Unit 3, Random Variables and Distribution

Content Area: **Math**
Course(s): **Statistics H**
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Title Section

Department of Curriculum and Instruction

Belleville Public Schools

Curriculum Guide

Statistics H, Grade 11,12

Random Variables and Distributions

Belleville Board of Education

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Unit Overview

in this unit students will analyze the nature of and applications involving random variables and their associated probability distributions.

Enduring Understanding

Probability is the basis for statistical inference

There is a difference between a probability distribution function verses a cumulative distribution function.

Discrete probability distributions give an estimate of the probability of the outcome of an event.

All tests contain a margin of error that produces false positives.

The normal curve can be standardized for application in a wide variety of situations where the mean = 0 and the standard deviation = 1.

The normal distribution and central limit theorem are essential when analyzing data samples.

Essential Questions

How can we interpret a probability distribution?

How can modeling predict the future?

What is a normal distribution?

How can you use the Binomial Distribution to determine the probability of an event occurring on a given attempt?

Why is the Normal Distribution so important to statisticians and useful in the real-world?

Exit Skills

Identify type discrete and continuous random variables.

Mean, variance standard deviation, and expected value for a discrete random variable.

Binomial probability distribution.

Poisson, multinomial, geometric and hypergeometric distribution.

Normal distributions.

Applications of the Normal Distribution.

The Central Limit Theorem.

The Normal approximation to the Binomial Distribution.

New Jersey Student Learning Standards (NJSLS)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.S-MD.A.1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
MA.S-MD.A.2	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
MA.S-MD.A.3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
MA.S-MD.A.4	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.
MA.S-MD.B.5	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
MA.S-MD.B.6	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number

	generator).
MA.S-MD.B.7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Interdisciplinary Connections

9.3.12.BM.1	Utilize mathematical concepts, skills and problem solving to obtain necessary information for decision-making in business.
9.3.12.FN.1	Utilize mathematical concepts, skills and problem solving to obtain necessary information for decision making in the finance industry.
9.3.ST.2	Use technology to acquire, manipulate, analyze and report data.
9.3.12.BM.6	Implement, monitor and evaluate business processes to ensure efficiency and quality results.
9.3.ST-SM.2	Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.
9.3.ST-SM.3	Analyze the impact that science and mathematics has on society.
9.3.ST-SM.4	Apply critical thinking skills to review information, explain statistical analysis, and to translate, interpret and summarize research and statistical data.
9.3.GV-REV.3	Design, develop, operate and review data analysis systems and procedures to minimize and eliminate revenue-related financial problems.

Learning Objectives

Understanding the concept of a random variable.

Find the mean , variance, standard deviation, and expected value for a discrete random variable.

Apply the binomial probability distribution, which give us the probability of obtaining a specified number of successes when an experiment is performs n times.

Understand and apply the Poisson, multinomial, geometric and hypergeometric distribution functions.

Identify symmetric or skewed distributions of data.

Identify properties of the normal distribution curve.

Find the area under the standard normal distribution.

Use the normal distribution to calculate probabilities.

Applying the normal distribution to a variety of situations by employing z-scores utilizing the standardize z-scores and the standard normal distribution.

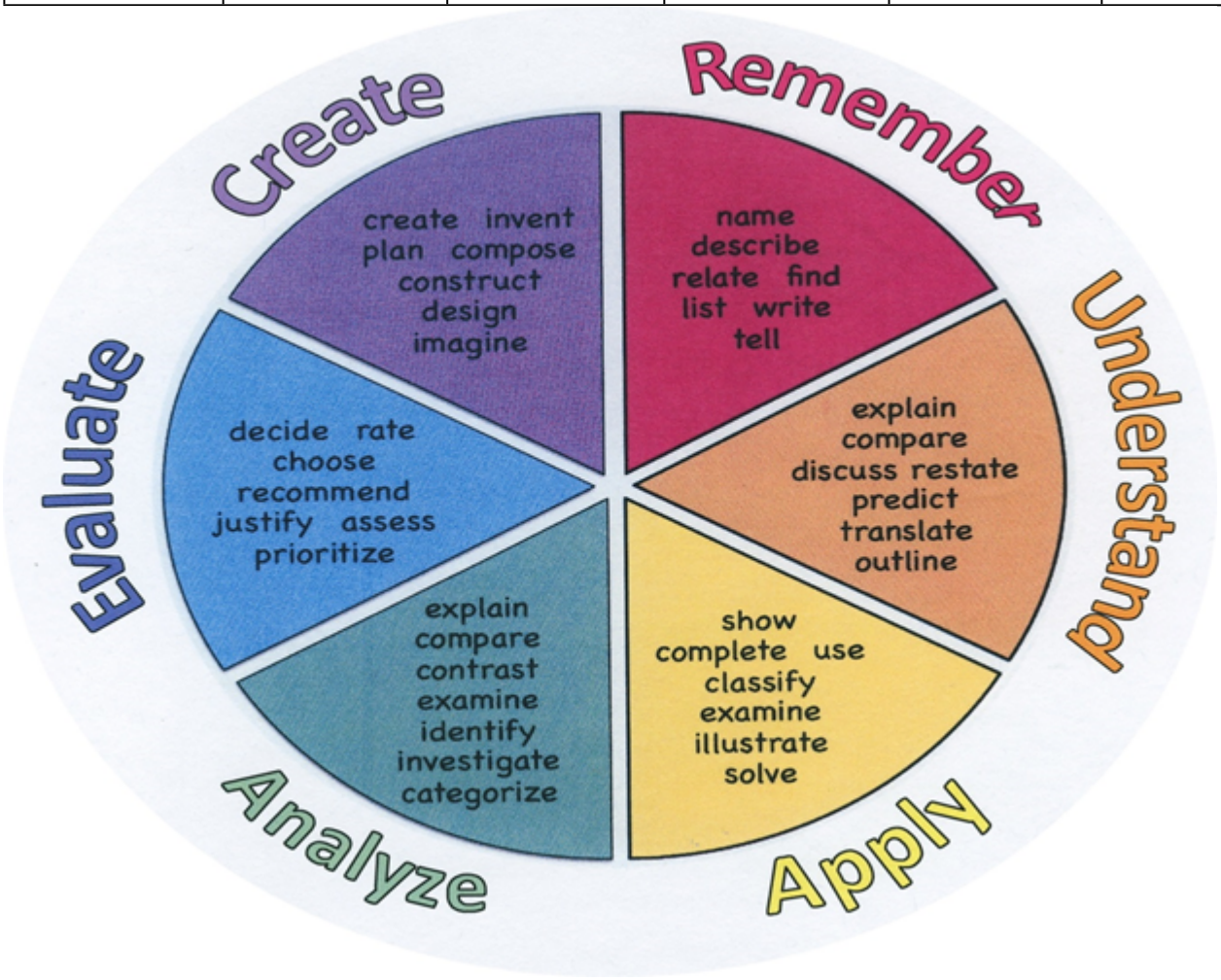
Apply the Central Limit Theorem to solve problems sample means for large data.

Use the normal approximation to compute binomial probabilities by transforming this discrete variable to a continuous variable and using the standard normal distribution

Action Verbs: Below are examples of action verbs associated with each level of the Revised Bloom's Taxonomy.

Remember	Understand	Apply	Analyze	Evaluate	Create
Choose	Classify	Choose	Categorize	Appraise	Combine
Describe	Defend	Dramatize	Classify	Judge	Compose
Define	Demonstrate	Explain	Compare	Criticize	Construct
Label	Distinguish	Generalize	Differentiate	Defend	Design
List	Explain	Judge	Distinguish	Compare	Develop
Locate	Express	Organize	Identify	Assess	Formulate
Match	Extend	Paint	Infer	Conclude	Hypothesize
Memorize	Give Examples	Prepare	Point out	Contrast	Invent
Name	Illustrate	Produce	Select	Critique	Make
Omit	Indicate	Select	Subdivide	Determine	Originate
Recite	Interrelate	Show	Survey	Grade	Organize
Select	Interpret	Sketch	Arrange	Justify	Plan
State	Infer	Solve	Breakdown	Measure	Produce
Count	Match	Use	Combine	Rank	Role Play
Draw	Paraphrase	Add	Detect	Rate	Drive
Outline	Represent	Calculate	Diagram	Support	Devise
Point	Restate	Change	Discriminate	Test	Generate
Quote	Rewrite	Classify	Illustrate		Integrate
Recall	Select	Complete	Outline		Prescribe
Recognize	Show	Compute	Point out		Propose
Repeat	Summarize	Discover	Separate		Reconstruct
Reproduce	Tell	Divide			Revise
	Translate	Examine			Rewrite
	Associate	Graph			Transform
	Compute	Interpolate			
	Convert	Manipulate			

	Discuss Estimate Extrapolate Generalize Predict	Modify Operate Subtract			



Suggested Activities & Best Practices

Graphing Calculators:TI-84 (Calculate binomial coefficients, Graph binomila probability distributions, calculagete geometric probabilities)

Starnes, The Practice of Statistics, 5e, Student Resources(online textbook, study guides, worksheets)

http://www.macmillanlearning.com/catalog/studentresources/tps5e

Statistical Lesson Resources:

http://www.apstatsmonkey.com/StatsMonkey/Statsmonkey.html

Actuarial Foundation lessons:

http://www.actuarialfoundation.org/programs/youth/hs-stats.shtml

Statistic Tutorials

https://stattrek.com/

Census Bureau:

https://www.census.gov/about/what.html

Videos:

http://www.learner.org/resources/series65.html?pop=yesd&pid=140#

apstatsguy.com

American Statistical Association:

http://www.amstat.org/

Desmos

https://learn.desmos.com/statistics

Assessment Evidence - Checking for Understanding (CFU)

Starnes, The Practice of Statistics 5e, Asssessments (Summative)

Edulastic Formative Assessments (Formative):
https://app.edulastic.com/#renderResource/close/Mjk0MjE2ODUwOA%3D%3D

Exit tickets: Google Forms, Edulastic, paper & pencil (F0rmative)

Common Benchmarks on OnCourse:#2/3 (Benchmark)

- Admit Tickets
- Anticipation Guide
- Common Benchmarks
- Compare & Contrast
- Create a Multimedia Poster
- DBQ's
- Define
- Describe
- Evaluate
- Evaluation rubrics
- Exit Tickets
- Explaining
- Fist- to-Five or Thumb-Ometer
- Illustration
- Journals
- KWL Chart
- Learning Center Activities
- Multimedia Reports
- Newspaper Headline
- Outline
- Question Stems
- Quickwrite
- Quizzes
- Red Light, Green Light
- Self- assessments
- Socratic Seminar
- Study Guide
- Surveys
- Teacher Observation Checklist
- Think, Pair, Share
- Think, Write, Pair, Share

- Top 10 List
- Unit review/Test prep
- Unit tests
- Web-Based Assessments
- Written Reports

Primary Resources & Materials

Starnes, The Practice of Statistics, 5e textbook, ebook

The Practice of Statistics digital resources

Ancillary Resources

TI-84 Graphing Calculator

ALEKS

Technology Infusion

- Youtube
- Khan academy
- Google Classroom
- GSuite
- Kutasoftware
- PodCasts
- Kahoot
- Twitter
- Ted Talks
- ALEKS
- QR Barcode Generator
- Calculator/Graphing calculator
- Flipgrid
- Peardeck
- Edulastic
- McGraw-Hill Education
- TI - 84 graphing Calculator
- Desmos.com

- Geogebra.org



Alignment to 21st Century Skills & Technology

Mastery and infusion of **21st Century Skills & Technology** and their Alignment to the core content areas is essential to student learning. The core content areas include:

- English Language Arts;
- Mathematics;
- Science and Scientific Inquiry (Next Generation);
- Social Studies, including Economics;
- Technology;

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.
TECH.8.1.12.A.3	Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.
TECH.8.1.12.F.CS1	Identify and define authentic problems and significant questions for investigation.

21st Century Skills/Interdisciplinary Themes

- English Language Arts;
- Mathematics;
- Science and Scientific Inquiry (Next Generation);
- Social Studies/Economics;
- Technology

- Communication and Collaboration
- Creativity and Innovation
- Critical thinking and Problem Solving
- ICT (Information, Communications and Technology) Literacy
- Information Literacy
- Life and Career Skills

- Media Literacy

21st Century Skills

- Civic Literacy
- Financial, Economic, Business and Entrepreneurial Literacy
- Global Awareness

Differentiation

Graphing calculator(Ti-84)

Differentiate by giving choice of assignments

Alternative assessments

Flexible grouping

Study Guides

Khan Academy statistics lessons

Differentiations:

- Small group instruction
- Small group assignments
- Extra time to complete assignments
- Pairing oral instruction with visuals
- Repeat directions
- Use manipulatives
- Center-based instruction
- Token economy
- Study guides
- Teacher reads assessments allowed
- Scheduled breaks
- Rephrase written directions
- Multisensory approaches
- Additional time
- Preview vocabulary

- Preview content & concepts
- Story guides
- Behavior management plan
- Highlight text
- Student(s) work with assigned partner
- Visual presentation
- Assistive technology
- Auditory presentations
- Large print edition
- Dictation to scribe
- Small group setting

Hi-Prep Differentiations:

- Alternative formative and summative assessments
- Choice boards
- Games and tournaments
- Group investigations
- Guided Reading
- Independent research and projects
- Interest groups
- Learning contracts
- Leveled rubrics
- Literature circles
- Multiple intelligence options
- Multiple texts
- Personal agendas
- Project-based learning
- Problem-based learning
- Stations/centers
- Think-Tac-Toes
- Tiered activities/assignments
- Tiered products
- Varying organizers for instructions

Lo-Prep Differentiations

- Choice of books or activities
- Cubing activities
- Exploration by interest
- Flexible grouping
- Goal setting with students
- Jigsaw
- Mini workshops to re-teach or extend skills
- Open-ended activities
- Think-Pair-Share
- Reading buddies
- Varied journal prompts
- Varied supplemental materials

Special Education Learning (IEP's & 504's)

Graphing calculator(TI-84)

Khan Academy statistics lessons

Use data visualization software to reason about any data variation observed

Modify data used

Verbal analysis of data

Alternative assessments

Flexible grouping

Study Guides

Choice of assignments(choice boards, choiice tables)

- printed copy of board work/notes provided
- additional time for skill mastery
- assistive technology
- behavior management plan
- Center-Based Instruction
- check work frequently for understanding
- computer or electronic device utilizes

- extended time on tests/ quizzes
- have student repeat directions to check for understanding
- highlighted text visual presentation
- modified assignment format
- modified test content
- modified test format
- modified test length
- multi-sensory presentation
- multiple test sessions
- preferential seating
- preview of content, concepts, and vocabulary
- Provide modifications as dictated in the student's IEP/504 plan
- reduced/shortened reading assignments
- Reduced/shortened written assignments
- secure attention before giving instruction/directions
- shortened assignments
- student working with an assigned partner
- teacher initiated weekly assignment sheet
- Use open book, study guides, test prototypes

English Language Learning (ELL)

Graphing calculator(TI-84)

Khan Academy English or Spanish website https://es.khanacademy.org/math/probability

Use data visualization software to reason about any data variation observed

Modify data used

Verbal analysis of data

Alternative assessments

Flexible grouping

Study Guides

Choice of assignments

- teaching key aspects of a topic. Eliminate nonessential information
- using videos, illustrations, pictures, and drawings to explain or clarif
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning;
- allowing students to correct errors (looking for understanding)
- allowing the use of note cards or open-book during testing
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using computer word processing spell check and grammar check features
- using true/false, matching, or fill in the blank tests in lieu of essay tests

At Risk

Graphing calculator(TI-84)

Use of manipulatives and "hands-on" activities

Differentiate assignments giving choice of data based on student interest

Khan Academy Statistics lessons

Use data visualization software to reason about any data variation observed

Modify data used

Verbal analysis of data

Alternative assessments

Flexible grouping

- allowing students to correct errors (looking for understanding)
- teaching key aspects of a topic. Eliminate nonessential information
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning
- allowing students to select from given choices
- allowing the use of note cards or open-book during testing
- collaborating (general education teacher and specialist) to modify vocabulary, omit or modify items to reflect objectives for the student, eliminate sections of the test, and determine how the grade will be determined prior to giving the test.
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- marking students' correct and acceptable work, not the mistakes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using authentic assessments with real-life problem-solving
- using true/false, matching, or fill in the blank tests in lieu of essay tests
- using videos, illustrations, pictures, and drawings to explain or clarify

Talented and Gifted Learning (T&G)

Graphing calculator(TI-84)

Offer activities and problems that extend beyond the current assignments

AP Statistic tasks

Differentiate assinments giving more challenging one; or a task in which data is tailored to students' interests

Statistics software:

https://www.statcato.org/

Khan Academy Statistics lessons English/Spanish

Choice of Data:

http://www.statcrunch.com/

- Above grade level placement option for qualified students
- Advanced problem-solving

- Allow students to work at a faster pace
- Cluster grouping
- Complete activities aligned with above grade level text using Benchmark results
- Create a blog or social media page about their unit
- Create a plan to solve an issue presented in the class or in a text
- Debate issues with research to support arguments
- Flexible skill grouping within a class or across grade level for rigor
- Higher order, critical & creative thinking skills, and discovery
- Multi-disciplinary unit and/or project
- Teacher-selected instructional strategies that are focused to provide challenge, engagement, and growth opportunities
- Utilize exploratory connections to higher-grade concepts
- Utilize project-based learning for greater depth of knowledge

Sample Lesson

Using the template below, please develop a **Sample Lesson** for the first unit only.

Unit Name:

NJSLS:

Interdisciplinary Connection:

Statement of Objective:

Anticipatory Set/Do Now:

Learning Activity:

Student Assessment/CFU's:

Materials:

21st Century Themes and Skills:

Differentiation/Modifications:

Integration of Technology: