

Unit 2, Congruence

Content Area: **Math**
Course(s): **Geometry**
Time Period:
Length: **43 Days , Grades 9,10**
Status: **Published**

Title Section

Department of Curriculum and Instruction



Belleville Public Schools

Curriculum Guide

Geometry A

Congruence

Belleville Board of Education

102 Passaic Avenue

Belleville, NJ 07109

Prepared by: Annamaria Contella

Dr. Richard Tomko, Ph.D., M.J., Superintendent of Schools

Ms. LucyAnn Demikoff, Director of Curriculum and Instruction K-12

Ms. Nicole Shanklin, Director of Elementary Education K-8, ESL Coordinator K-12

Mr. George Droste, Director of Secondary Education

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Unit Overview

In this unit, students will establish the definition of congruence through transformations and apply the concept of congruence when proving theorems about triangles and parallelograms. They will explore and establish the criteria for triangle congruence (SSS, SAS, ASA...). Students will apply formal reasoning to prove theorems—using a variety of formats—and solve problems about lines, angles, triangles, quadrilaterals, and other polygons. Students will apply reasoning to complete geometric constructions and explain why constructions work.

Enduring Understanding

Analyze geometric relationships in order to make and verify conjectures involving triangles.

Apply the concept of congruence to justify properties of figures and solve problems.

Identify and use perpendicular bisectors, angle bisectors, medians and altitudes of triangles.

Apply properties of inequalities relating to measures of angles and sides of triangles.

Use indirect proof with algebra and geometry.

Apply the triangle inequality theorem and SSS, SAS inequalities.

Investigate interior and exterior angles of polygons.

Recognize and apply the properties of parallelograms, rectangles, rhombi, squares, and trapezoids.

Position quadrilaterals for use in coordinate proofs

Essential Questions

In what ways can we classify triangles?

What does it mean to apply the Angle Sum Theorem and the Exterior Angle Theorem?

How do you identify corresponding parts of congruent triangles?

How do you test triangles for congruence using SSS, SAS, ASA, HL and AAS?

Does the circumcenter of a triangle have to be inside the triangle?

What does the Exterior Angle Theorem state?

How does indirect reasoning work? What is the name for a proof arrived at through indirect reasoning?

What steps would you take to prove that a perpendicular segment is the shortest distance from a point to a line?

Under what circumstances would you choose to use the SAS Inequality Theorem instead of the SSS Inequality Theorem?

How can you prove the Interior Angle Sum Theorem?

If there is a parallelogram with two diagonals that bisect, do the diagonals share a midpoint?

Will the four segments formed by the diagonals intersection be congruent?

What are the tests for a parallelogram?

What type of figure has congruent diagonals? Why?

Is a rhombus always a square? Why or why not?

How are a trapezoid and a parallelogram the same? How are they different?

Why is it useful to place a figure onto a coordinate plane to prove theorems?

Exit Skills

By the end of Unit 2, Congruence, students will know:

How to identify and classify triangles by angles.

How to identify and classify triangles by sides.

How to apply the Angle Sum Theorem.

How to apply the Exterior Angle Theorem.

How to name and label corresponding parts of congruent triangles.

How to identify congruence transformations.

How to use the SSS, SAS, ASA, AAS postulates to test for triangle congruence.

How to use the HL theorem to test for triangle congruence.

How to use properties of isosceles triangles.

How to use properties of equilateral triangles.

How to identify and use perpendicular bisectors and angle bisectors in triangles.

How to identify and use medians and altitudes in triangles.

How to recognize and apply properties of inequalities to the measures of angles of a triangle.

How to use indirect proof with geometry.

How to apply the Triangle Inequality Theorem.

How to determine the shortest distance between a point and a line.

How to apply the SAS Inequality.

How to apply the SSS Inequality.

How to find the sum of the measures of the interior angles of a polygon.

How to find the sum of the measures of the exterior angles of a polygon.

How to recognize and apply properties of the sides and angles of parallelograms.

How to recognize and apply properties of diagonals of parallelograms.

How to recognize the conditions that ensure a quadrilateral is a parallelogram.

How to prove that a set of points forms a parallelogram in the coordinate plane.

How to recognize and apply properties of rectangles.

How to determine whether parallelograms are rectangles.

How to recognize and apply properties of rhombi and squares.

How to recognize and apply properties of trapezoids.

How to solve problems involving medians of trapezoids.

How to position and label quadrilaterals for the use in coordinate planes.

How to prove theorems using coordinate proofs.

New Jersey Student Learning Standards (NJSL)

MA.G-CO.B.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
MA.G-CO.B.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
MA.G-CO.C.9	Prove theorems about lines and angles.
MA.G-CO.C.10	Prove theorems about triangles.
MA.G-CO.C.11	Prove theorems about parallelograms.
MA.G-CO.D.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.G-GPE.B.4	Use coordinates to prove simple geometric theorems algebraically.
MA.G-GPE.B.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
MA.G-SRT.B.4	Prove theorems about triangles.
MA.G-SRT.B.5	Use congruence and similarity criteria for triangles to solve problems and to prove

relationships in geometric figures.

Interdisciplinary Connections

12.9.3.ST-ET.1	Use STEM concepts and processes to solve problems involving design and/or production.
12.9.3.ST-SM.2	Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.
LA.W.9-10.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
9-12.HS-PS1-4.2	Modeling in 9–12 builds on K–8 and progresses to using, synthesizing, and developing models to predict and show relationships among variables between systems and their components in the natural and designed worlds.
9-12.HS-PS1-3.3	Planning and Carrying Out Investigations
9-12.HS-PS2-4.5	Mathematical and computational thinking at the 9–12 level builds on K–8 and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.

Learning Objectives

Identify and classify triangles by angles.

Identify and classify triangles by sides.

Apply the Angle Sum Theorem.

Apply the Exterior Angle Theorem.

Determine, identify, and label corresponding parts of congruent triangles.

Identify congruence transformations.

Use the SSS, SAS, ASA, AAS to test for triangle congruence.

Use the HL Theorem to test for triangle congruence.

Use and apply properties of isosceles triangles .

Use and apply properties of equilateral triangles.

Identify and use perpendicular bisectors and angle bisectors in triangles.

Identify and use medians and altitudes in triangles.

Recognize and apply properties of inequalities to the measures of angles of a triangle.

Use indirect proof with geometry.

Use and apply the Triangle Inequality Theorem.

Determine the shortest distance between a point and a line.

Apply the SAS Inequality.

Apply the SSS Inequality.

Determine the sum of the measures of the interior angles of a polygon.

Determine the sum of the measures of the exterior angles of a polygon.

Recognize and apply properties of the sides and angles of parallelograms.

Recognize and apply properties of diagonals of parallelograms.

Recognize the conditions that ensure a quadrilateral is a parallelogram.

Prove that a set of points forms a parallelogram in the coordinate plane.

Recognize and apply properties of rectangles.

Determine whether parallelograms are rectangles.

Recognize and apply properties of rhombi and squares.

Recognize and apply properties of trapezoids.

Solve problems involving medians of trapezoids.

Position and label quadrilaterals for the use in coordinate planes.

Prove theorems using coordinate proofs.

Action Verbs: Below are examples of action verbs associated with each level of the Revised Bloom's Taxonomy.

Remember	Understand	Apply	Analyze	Evaluate	Create
Choose	Classify	Choose	Categorize	Appraise	Combine
Describe	Defend	Dramatize	Classify	Judge	Compose
Define	Demonstrate	Explain	Compare	Criticize	Construct
Label	Distinguish	Generalize	Differentiate	Defend	Design
List	Explain	Judge	Distinguish	Compare	Develop
Locate	Express	Organize	Identify	Assess	Formulate
Match	Extend	Paint	Infer	Conclude	Hypothesize
Memorize	Give Examples	Prepare	Point out	Contrast	Invent
Name	Illustrate	Produce	Select	Critique	Make
Omit	Indicate	Select	Subdivide	Determine	Originate
Recite	Interrelate	Show	Survey	Grade	Organize
Select	Interpret	Sketch	Arrange	Justify	Plan
State	Infer	Solve	Breakdown	Measure	Produce
Count	Match	Use	Combine	Rank	Role Play
Draw	Paraphrase	Add	Detect	Rate	Drive
Outline	Represent	Calculate	Diagram	Support	Devise
Point	Restate	Change	Discriminate	Test	Generate
Quote	Rewrite	Classify	Illustrate		Integrate
Recall	Select	Complete	Outline		Prescribe
Recognize	Show	Compute	Point out		Propose
Repeat	Summarize	Discover	Separate		Reconstruct
Reproduce	Tell	Divide			Revise
	Translate	Examine			Rewrite
	Associate	Graph			Transform
	Compute	Interpolate			
	Convert	Manipulate			
	Discuss	Modify			
	Estimate	Operate			
	Extrapolate	Subtract			
	Generalize				
	Predict				



Suggested Activities & Best Practices

Textbook, eAssessment, supplemental materials:

<https://my.mheducation.com/login>

AI Assessment and Learning System:

<https://www.aleks.com/>

Mindset:


<https://www.youtube.com/watch?v=3icoSeGqQtY>

<http://www.youcubed.org/wp-content/uploads/Positive-Classroom-Norms2.pdf>

Coaching Corner:

<https://sites.google.com/belleville.k12.nj.us/thecoachingcorner/home>

Videos:

<https://www.mathantics.com/> 

<https://mashupmath.com/high-school-math-lessons>

Constructions:

<http://www.mathopenref.com/tocs/constructionstoc.html>

Geometry Activities:

<https://math.rice.edu/~lanius/Geom/>

Geogebra:

<https://www.geogebra.org/a/7?lang=en>

Desmos:

<https://teacher.desmos.com/activitybuilder/custom/5664e067eb08d9501576caa0>

Kahoot:

<https://create.kahoot.it/share/sss-sas-aas-asa-postulates/b2372ee9-a0ca-4eb8-955a-af8c966e283>

Assessment Evidence - Checking for Understanding (CFU)

Glencoe McGraw Hill : Chapter Assessments, Midchapter Assessments (Summative) -

<https://connected.mcgraw->

hill.com/c2j/assetBuckets.assess.do?bookId=DFRTR2RBH9YT25W7OSMM6J3XM1&selectedCategoryId=3KTMO6D7VZ6SJ4YD3XNOQB3O44

EAssessment test generator (Summative): <https://assess.k12.mhedu.com/Instructor/TestGenerator.aspx>

Edulastic Formative Assessments (Formative):

<https://app.edulastic.com/#renderResource/close/Mjk0MjE2ODUwOA%3D%3D>

Common benchmarks on OnCourse (Benchmark)

"Do Now/Exit Ticket" Activity (Formative)

- Admit Tickets
- Anticipation Guide
- Common Benchmarks
- Compare & Contrast
- Create a Multimedia Poster
- DBQ's
- Define
- Describe
- Evaluate
- Evaluation rubrics
- Exit Tickets
- Explaining
- Fist- to-Five or Thumb-Ometer
- Illustration
- Journals
- KWL Chart
- Learning Center Activities
- Multimedia Reports
- Newspaper Headline
- Outline
- Question Stems
- Quickwrite
- Quizzes
- Red Light, Green Light
- Self- assessments
- Socratic Seminar
- Study Guide
- Surveys
- Teacher Observation Checklist
- Think, Pair, Share

- Think, Write, Pair, Share
- Top 10 List
- Unit review/Test prep
- Unit tests
- Web-Based Assessments
- Written Reports

Primary Resources & Materials

Glencoe McGraw-Hill Geometry 2014

Practice Glencoe Geometry

Study Guide Glencoe Geometry

Ancillary Resources

ALEKS

The Glencoe Personal Tutor Plus

The Glencoe Personal Tutor Plus(Spanish)

Kutasoftware Geometry

Technology Infusion

Create and have students complete exit tickets using Edulastic

{ <https://app.edulastic.com/#renderResource/close/Mjk0MjE2ODUwOA%3D%3D> } or Google forms

Create classes on Google classroom and post assignments, monitor student progress, and offer feedback.

Use geogebra to model problems.

Other technology that can be infused into this unit to enhance learning may include

- Youtube
- Khan academy
- Google Classroom

- GSuite
- Kutasoftware
- PodCasts
- Skype
- Twitter
- Ted Talks
- ALEKS
- QR Barcode Generator
- Calculator/Graphing calculator
- Flipgrid
- Peardeck
- Edulastic
- McGraw-Hill Education
- Desmos.com
- Geogebra.org

Win 8.1 Apps/Tools Pedagogy Wheel

Podcasts
 Photostory 3
 Kid Story Builder
 Music Maker Jam
 Paint A Story
 Office 365
 MS PowerPoint
 Stack 'Em Up
 NqSquared Numbers
 Physamajig
 Xylophone 8

Wikipedia
 Skydrive
 Lync
 SkyMap
 Skype
 Office 365
 Puzzle Touch
 Easy QR
 Memorylage
 Life Moments
 Word Cloud Maker

Where's Waldo?
 MS Excel
 Flipboard
 Office 365
 Nova Mindmapping

Ted Talks
 Record Voice Pen



Originally taken from <http://www.coetail.com/vzimmer/files/2013/02/iPadagogy-Wheel.001.jpg>
 And adapted for Windows 8.1 devices by Charlotte Beckhurst @CharBeckhurst

Alignment to 21st Century Skills & Technology

Develop mathematical thinking using real world problems in the Glencoe Interactive Student Guide Workbook https://catalog.mcgraw-hill.com/repository/private_data/DOC/50001167/94/30.pdf

Mastery and infusion of **21st Century Skills & Technology** and their Alignment to the core content areas is essential to student learning. The core content areas include:

- English Language Arts;
- Mathematics;
- Science and Scientific Inquiry (Next Generation);
- Technology;
- Visual and Performing Arts.

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.
TECH.8.1.12.A.3	Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.
TECH.8.1.12.F.CS1	Identify and define authentic problems and significant questions for investigation.

21st Century Skills/Interdisciplinary Themes

Glencoe -McGrawHill Resources:

Chapter Projects - Map Your Town, Scientific Method, Function of Lines in Construction, Classifying Triangles, Architecture: Triangular Design, Game Time, Picture This Photography & Tessellations, Graduation Planning, Time or Rebound

- Communication and Collaboration
- Creativity and Innovation
- Critical thinking and Problem Solving
- ICT (Information, Communications and Technology) Literacy

- Information Literacy
- Life and Career Skills
- Media Literacy

21st Century Skills

- Civic Literacy
- Environmental Literacy
- Financial, Economic, Business and Entrepreneurial Literacy
- Global Awareness
- Health Literacy

Differentiation

Glencoe-McGrawHill Resources:

Chapter Openers Animation

Student Anticipation Guide

Student Anticipation Guide(Spanish)

Teaching with Foldables

Math Triumphs: Foundations for Geometry

Interactive Student Guide

Personal Tutor

Personal Tutor (Spanish)

Kutasoftware Geometry

Differentiations:

- Small group instruction
- Small group assignments
- Extra time to complete assignments
- Pairing oral instruction with visuals
- Repeat directions

- Use manipulatives
- Center-based instruction
- Token economy
- Study guides
- Teacher reads assessments allowed
- Scheduled breaks
- Rephrase written directions
- Multisensory approaches
- Additional time
- Preview vocabulary
- Preview content & concepts
- Story guides
- Behavior management plan
- Highlight text
- Student(s) work with assigned partner
- Visual presentation
- Assistive technology
- Auditory presentations
- Large print edition
- Dictation to scribe
- Small group setting

Hi-Prep Differentiations:

- Alternative formative and summative assessments
- Choice boards
- Games and tournaments
- Group investigations
- Guided Reading
- Independent research and projects
- Interest groups
- Learning contracts
- Leveled rubrics
- Literature circles
- Multiple intelligence options
- Multiple texts
- Personal agendas
- Project-based learning
- Problem-based learning
- Stations/centers
- Think-Tac-Toes
- Tiered activities/assignments
- Tiered products
- Varying organizers for instructions

Lo-Prep Differentiations

- Choice of books or activities
- Cubing activities
- Exploration by interest
- Flexible grouping
- Goal setting with students
- Jigsaw

- Mini workshops to re-teach or extend skills
- Open-ended activities
- Think-Pair-Share
- Reading buddies
- Varied journal prompts
- Varied supplemental materials

Special Education Learning (IEP's & 504's)

Glencoe-McGrawHill Resources:

Chapter Openers Animation

Student Anticipation Guide

Teaching with Foldables

Math Triumphs: Foundations for Geometry

Personal Tutor

Kutasoftware Geometry

Interactive Student Notebooks

- printed copy of board work/notes provided
- additional time for skill mastery
- assistive technology
- behavior management plan
- Center-Based Instruction
- check work frequently for understanding
- computer or electronic device utilizes
- extended time on tests/ quizzes
- have student repeat directions to check for understanding
- highlighted text visual presentation

- modified assignment format
- modified test content
- modified test format
- modified test length
- multiple test sessions
- multi-sensory presentation
- preferential seating
- preview of content, concepts, and vocabulary
- Provide modifications as dictated in the student's IEP/504 plan
- reduced/shortened reading assignments
- Reduced/shortened written assignments
- secure attention before giving instruction/directions
- shortened assignments
- student working with an assigned partner
- teacher initiated weekly assignment sheet
- Use open book, study guides, test prototypes

English Language Learning (ELL)

Glencoe-McGrawHill Resources:

Chapter Openers Animation

Student Anticipation Guide(English & Spanish)

Teaching with Foldables

Math Triumphs: Foundations for Geometry

Interactive Student Guide

Personal Tutor (English and Spanish)

Kutasoftware Geometry

Khan Academy - Geometria Spanish Website

<https://es.khanacademy.org/math/geometry-home>

- teaching key aspects of a topic. Eliminate nonessential information
- using videos, illustrations, pictures, and drawings to explain or clarify
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning;
- allowing students to correct errors (looking for understanding)

- allowing the use of note cards or open-book during testing
- decreasing the amount of work presented or required
- having peers take notes or providing a copy of the teacher's notes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using computer word processing spell check and grammar check features
- using true/false, matching, or fill in the blank tests in lieu of essay tests

At Risk

Glencoe-McGrawHill Resources:

Chapter Projects

Chapter Openers Animation

Student Anticipation Guide

Student Anticipation Guide(Spanish)

Teaching with Foldables

Math Triumphs: Foundations for Geometry

Interactive Student Guide

Personal Tutor

Personal Tutor (Spanish)

Kutasoftware Geometry

Khan Academy Geometry Lessons

- allowing students to correct errors (looking for understanding)
- teaching key aspects of a topic. Eliminate nonessential information
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning
- allowing students to select from given choices
- allowing the use of note cards or open-book during testing
- collaborating (general education teacher and specialist) to modify vocabulary, omit or modify items to reflect objectives for the student, eliminate sections of the test, and determine how the grade will be determined prior to giving the test.
- decreasing the amount of work presented or required
- having peers take notes or providing a copy of the teacher's notes
- marking students' correct and acceptable work, not the mistakes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using authentic assessments with real-life problem-solving
- using true/false, matching, or fill in the blank tests in lieu of essay tests
- using videos, illustrations, pictures, and drawings to explain or clarify

Talented and Gifted Learning (T&G)

Glencoe-McGrawHill Resources:

Chapter Openers Animation

Student Anticipation Guide

Student Anticipation Guide(Spanish)

Teaching with Foldables

Interactive Student Guide

Personal Tutor

Personal Tutor (Spanish)

Math Forum

Kutasoftware Geometry

Khan Academy Lessons

SAT/ACT practice problems

- Above grade level placement option for qualified students
- Advanced problem-solving
- Allow students to work at a faster pace
- Cluster grouping
- Complete activities aligned with above grade level text using Benchmark results
- Create a blog or social media page about their unit
- Create a plan to solve an issue presented in the class or in a text
- Debate issues with research to support arguments
- Flexible skill grouping within a class or across grade level for rigor
- Higher order, critical & creative thinking skills, and discovery
- Multi-disciplinary unit and/or project
- Teacher-selected instructional strategies that are focused to provide challenge, engagement, and growth opportunities
- Utilize exploratory connections to higher-grade concepts
- Utilize project-based learning for greater depth of knowledge

Sample Lesson

Using the template below, please develop a **Sample Lesson** for the first unit only.

Unit Name:

NJSLS:

Interdisciplinary Connection:

Statement of Objective:

Anticipatory Set/Do Now:

Learning Activity:

Student Assessment/CFU's:

Materials:

21st Century Themes and Skills:

Differentiation/Modifications:

Integration of Technology: