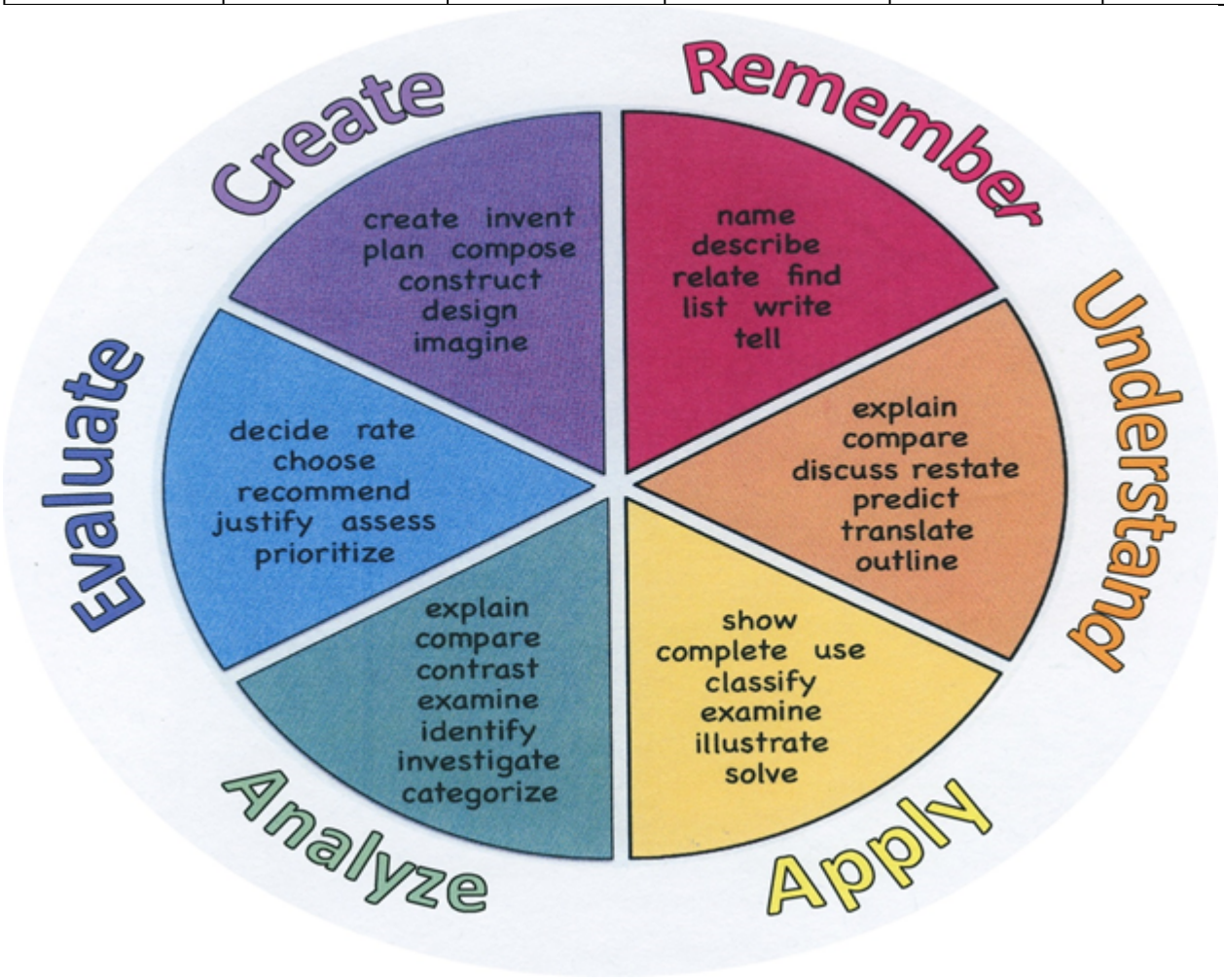
Students will be able to:

- Find values of trigonometric functions for acute angles of right triangles.
- Use trigonometric functions to solve right triangles.
- Convert degree measures of angles to radian measures, and vice versa.
- Identify coterminal angles.
- Find arc length and area of a sector.
- Use angle measures to solve real-world problems.
- Find values of trigonometric functions for any angle using the unit circle.
- Graph transformations of the sine and cosine functions.
- Use sinusoidal functions to solve problems.
- Graph tangent and reciprocal trigonometric functions.
- Graph damped trigonometric functions.
- Evaluate and graph inverse trigonometric functions.
- Find compositions of trigonometric functions.
- Solve oblique triangles by using the Law of Sines or the Law of Cosines.
- Find areas of oblique triangles.
- Identify and use basic trigonometric identities to find trigonometric values.
- Use basic trigonometric identities to simplify and rewrite trigonometric expressions.
- Verify trigonometric identities.
- Determine whether equations are identities.
- Solve trigonometric equations using algebraic techniques.
- Solve trigonometric equations using basic identities.
- Use sum and difference identities to evaluate trigonometric functions.
- Use sum and difference identities to solve trigonometric equations.
- Use double-angle, power-reducing, and half-angle identities to evaluate trigonometric expressions and solve trigonometric equations.
- Use product-to-sum identities to evaluate trigonometric expressions and solve trigonometric equations.

Action Verbs: Below are examples of action verbs associated with each level of the Revised Bloom's Taxonomy.

Remember	Understand	Apply	Analyze	Evaluate	Create
Choose	Classify	Choose	Categorize	Appraise	Combine
Describe	Defend	Dramatize	Classify	Judge	Compose
Define	Demonstrate	Explain	Compare	Criticize	Construct
Label	Distinguish	Generalize	Differentiate	Defend	Design
List	Explain	Judge	Distinguish	Compare	Develop
Locate	Express	Organize	Identify	Assess	Formulate
Match	Extend	Paint	Infer	Conclude	Hypothesize

Memorize Name Omit Recite Select State Count Draw Outline Point Quote Recall Recognize Repeat Reproduce	Give Examples Illustrate Indicate Interrelate Interpret Infer Match Paraphrase Represent Restate Rewrite Select Show Summarize Tell Translate Associate Compute Convert Discuss Estimate Extrapolate Generalize Predict	Prepare Produce Select Show Sketch Solve Use Add Calculate Change Classify Complete Compute Discover Divide Examine Graph Interpolate Manipulate Modify Operate Subtract	Point out Select Subdivide Survey Arrange Breakdown Combine Detect Diagram Discriminate Illustrate Outline Point out Separate	Contrast Critique Determine Grade Justify Measure Rank Rate Support Test	Invent Make Originate Organize Plan Produce Role Play Drive Devise Generate Integrate Prescribe Propose Reconstruct Revise Rewrite Transform



Suggested Activities & Best Practices

- Online textbook practice problems, study guides, and worksheets
- Desmos Classroom Activities, such as "Graphing the Sine Function using Amplitude, Period, and Vertical Translation" activity
- Higher-order thinking tasks, such as Illustrative Mathematics task "Modeling the Carousel Motion"
- Practice activities from ALEKS, KUTA Software, Khan Academy, etc., such as "Inverse Trigonometric Functions"

Assessment Evidence - Checking for Understanding (CFU)

- Regular Exit Tickets to assess individual learning objectives (Formative)
- Quizzes to assess groups of learning objectives - at least one quiz for each chapter (Chapters 4 and 5) (Summative)
- Chapter Tests given at least once per chapter - at least 2 tests in this unit (Chapters 4 and 5) (Summative)
- Common Quarterly/Benchmark Exams - Quarter 3 Exam for this unit (Benchmark)
- Web-Based Assessments (using Google Forms, ALEKS, Edulastic, Khan Academy, etc.) (Formative/Summative)

- Admit Tickets
- Anticipation Guide
- Common Benchmarks
- Compare & Contrast
- Create a Multimedia Poster
- Define
- Describe
- Evaluate
- Evaluation rubrics
- Exit Tickets
- Explaining
- Fist- to-Five or Thumb-Ometer
- Illustration
- KWL Chart
- Learning Center Activities
- Newspaper Headline
- Outline
- Question Stems
- Quickwrite
- Quizzes
- Red Light, Green Light
- Self- assessments

- Socratic Seminar
- Study Guide
- Surveys
- Teacher Observation Checklist
- Think, Pair, Share
- Think, Write, Pair, Share
- Top 10 List
- Unit review/Test prep
- Unit tests
- Web-Based Assessments

Primary Resources & Materials

- Glencoe McGraw-Hill Precalculus 2014
- Practice Glencoe Precalculus
- Study Guide Glencoe Precalculus
- connected.mcgraw-hill.com

Ancillary Resources

- Glencoe McGraw-Hill Algebra 2 2014
- ALEKS
- Kuta Software

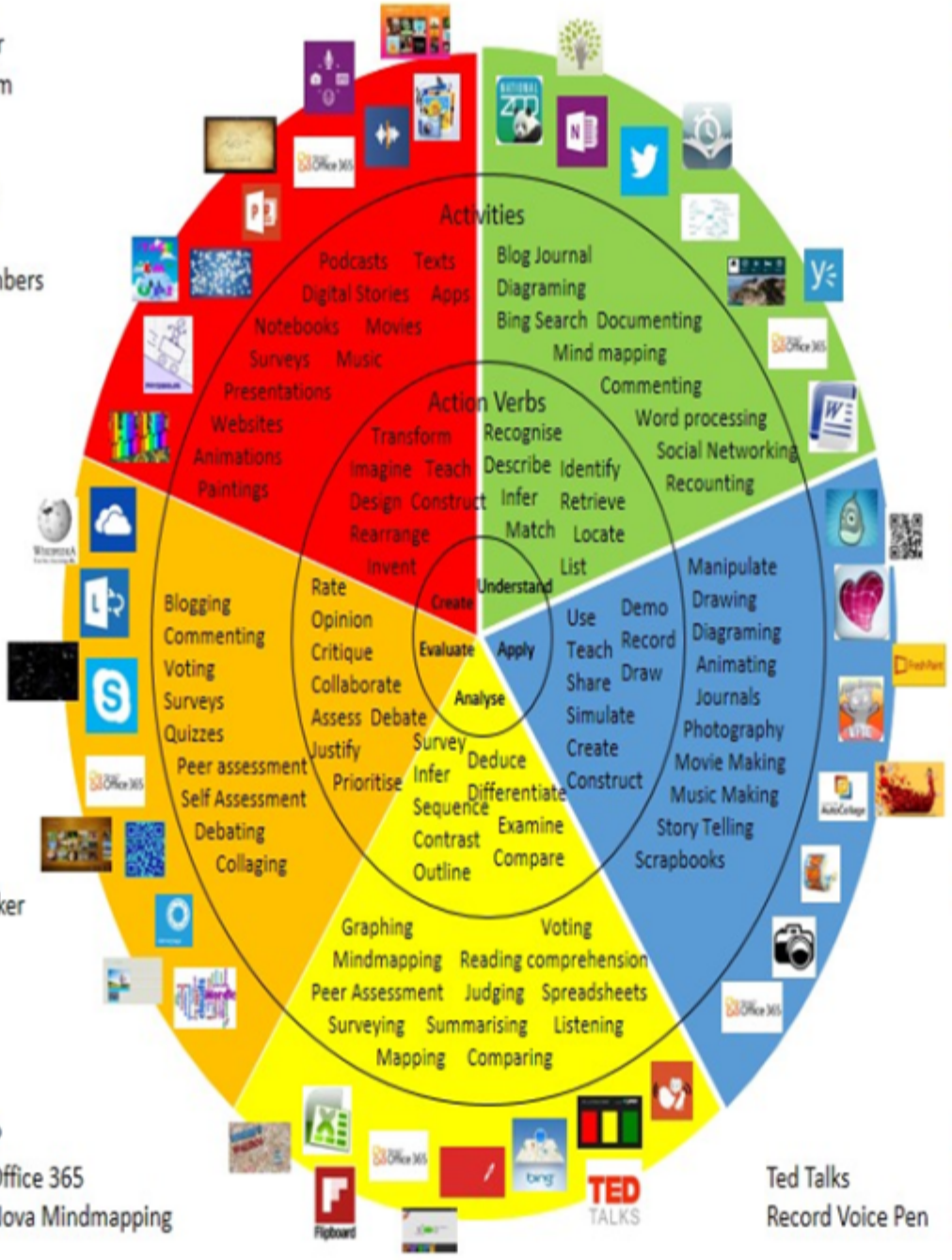
Technology Infusion

- Smart TV - Display and interact with lessons and activities
- Chromebooks - students access activities, slides, and practice problems
- Google Classroom - Slides, Forms, Drive, etc.
- ALEKS - Students practice individual learning objectives such as "Converting between degree and radian mode"
- Desmos - Students interact with classroom activities or use graphing software to graph and analyze functions
- YouTube - Students watch videos to deepen understanding of specific concepts throughout the unit
- Khan Academy - Students practice individual learning objectives, such as "Finding trig values using trig identities"
- Calculator/Graphing calculator - Students perform calculations or graph and analyze functions
- Edulastic - Students complete assessments and checks for understanding
- KUTA Software - Teacher generates a variety of assessments and practice problems for individual learning objectives or groups of learning objectives
- Pear Deck - Teacher presents information through an interactive slide show presentation

Win 8.1 Apps/Tools Pedagogy Wheel

Podcasts
Photostory 3
Kid Story Builder
Music Maker Jam
Paint A Story
Office 365
MS PowerPoint
Stack 'Em Up
NqSquared Numbers
Physamajig
Xylophone 8

Wikipedia
Skydrive
Lync
SkyMap
Skype
Office 365
Puzzle Touch
Easy QR
Memorylage
Life Moments
Word Cloud Maker



Where's Waldo?
MS Excel Office 365
Flipboard Nova Mindmapping

Ted Talks
Record Voice Pen

Originally taken from http://www.coetail.com/vzimmer/files/2013/02/iPadagogy-Wheel.001.jpg
And adapted for Windows 8.1 devices by Charlotte Beckhurst @CharBeckhurst

Alignment to 21st Century Skills & Technology

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP2.1	Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP4.1	Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others' time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP6.1	Career-ready individuals regularly think of ideas that solve problems in new and different ways, and they contribute those ideas in a useful and productive manner to improve their organization. They can consider unconventional ideas and suggestions as solutions to issues, tasks or problems, and they discern which ideas and suggestions will add greatest value. They seek new methods, practices, and ideas from a variety of sources and seek to apply those ideas to their own workplace. They take action on their ideas and understand how to bring innovation to an organization.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP8.1	Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP11.1	Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.
TECH.8.1.12	Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

TECH.8.1.12.A	Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems and operations.
TECH.8.1.12.A.3	Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.
TECH.8.1.12.A.CS1	Understand and use technology systems.

21st Century Skills/Interdisciplinary Themes

- Communication and Collaboration
- Creativity and Innovation
- Critical thinking and Problem Solving
- ICT (Information, Communications and Technology) Literacy
- Information Literacy
- Life and Career Skills
- Media Literacy

21st Century Skills

- Civic Literacy
- Financial, Economic, Business and Entrepreneurial Literacy
- Global Awareness

Differentiation

- Small group instruction - Teacher utilizes small groups to remediate or enrich specific topics with different groups of students, as necessary.
- Study guides - Teacher provides students with study guides prior to quizzes and tests.
- Problem-based learning - Teacher introduces topics to students as part of a project, such as creating functions to model circular motion.
- Open-ended activities - Students complete activities with multiple entry points and more than one possible solution.

Differentiations:

- Small group instruction
- Small group assignments
- Extra time to complete assignments
- Pairing oral instruction with visuals
- Repeat directions

- Use manipulatives
- Center-based instruction
- Study guides
- Scheduled breaks
- Rephrase written directions
- Multisensory approaches
- Additional time
- Preview vocabulary
- Preview content & concepts
- Behavior management plan
- Highlight text
- Student(s) work with assigned partner
- Visual presentation
- Assistive technology
- Auditory presentations
- Small group setting

Hi-Prep Differentiations:

- Alternative formative and summative assessments
- Choice boards
- Games and tournaments
- Group investigations
- Learning contracts
- Multiple intelligence options
- Personal agendas
- Project-based learning
- Problem-based learning
- Stations/centers
- Tiered activities/assignments
- Tiered products
- Varying organizers for instructions

Lo-Prep Differentiations

- Exploration by interest
- Flexible grouping
- Goal setting with students
- Jigsaw
- Mini workshops to re-teach or extend skills
- Open-ended activities
- Think-Pair-Share
- Varied supplemental materials

Special Education Learning (IEP's & 504's)

- Provide modifications as dictated in student's IEP/504 - Teacher modifies tests/assesments as necessary.
- Additional time for skill mastery - Teacher allows students additional time to master particular learning objectives.
- Center-Based Instruction - Teacher utilizes different sets of stations/centers in order to differentiate and provide students with varied learning settings.
- Modify assignments/tests - Teacher modifies tests/assesments by, for example, writing multi-part answers for questions that require students to complete multiple steps.
- Utilize computers or electronic devices - Teacher uses chromebooks and smart TV to provide students with visualizations of graphs/models and allow students to interact with them them.
- Extended time on tests/quizzes - Teacher allows students to have extended time on tests/quizzes as dictated by their IEP/504.
- Use of calculator on tests/quizzes - Students are allowed to use calculators on tests/quizzes.
- Use of study guide, reference sheets, or notes on tests/quizzes - Teacher allows students to use reference sheets or study guides on tests/quizes that contain information such as a list of trig identities.

- printed copy of board work/notes provided
- additional time for skill mastery
- assistive technology
- behavior management plan
- Center-Based Instruction
- check work frequently for understanding
- computer or electronic device utilizes
- extended time on tests/ quizzes
- have student repeat directions to check for understanding
- highlighted text visual presentation
- modified assignment format
- modified test content
- modified test format
- modified test length
- multi-sensory presentation
- multiple test sessions
- preferential seating
- preview of content, concepts, and vocabulary
- Provide modifications as dictated in the student's IEP/504 plan
- secure attention before giving instruction/directions
- shortened assignments
- student working with an assigned partner
- teacher initiated weekly assignment sheet
- Use open book, study guides, test prototypes

English Language Learning (ELL)

- Using videos, illustrations, pictures and drawings to explain or clarify - Teacher provides tools such as visualizations of

graphs that students can interact with.

- Eliminate nonessential information - Teacher explains concepts using only the vocabulary that is essential to understand a concept.
- Tutoring by peers - Teacher allows peers to explain concepts to ELL students.
- Allow students to correct errors - Teacher allows students to gain back points by correcting their errors on a test/quiz.
- Modify assignments/tests - Teacher modifies tests/assesments by, for example, writing multi-part answers for questions that require students to complete multiple steps.
- Use of study guide, reference sheets, or notes on tests/quizzes - Teacher allows students to use reference sheets or study guides on tests/quizes that contain information such as a list of trig identities.

- teaching key aspects of a topic. Eliminate nonessential information
- using videos, illustrations, pictures, and drawings to explain or clarif
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning;
- allowing students to correct errors (looking for understanding)
- allowing the use of note cards or open-book during testing
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- modifying tests to reflect selected objectives
- providing study guides
- reducing the number of answer choices on a multiple choice test
- tutoring by peers

At Risk

- Decrease the amount of work presented or required - Teacher allows students to submit less work, for example, only complete 3 out of the 5 practice problems for a specific learning objective.
- Using videos, illustrations, pictures, and drawings to explain or clarify - Teacher provides tools such as visualizations of graphs that students can interact with.
- Tutoring by peers - Teacher allows peers to explain concepts to at risk students.
- Providing study guides - Teacher provides students with study guides prior to quizzes and tests. For example, a variety of practice problems related to the topics being assessed.
- Allowing students to correct errors - Teacher allows students to gain back points by correcting their errors on a test/quiz.
- Allowing students to select from given choices - Teacher gives students a choice of activities to complete, such as draw a graph, create an equation, or write a sentence to model a situation.
- Allowing the use of notes, study guides, or reference sheets on tests/quizzes - Teacher allows students to use reference sheets or study guides on tests/quizes that contain information such as a list of trig identities.

- allowing students to correct errors (looking for understanding)
- teaching key aspects of a topic. Eliminate nonessential information
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning
- allowing students to select from given choices
- allowing the use of note cards or open-book during testing
- collaborating (general education teacher and specialist) to modify vocabulary, omit or modify items to reflect objectives for the student, eliminate sections of the test, and determine how the grade will be

determined prior to giving the test.

- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- marking students' correct and acceptable work, not the mistakes
- modifying tests to reflect selected objectives
- providing study guides
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using authentic assessments with real-life problem-solving
- using videos, illustrations, pictures, and drawings to explain or clarify

Talented and Gifted Learning (T&G)

- Create a plan to solve an issue presented in the class - Teacher allows students to use their understanding of functions to model a specific problem of their choosing.
- Complete activities alligned with above grade level standards - Students learn concepts that are more complex, such as graphing using polar coordinates.
- Utilize problem-based learning for greater depth of knowledge - Teacher introduces topics to students as part of a project, such as creating functions to model circular motion.
- Allow students to work at a faster pace - Teacher provides resources for students to move ahead if they are able to demonstrate mastery of learning objectives at a faster pace.

- Above grade level placement option for qualified students
- Advanced problem-solving
- Allow students to work at a faster pace
- Cluster grouping
- Complete activities aligned with above grade level text using Benchmark results
- Create a plan to solve an issue presented in the class or in a text
- Flexible skill grouping within a class or across grade level for rigor
- Higher order, critical & creative thinking skills, and discovery
- Multi-disciplinary unit and/or project
- Teacher-selected instructional strategies that are focused to provide challenge, engagement, and growth opportunities
- Utilize exploratory connections to higher-grade concepts
- Utilize project-based learning for greater depth of knowledge

Sample Lesson

Using the template below, please develop a **Sample Lesson** for the first unit only.

Unit Name:

NJSLS:

Interdisciplinary Connection:

Statement of Objective:

Anticipatory Set/Do Now:

Learning Activity:

Student Assessment/CFU's:

Materials:

21st Century Themes and Skills:

Differentiation/Modifications:

Integration of Technology: