Unit 5: Conic Sections & Trigonometry

Content Area: **Math**
Course(s): **Algebra 2H**
Time Period:
Length: **30 days, Grades 10-12**
Status: **Published**

Title Section

Department of Curriculum and Instruction

Belleville Public Schools

Curriculum Guide

ALGEBRA 2 HONORS, GRADES 10-12

UNIT 5: CONIC SECTIONS & TRIGONOMETRY

Belleville Board of Education

102 Passaic Avenue

Belleville, NJ 07109

Prepared by: Mrs. Lily Marietto

Dr. Richard Tomko, Ph.D., M.J., Superintendent of Schools

Ms. LucyAnn Demikoff, Director of Curriculum and Instruction K-12

Ms. Nicole Shanklin, Director of Elementary Education K-8, ESL Coordinator K-12

Mr. George Droste, Director of Secondary Education

Board Approved: September 23, 2019

Unit Overview

In this unit:

- students will study properties and characteristics of conic sections. Students will start by applying the distance and midpoint formulas, then learn how to graph and write equations for parabolas, circles, ellipses, and hyperbolas.
- students will study angles, the unit circle, and the trigonometric functions. Students will first study acute angles in triangles and on the unit circle, then apply their understanding of angles and the trigonometric functions to angles in radian measure and different types of angles on the unit circle. They will graph the trigonometric functions sine, cosine, and tangent, and their inverse. Then they will explore phase shifts and transformations of these functions through graphs and equations of the functions.

Enduring Understanding

- Use equations to model real-world situations.
- Choose the proper function type to represent a relation.
- Graph different types of parent functions and their transformations.
- Transform and simplify many different types of equations.

- Simplify expressions, using a variety of properties and strategies.
- Rearrange equations to highlight useful information, such as the vertex of a parabola.
- Identify the relationship between a graph and an equation and be able to translate one into the other.
- Derive equations given specific information about a graph or a relationship between quantities.
- Understand the relationship between the unit circle, radian measure, degree measure, and the trigonometric functions.

Essential Questions

- What are conic sections and why are they called conic sections?
- In what situations could you use a conic section to model a relation?
- What are the different parts of a parabola called?
- What are concentric circles?
- Why is it important to know the parts of an ellipse?
- Which is the only conic section that can be a function?
- How can you describe a plane that forms a circle?
- How can you tell conic sections apart?
- How does changing parts of an equation change the graph of the function?
- What is radian measure? How does it relate to degree measure of an angle?
- What are the trigonometric functions?
- In what situations could trigonometric functions relate to the real world?
- What is a periodic function?
- What are the special right triangles and why are they important in trigonometry?

Exit Skills

By the end of Unit 5, Algebra 2 Honors students will know:

- How to sketch graphs of conic sections to relate simple parameter changes in the equation to corresponding changes in the graph.
- How to write an equation to represent a given conic section.
- How to describe a conic section as the intersection of a plane and a cone.
- How to identify symmetries from graphs of conic sections.
- How to identify the conic section from a given equation.
- How to explore trigonometric functions, first in acute angles in standard form, and also for points on the unit circle.
- How to use radian measure to describe angles on a unit circle.
- How to develop inverses for the sine, cosine, and tangent functions.
- How to use trigonometric functions to explore amplitude and period.
- How to investigate phase shifts and vertical shifts in the graphs of trigonometric functions.

New Jersey Student Learning Standards (NJSLS)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
MA.G-GPE.A	Translate between the geometric description and the equation for a conic section
MA.G-GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
MA.G-GPE.A.2	Derive the equation of a parabola given a focus and directrix.
MA.G-GPE.A.3	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
MA.F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
MA.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
MA.F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Interdisciplinary Connections

9-12.HS-ETS1-4.4.1	Models (e.g., physical, mathematical, computer models) can be used to simulate systems

	and interactions— including energy, matter, and information flows— within and between systems at different scales.
9-12.HS-ETS1-4.5	Using Mathematics and Computational Thinking
9-12.HS-ETS1-4.5.1	Use mathematical models and/or computer simulations to predict the effects of a design solution on systems and/or the interactions between systems.
9-12.HS-PS1-8.2.1	Develop a model based on evidence to illustrate the relationships between systems or between components of a system.

Learning Objectives

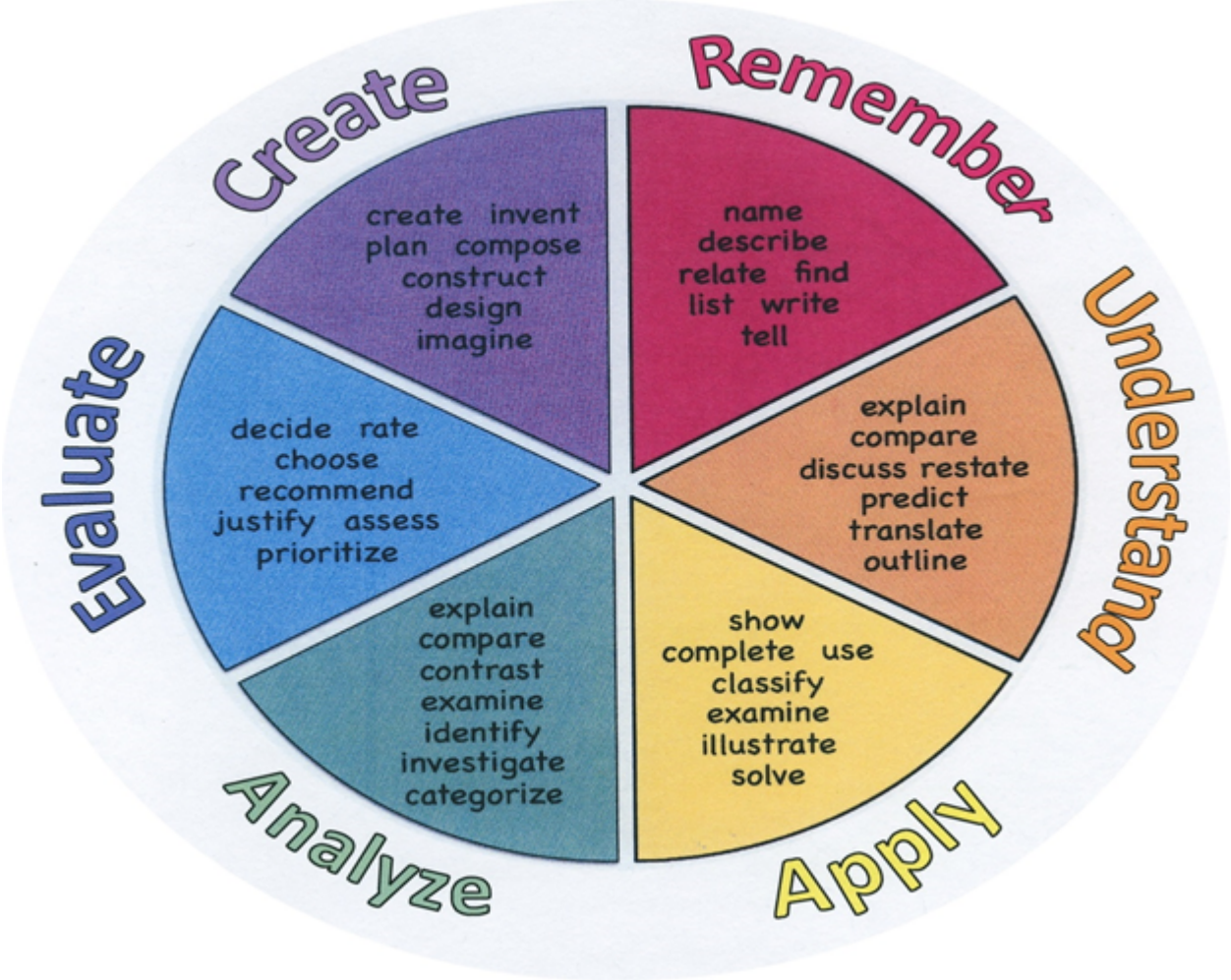
Students will be able to:

- find the midpoint of a segment and the distance between to points on the coordinate plane.
- write equations of parabolas and graph parabolas.
- write equations of circles and graph circles.
- write equations of ellipses and graph ellipses.
- write equations of hyperbolas and graph hyperbolas.
- use trigonometric functions to find side lengths and angle measures of right triangles.
- draw and find angles in standard position.
- convert between degree measures and radian measures.
- find values of trigonometric functions using general angles and reference angles.
- find values of trigonometric functions based on the unit circle.
- evaluate trigonometric functions using the properties of periodic functions.
- describe and graph the sine, cosine, tangent, and other trigonometric functions.
- graph horizontal translations of trigonometric graphs and find phase shifts.
- graph vertical translations of trigonometric graphs.

Action Verbs: Below are examples of action verbs associated with each level of the Revised Bloom's Taxonomy.

Remember	Understand	Apply	Analyze	Evaluate	Create
Choose	Classify	Choose	Categorize	Appraise	Combine
Describe	Defend	Dramatize	Classify	Judge	Compose
Define	Demonstrate	Explain	Compare	Criticize	Construct
Label	Distinguish	Generalize	Differentiate	Defend	Design
List	Explain	Judge	Distinguish	Compare	Develop
Locate	Express	Organize	Identify	Assess	Formulate
Match	Extend	Paint	Infer	Conclude	Hypothesize
Memorize	Give Examples	Prepare	Point out	Contrast	Invent
Name	Illustrate	Produce	Select	Critique	Make
Omit	Indicate	Select	Subdivide	Determine	Originate
Recite	Interrelate	Show	Survey	Grade	Organize
Select	Interpret	Sketch	Arrange	Justify	Plan
State	Infer	Solve	Breakdown	Measure	Produce
Count	Match	Use	Combine	Rank	Role Play
Draw	Paraphrase	Add	Detect	Rate	Drive
Outline	Represent	Calculate	Diagram	Support	Devise
Point	Restate	Change	Discriminate	Test	Generate
Quote	Rewrite	Classify	Illustrate		Integrate
Recall	Select	Complete	Outline		Prescribe
Recognize	Show	Compute	Point out		Propose
Repeat	Summarize	Discover	Separate		Reconstruct
Reproduce	Tell	Divide			Revise
	Translate	Examine			Rewrite

	Associate Compute Convert Discuss Estimate Extrapolate Generalize Predict	Graph Interpolate Manipulate Modify Operate Subtract			Transform



Suggested Activities & Best Practices

- Online textbook practice problems, study guides, and worksheets
- Desmos Classroom Activities, such as "Building Conic Sections" activity
- Higher-order thinking tasks, such as Illustrative Mathematics task "Windmills Everywhere"
- Practice activities from ALEKS, KUTA Software, Khan Academy, etc., such as "Trig values of special angles"

Assessment Evidence - Checking for Understanding (CFU)

- Regular Exit Tickets to assess individual learning objectives (Formative)
- Quizzes to assess groups of learning objectives - at least one quiz for each chapter (Chapters 9 and 12) (Summative)
- Chapter Tests given at least once per chapter - at least 2 tests in this unit (Chapters 9 and 12) (Summative)
- Common Quarterly/Benchmark Exams - Quarter 4 Exam for this unit (Benchmark)
- Web-Based Assessments (using Google Forms, ALEKS, Edulastic, Khan Academy, etc.) (Formative/Summative)

- Admit Tickets
- Anticipation Guide
- Common Benchmarks
- Compare & Contrast
- Create a Multimedia Poster
- DBQ's
- Define
- Describe
- Evaluate
- Evaluation rubrics
- Exit Tickets
- Explaining
- Fist- to-Five or Thumb-Ometer
- Illustration
- Journals
- KWL Chart
- Learning Center Activities
- Multimedia Reports
- Newspaper Headline
- Outline
- Question Stems
- Quickwrite
- Quizzes
- Red Light, Green Light
- Self- assessments
- Socratic Seminar
- Study Guide
- Surveys
- Teacher Observation Checklist
- Think, Pair, Share
- Think, Write, Pair, Share
- Top 10 List
- Unit review/Test prep

- Unit tests
- Web-Based Assessments
- Written Reports

Primary Resources & Materials

- Glencoe McGraw-Hill Algebra 2 2014
- Glencoe McGraw-Hill Algebra 2 2010
- Practice Glencoe Algebra 2
- Study Guide Glencoe Algebra 2
- connected.mcgraw-hill.com

Ancillary Resources

- McDougal Littell Algebra 2 & Trigonometry 2011
- McDougal Littell Algebra 2 & Trigonometry Resource Files 2011
- Prentice Hall Algebra 2 2011
- Kuta Software

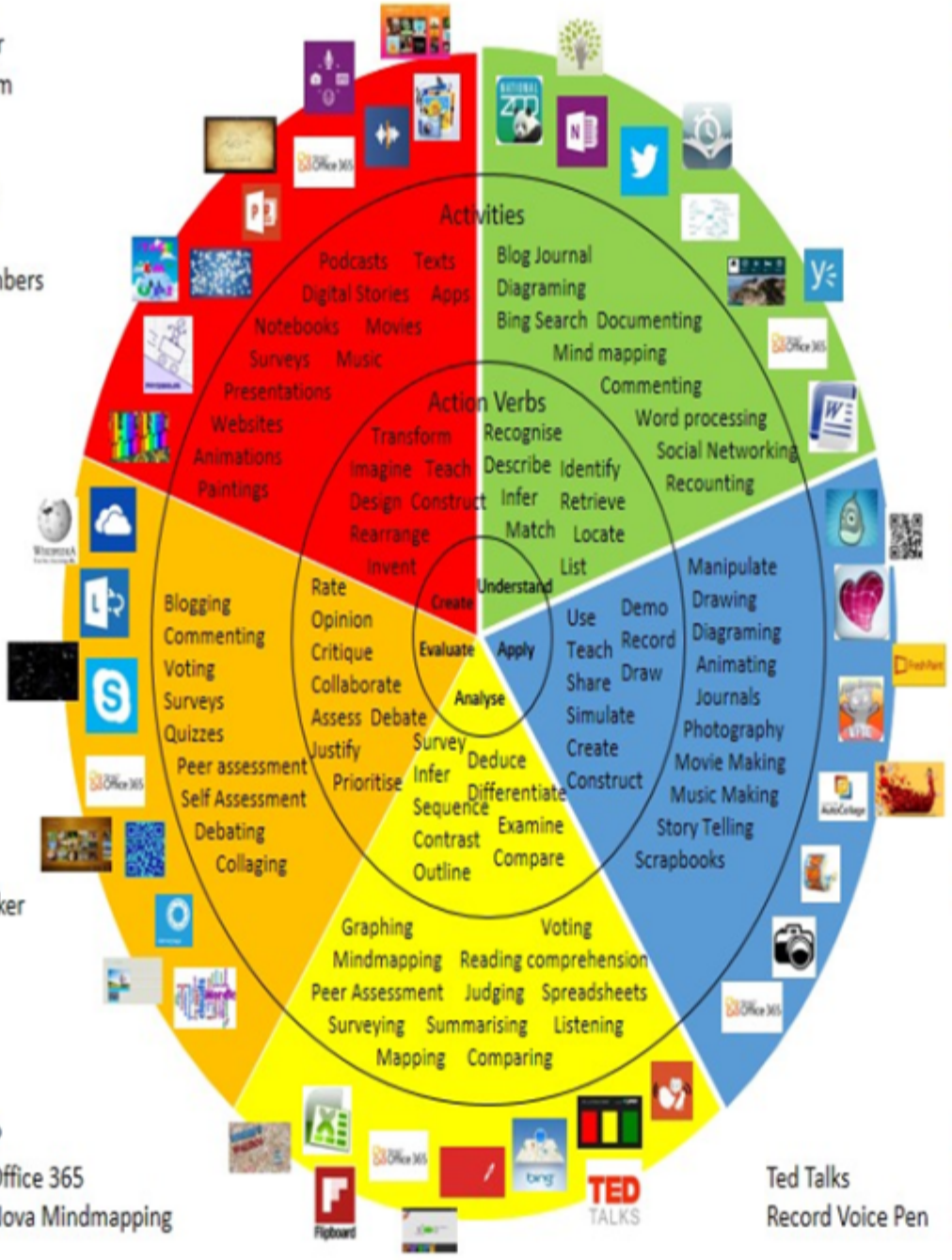
Technology Infusion

- Smart TV - Display and interact with lessons and activities
- Chromebooks - students access activities, slides, and practice problems
- Google Classroom - Slides, Forms, Drive, etc.
- ALEKS - Students practice individual learning objectives such as "Write an equation of a circle given its center and a point on the circle"
- Desmos - Students interact with classroom activities or use graphing software to graph and analyze functions
- YouTube - Students watch videos to deepen understanding of specific concepts throughout the unit
- Khan Academy - Students practice individual learning objectives, such as "Period of sinusoidal functions from a graph"
- Calculator/Graphing calculator - Students perform calculations or graph and analyze functions
- Edulastic - Students complete assessments and checks for understanding
- KUTA Software - Teacher generates a variety of assessments and practice problems for individual learning objectives or groups of learning objectives
- Pear Deck- Teacher presents information through an interactive slide show presentation

Win 8.1 Apps/Tools Pedagogy Wheel

Podcasts
Photostory 3
Kid Story Builder
Music Maker Jam
Paint A Story
Office 365
MS PowerPoint
Stack 'Em Up
NqSquared Numbers
Physamajig
Xylophone 8

Wikipedia
Skydrive
Lync
SkyMap
Skype
Office 365
Puzzle Touch
Easy QR
Memorylage
Life Moments
Word Cloud Maker



Where's Waldo?
MS Excel Office 365
Flipboard Nova Mindmapping

Ted Talks
Record Voice Pen

Originally taken from http://www.coetail.com/vzimmer/files/2013/02/iPadagogy-Wheel.001.jpg
And adapted for Windows 8.1 devices by Charlotte Beckhurst @CharBeckhurst

Alignment to 21st Century Skills & Technology

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP2.1	Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP4.1	Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others' time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP6.1	Career-ready individuals regularly think of ideas that solve problems in new and different ways, and they contribute those ideas in a useful and productive manner to improve their organization. They can consider unconventional ideas and suggestions as solutions to issues, tasks or problems, and they discern which ideas and suggestions will add greatest value. They seek new methods, practices, and ideas from a variety of sources and seek to apply those ideas to their own workplace. They take action on their ideas and understand how to bring innovation to an organization.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP7.1	Career-ready individuals are discerning in accepting and using new information to make decisions, change practices or inform strategies. They use reliable research process to search for new information. They evaluate the validity of sources when considering the use and adoption of external information or practices in their workplace situation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP8.1	Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP11.1	Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.
TECH.8.1.12.F	Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

TECH.8.1.12.F.CS3 Collect and analyze data to identify solutions and/or make informed decisions.

21st Century Skills/Interdisciplinary Themes

- Communication and Collaboration
- Information Literacy
- Media Literacy
- ICT (Information, Communications and Technology) Literacy
- Life and Career Skills
- Creativity and Innovation
- Critical Thinking and Problem Solving

- Communication and Collaboration
- Creativity and Innovation
- Critical thinking and Problem Solving
- ICT (Information, Communications and Technology) Literacy
- Information Literacy
- Life and Career Skills
- Media Literacy

21st Century Skills

- Global Awareness
- Financial, Economic, Business, and Entrepeneurial Literacy
- Civic Literacy

- Civic Literacy
- Environmental Literacy
- Financial, Economic, Business and Entrepreneurial Literacy
- Global Awareness
- Health Literacy

Differentiation

- Small group instruction - Teacher utilizes small groups to remediate or enrich specific topics with different groups of students, as necessary.
- Study guides - Teacher provides students with study guides prior to quizzes and tests.
- Problem-based learning - Teacher introduces topics to students as part of a project, such as creating functions to model height of an object above the ground.
- Open-ended activities - Students complete activities with multiple entry points and more than one possible solution.

Differentiations:

- Small group instruction
- Small group assignments

- Extra time to complete assignments
- Pairing oral instruction with visuals
- Repeat directions
- Use manipulatives
- Center-based instruction
- Token economy
- Study guides
- Teacher reads assessments allowed
- Scheduled breaks
- Rephrase written directions
- Multisensory approaches
- Additional time
- Preview vocabulary
- Preview content & concepts
- Story guides
- Behavior management plan
- Highlight text
- Student(s) work with assigned partner
- Visual presentation
- Assistive technology
- Auditory presentations
- Large print edition
- Dictation to scribe
- Small group setting

Hi-Prep Differentiations:

- Alternative formative and summative assessments
- Choice boards
- Games and tournaments
- Group investigations
- Guided Reading
- Independent research and projects
- Interest groups
- Learning contracts
- Leveled rubrics
- Literature circles
- Multiple intelligence options
- Multiple texts
- Personal agendas
- Project-based learning
- Problem-based learning
- Stations/centers
- Think-Tac-Toes
- Tiered activities/assignments
- Tiered products
- Varying organizers for instructions

Lo-Prep Differentiations

- Choice of books or activities
- Cubing activities
- Exploration by interest

- Flexible grouping
- Goal setting with students
- Jigsaw
- Mini workshops to re-teach or extend skills
- Open-ended activities
- Think-Pair-Share
- Reading buddies
- Varied journal prompts
- Varied supplemental materials

Special Education Learning (IEP's & 504's)

- Provide modifications as dictated in student's IEP/504 - Teacher modifies tests/assesments as necessary.
- Additional time for skill mastery - Teacher allows students additional time to master particular learning objectives.
- Center-Based Instruction - Teacher utilizes different sets of stations/centers in order to differentiate and provide students with varied learning settings.
- Modify assignments/tests - Teacher modifies tests/assesments by, for example, writing multi-part answers for questions that require students to complete multiple steps.
- Utilize computers or electronic devices - Teacher uses chromebooks and smart TV to provide students with visualizations of graphs/models and allow students to interact with them them.
- Extended time on tests/quizzes - Teacher allows students to have extended time on tests/quizzes as dictated by their IEP/504.
- Use of calculator on tests/quizzes - Students are allowed to use calculators on tests/quizzes.
- Use of study guide, reference sheets, or notes on tests/quizzes - Teacher allows students to use reference sheets or study guides on tests/quizes that contain information such as standard form of conic sections and trigonometric functions.

- printed copy of board work/notes provided
- additional time for skill mastery
- assistive technology
- behavior management plan
- Center-Based Instruction
- check work frequently for understanding
- computer or electronic device utilizes
- extended time on tests/ quizzes
- have student repeat directions to check for understanding
- highlighted text visual presentation
- modified assignment format
- modified test content
- modified test format
- modified test length

- multi-sensory presentation
- multiple test sessions
- preferential seating
- preview of content, concepts, and vocabulary
- Provide modifications as dictated in the student's IEP/504 plan
- reduced/shortened reading assignments
- Reduced/shortened written assignments
- secure attention before giving instruction/directions
- shortened assignments
- student working with an assigned partner
- teacher initiated weekly assignment sheet
- Use open book, study guides, test prototypes

English Language Learning (ELL)

- Using videos, illustrations, pictures and drawings to explain or clarify - Teacher provides tools such as visualizations of graphs that students can interact with.
- Eliminate nonessential information - Teacher explains concepts using only the vocabulary that is essential to understand a concept.
- Tutoring by peers - Teacher allows peers to explain concepts to ELL students.
- Allow students to correct errors - Teacher allows students to gain back points by correcting their errors on a test/quiz.
- Modify assignments/tests - Teacher modifies tests/assesments by, for example, writing multi-part answers for questions that require students to complete multiple steps.
- Use of study guide, reference sheets, or notes on tests/quizzes - Teacher allows students to use reference sheets or study guides on tests/quizes that contain information such as standard form of conic sections and trigonometric functions.

- teaching key aspects of a topic. Eliminate nonessential information
- using videos, illustrations, pictures, and drawings to explain or clarif
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student’s learning;
- allowing students to correct errors (looking for understanding)
- allowing the use of note cards or open-book during testing
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher’s notes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using computer word processing spell check and grammar check features
- using true/false, matching, or fill in the blank tests in lieu of essay tests

At Risk

- Decrease the amount of work presented or required - Teacher allows students to submit less work, for example, only complete 3 out of the 5 practice problems for a specific learning objective.
- Using videos, illustrations, pictures, and drawings to explain or clarify - Teacher provides tools such as visualizations of graphs that students can interact with.
- Tutoring by peers - Teacher allows peers to explain concepts to at risk students.
- Providing study guides - Teacher provides students with study guides prior to quizzes and tests. For example, a variety of practice problems related to the topics being assessed.
- Allowing students to correct errors - Teacher allows students to gain back points by correcting their errors on a test/quiz.
- Allowing students to select from given choices - Teacher gives students a choice of activities to complete, such as draw a graph, create an equation, or write a sentence to model a situation.
- Allowing the use of notes, study guides, or reference sheets on tests/quizzes - Teacher allows students to use reference sheets or study guides on tests/quizes that contain information such as standard form of conic sections and trigonometric functions.

- allowing students to correct errors (looking for understanding)
- teaching key aspects of a topic. Eliminate nonessential information
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning
- allowing students to select from given choices
- allowing the use of note cards or open-book during testing
- collaborating (general education teacher and specialist) to modify vocabulary, omit or modify items to reflect objectives for the student, eliminate sections of the test, and determine how the grade will be determined prior to giving the test.
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- marking students' correct and acceptable work, not the mistakes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using authentic assessments with real-life problem-solving
- using true/false, matching, or fill in the blank tests in lieu of essay tests
- using videos, illustrations, pictures, and drawings to explain or clarify

Talented and Gifted Learning (T&G)

- Create a plan to solve an issue presented in the class - Teacher allows students to use their understanding of functions to model a specific problem of their choosing.
- Complete activities alligned with above grade level standards - Students graph functions that are more complex, such as inverses of trigonometric functions.
- Utilize problem-based learning for greater depth of knowledge - Teacher introduces topics to students as part of a project, such as creating functions to model movement of an object in circular motion.
- Allow students to work at a faster pace - Teacher provides resources for students to move ahead if they are able to demonstrate mastery of learning objectives at a faster pace.

- Above grade level placement option for qualified students
- Advanced problem-solving
- Allow students to work at a faster pace
- Cluster grouping
- Complete activities aligned with above grade level text using Benchmark results
- Create a blog or social media page about their unit
- Create a plan to solve an issue presented in the class or in a text
- Debate issues with research to support arguments
- Flexible skill grouping within a class or across grade level for rigor
- Higher order, critical & creative thinking skills, and discovery
- Multi-disciplinary unit and/or project
- Teacher-selected instructional strategies that are focused to provide challenge, engagement, and growth opportunities
- Utilize exploratory connections to higher-grade concepts
- Utilize project-based learning for greater depth of knowledge

Sample Lesson

Using the template below, please develop a **Sample Lesson** for the first unit only.

Unit Name:

NJSLS:

Interdisciplinary Connection:

Statement of Objective:

Anticipatory Set/Do Now:

Learning Activity:

Student Assessment/CFU's:

Materials:

21st Century Themes and Skills:

Differentiation/Modifications:

Integration of Technology: