

# Unit 4, Circles, Area & Volume

Content Area: **Math**  
Course(s): **Geometry**  
Time Period:  
Length: **43 Days**  
Status: **Published**

## **Title Section**

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## **Department of Curriculum and Instruction**



**Belleville Public Schools**

**Curriculum Guide**

## **Geometry A**

## **Circles, Area & Volume**

**Belleville Board of Education**

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Board Approved: September 23, 2019

## **Unit Overview**

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Students will explore and apply properties of circles as well as prove theorems dealing with tangents, secants, radians, arc lengths, and inscribed angles. Area and perimeter will be revisited and used to find areas and perimeters of irregular shapes as well as prove and apply properties of areas.

## **Enduring Understanding**

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Students will understand

the relationship between geometry and real life situations.

the different components of circles and how they relate to angle measurements.

that all circles are similar.

that angles inscribed on a diameter are right angles.

that the length of the arc intercepted by an angle is proportional to the radius.

that angles have properties in quadrilaterals inscribed in a circle.

the relationship between algebra and coordinate geometry.

that volume formulas can be used for cylinders, pyramids, cones, and spheres.

that 2-D and 3-D objects can be used to identify one another.

that Geometry helps us to understand the structure of space and the spatial relations around us

## **Essential Questions**

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What is the difference between a chord and a diameter of a circle?

How does the type of arc relate to its degree measure?

Through what point does a perpendicular bisector of a chord pass?

What is the relationship between the measure of an inscribed angle and the measure of its intercepted arc?

Why would two segments be congruent if they have a common endpoint and both are tangent to a circle?

What is the difference between chords and secants?

How is finding the measures of two chords that intersect inside a circle different from finding the measures of two chords that intersect outside the circle?

How would you write an equation for a circle, if you were given the diameter and the equations of the two tangent lines?

What is area and how do we find area of polygons and composite figures?

What is the difference between perimeter and area of polygons?

How can the coordinate plane and use of the distance formula be used to find area of polygons?

How do we find area of regular polygons?

What is the difference between circumference and area of circles?

What is geometric probability and how can area be used to find probabilities?

How are cross-sections of solids examined?

How do we find surface area of prisms, cylinders, pyramids, cones, and spheres?

Given a surface area, how can we work backwards to find a missing length of a solid.

How do we find volume of prisms, cylinders, pyramids, cones, and spheres?

How are the area formulas of polygons related to the volume formulas of solids?

How can solids be congruent or similar?

How do we find the ratio of areas and the ratio of volumes between similar solids?

**Exit Skills**

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By the end of this unit students will know:

How to identify and use parts of circles.

How to solve problems involving the circumference of a circle.

How to recognize major arcs, minor arcs, semicircles, & central angles and their measures.

How to find arc lengths.

How to recognize and use relationships between arcs & chords.

How to recognize and use relationships between chords and diameters.

How to find measures of inscribed angles.

How to find measures of angles of inscribed polygons.

How to use properties of tangents.

How to solve problems involving circumscribed polygons.

How to find measures of angles formed by lines intersecting on or inside a circle.

How to find measures of angles formed by lines intersecting outside the circle.

How to find measures of segments that intersect in the interior of the circle.

How to find measures of segments that intersect in the exterior of the circle.

How to write equations of circles & graph circles on a coordinate plane.

How to find perimeters and areas of parallelograms.

How to determine whether points on a coordinate plane define a parallelogram.

How to find areas of triangles.

How to find areas of trapezoids and rhombi.

How to find areas of regular polygons.

How to find areas of circles.

How to find areas of composite figures.

How to find areas of composite figures on the coordinate plane.

How to solve problems involving geometric probability.

How to solve problems involving sectors and segments of circles.

How to investigate cross sections of three-dimensional figures.

How to find lateral areas of prisms.

How to find surface area of prisms.

How to find lateral area of cylinders.

How to find surface area of cylinders.

How to find lateral area of regular pyramids.

How to find surface areas of regular pyramids.

How to find lateral areas of cones.

How to find surface area of cones.

How to recognize and define basic properties of spheres.

How to find surface areas of spheres.

How to find volumes of prisms.

How to find volumes of cylinders.

How to find volumes of pyramids.

How to find volumes of cones.

How to find volumes of spheres.

How to solve problems involving volumes of spheres.

How to identify congruent or similar solids.

How to state the properties of similar solids.

How to graph solids in space.

How to use the Distance and Midpoint formulas for points in space.

## **New Jersey Student Learning Standards (NJSLS)**

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MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.G-CO.D.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.



MA.G-C.A.1	Prove that all circles are similar.
MA.G-C.A.2	Identify and describe relationships among inscribed angles, radii, and chords.
MA.G-C.A.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
MA.G-C.A.4	Construct a tangent line from a point outside a given circle to the circle.
MA.G-C.B.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
MA.G-GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
MA.G-GPE.A.2	Derive the equation of a parabola given a focus and directrix.
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
MA.G-GMD.A.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
MA.G-GMD.A.2	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
MA.G-GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
MA.G-GMD.B.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

## Interdisciplinary Connections

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9.3.12.FN.1	Utilize mathematical concepts, skills and problem solving to obtain necessary information for decision making in the finance industry.
12.9.3.ST-ET.1	Use STEM concepts and processes to solve problems involving design and/or production.
12.9.3.ST-SM.2	Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.
LA.9-10.W.9-10.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence. Introduce precise claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that establishes clear relationships among claim(s), counterclaims, reasons, and evidence.
9-12.HS-PS1-4.2	Modeling in 9–12 builds on K–8 and progresses to using, synthesizing, and developing models to predict and show relationships among variables between systems and their components in the natural and designed worlds.
9-12.HS-PS1-3.3	Planning and Carrying Out Investigations
9-12.HS-PS2-4.5	Mathematical and computational thinking at the 9–12 level builds on K–8 and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for

statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.

## **Learning Objectives**

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Identify and use parts of circles.

Solve problems involving the circumference of circles.

Recognize major arcs, minor arcs, semicircles, central angles and their measures.

Find arc lengths.

Recognize and use relationships between arcs and chords.

Recognize and use relationships between chords and diameters.

Find measures of inscribed angles.

Find measures of inscribed polygons.

Use properties of tangents.

Solve problems involving circumscribed polygons.

Find measures of angles formed by lines intersecting on or inside a circle..

Find measures of angles formed by lines intersecting outside the circle.

Find measures of segments that intersect in the interior of the circle.

Find measures of segments that intersect in the exterior of the circle.

Write equations of circles and graphing circles on a coordinate plane.

Find perimeters and areas of parallelograms.

Determine whether points on a coordinate plane define a parallelogram.

Find areas of regular polygons.

Find areas of circles.

Find areas of composite figures.

Find areas of composite figures on the coordinate plane.

Solve problems involving geometric probability.

Solve problems involving sectors and segments of circles.

Investigate cross sections of three-dimensional figures.

Find lateral areas of prisms.

Find surface area of prisms.

Find lateral area of cylinders.

Find surface area of cylinders.

Find lateral area of regular pyramids.

Find surface areas of regular pyramids.

Find lateral areas of cones.

Find surface area of cones.

Recognize and define basic properties of spheres.

Find surface areas of spheres.

Find volumes of prisms.

Find volumes of cylinders.

Find volumes of pyramids.

Find volumes of cones.

Find volumes of spheres.

Solve problems involving volumes of spheres.

Identify congruent or similar solids.

State the properties of similar solids.

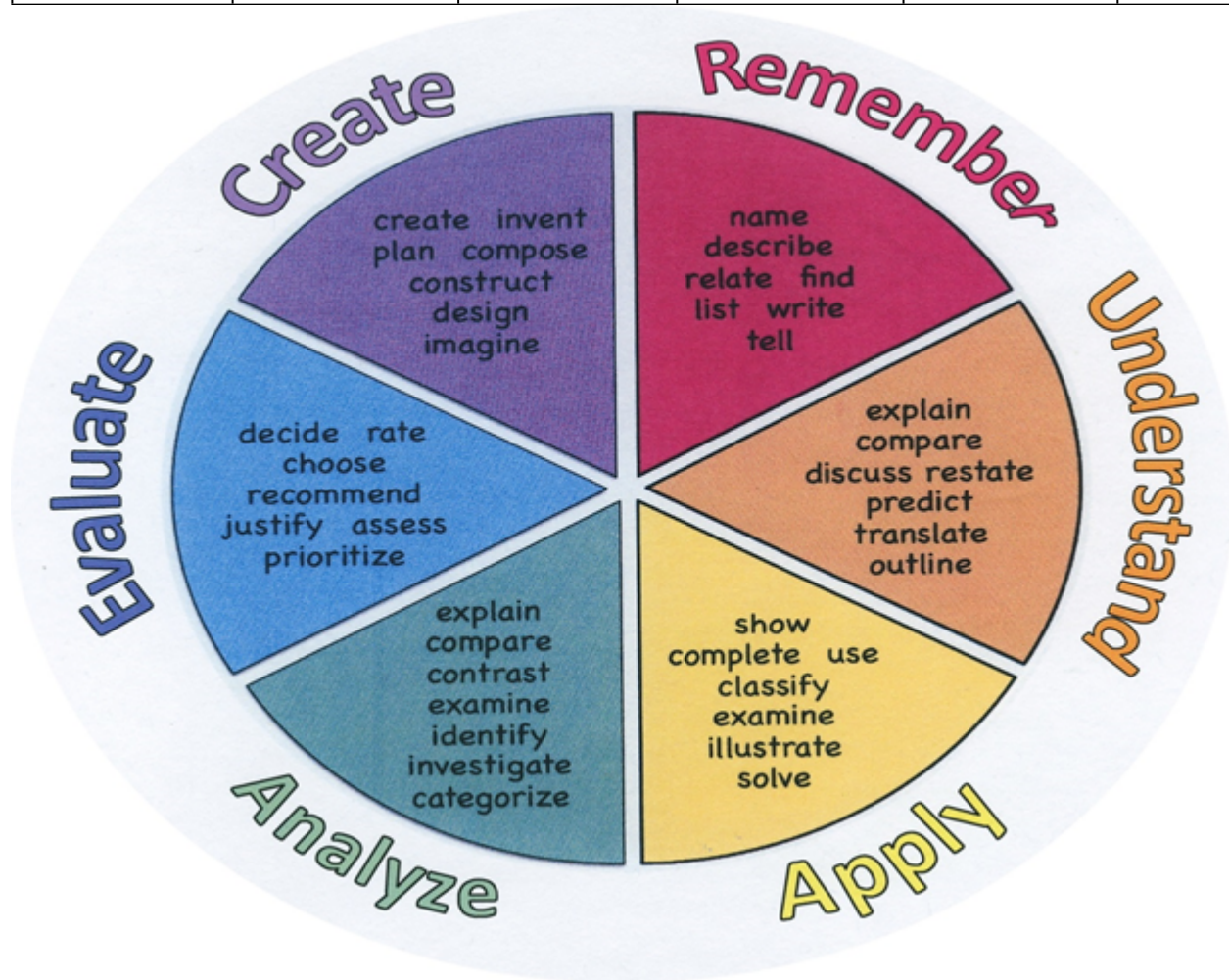
Graph solids in space.

Use the Distance and Midpoint formulas for points in space.

**Action Verbs:** Below are examples of action verbs associated with each level of the Revised Bloom's Taxonomy.

Remember	Understand	Apply	Analyze	Evaluate	Create
Choose	Classify	Choose	Categorize	Appraise	Combine
Describe	Defend	Dramatize	Classify	Judge	Compose
Define	Demonstrate	Explain	Compare	Criticize	Construct
Label	Distinguish	Generalize	Differentiate	Defend	Design
List	Explain	Judge	Distinguish	Compare	Develop
Locate	Express	Organize	Identify	Assess	Formulate
Match	Extend	Paint	Infer	Conclude	Hypothesize
Memorize	Give Examples	Prepare	Point out	Contrast	Invent
Name	Illustrate	Produce	Select	Critique	Make
Omit	Indicate	Select	Subdivide	Determine	Originate
Recite	Interrelate	Show	Survey	Grade	Organize
Select	Interpret	Sketch	Arrange	Justify	Plan
State	Infer	Solve	Breakdown	Measure	Produce
Count	Match	Use	Combine	Rank	Role Play
Draw	Paraphrase	Add	Detect	Rate	Drive
Outline	Represent	Calculate	Diagram	Support	Devise
Point	Restate	Change	Discriminate	Test	Generate
Quote	Rewrite	Classify	Illustrate		Integrate
Recall	Select	Complete	Outline		Prescribe
Recognize	Show	Compute	Point out		Propose
Repeat	Summarize	Discover	Separate		Reconstruct
Reproduce	Tell	Divide			Revise
	Translate	Examine			Rewrite
	Associate	Graph			Transform
	Compute	Interpolate			
	Convert	Manipulate			
	Discuss	Modify			

	Estimate Extrapolate Generalize Predict	Operate Subtract			
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## Suggested Activities & Best Practices

Textbook, eAssessment, supplemental materials:

<https://my.mheducation.com/login>

AI Assessment and Learning System:

<https://www.aleks.com/>

Mindset:

<https://www.youtube.com/watch?v=3icoSeGqQtY>

<http://www.youcubed.org/wp-content/uploads/Positive-Classroom-Norms2.pdf>

Coaching Corner:

<https://sites.google.com/belleville.k12.nj.us/thecoachingcorner/home>

Videos:

<https://www.mathantics.com/>



<https://mashupmath.com/high-school-math-lessons>

Constructions:

<http://www.mathopenref.com/tocs/constructionstoc.html>

Geometry Activities:

<https://math.rice.edu/~lanius/Geom/>

Geogebra:

<https://www.geogebra.org/a/7?lang=en>

Desmos:

<https://teacher.desmos.com/activitybuilder/custom/5664e067eb08d9501576caa0>

Kahoot:

<https://create.kahoot.it/>

## **Assessment Evidence - Checking for Understanding (CFU)**

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Glencoe McGraw Hill : Chapter Assessments, Midchapter Assessments (Summative) -

<https://connected.mcgraw-hill.com/c2j/assetBuckets.assess.do?bookId=DFRTR2RBH9YT25W7OSMM6J3XM1&selectedCategoryId=3KTMO6D7VZ6SJ4YD3XNOQB3O44>

EAssessment test generator (Summative): <https://assess.k12.mhedu.com/Instructor/TestGenerator.aspx>

Edulastic Formative Assessments (Formative):

<https://app.edulastic.com/#renderResource/close/Mjk0MjE2ODUwOA%3D%3D>

Common benchmarks on OnCourse (Benchmark)

"Do Now/Exit Ticket" Activity (Formative)

- Admit Tickets
- Anticipation Guide
- Common Benchmarks
- Compare & Contrast
- Create a Multimedia Poster
- DBQ's
- Define
- Describe
- Evaluate
- Evaluation rubrics
- Exit Tickets
- Explaining
- Fist- to-Five or Thumb-Ometer
- Illustration
- Journals
- KWL Chart
- Learning Center Activities
- Multimedia Reports
- Newspaper Headline
- Outline
- Question Stems
- Quickwrite



- Quizzes
- Red Light, Green Light
- Self- assessments
- Socratic Seminar
- Study Guide
- Surveys
- Teacher Observation Checklist
- Think, Pair, Share
- Think, Write, Pair, Share
- Top 10 List
- Unit review/Test prep
- Unit tests
- Web-Based Assessments
- Written Reports

## **Primary Resources & Materials**

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Glencoe McGraw-Hill Geometry 2014

Practice Glencoe Geometry

Study Guide Glencoe Geometry

## **Ancillary Resources**

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ALEKS

The Glencoe Personal Tutor Plus

The Glencoe Personal Tutor Plus(Spanish)

Kutasoftware Geometry

## Technology Infusion

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Create and have students complete exit tickets using Edulastic

{ <https://app.edulastic.com/#renderResource/close/Mjk0MjE2ODUwOA%3D%3D> } or Google forms

Create classes on Google classroom and post assignments, monitor student progress, and offer feedback.

Use geogebra to model problems.

Other technology that can be infused into this unit to enhance learning may include

- Youtube
- Khan academy
- Google Classroom
- GSuite
- Kutasoftware
- PodCasts
- Skype
- Twitter
- Ted Talks
- ALEKS
- QR Barcode Generator
- Calculator/Graphing calculator
- Flipgrid
- Peardeck
- Edulastic
- McGraw-Hill Education
- Desmos.com
- Geogebra.org

Originally taken from <http://www.coetall.com/vzimmer/files/2013/02/IPadagogy-Wheel.001.jpg>  
And adapted for Windows 8.1 devices by Charlotte Beckhurst @CharBeckhurst

Wikipedia  
Skydrive  
Lync  
SkyMap  
Skype  
Office 365  
Puzzle Touch  
Easy QR  
Memorylage  
Life Moments  
Word Cloud Maker

Ted Talks  
Record Voice Pen



## **Alignment to 21st Century Skills & Technology**

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Develop mathematical thinking using real world problems in the Glencoe Interactive Student Guide Workbook [https://catalog.mcgraw-hill.com/repository/private\\_data/DOC/50001167/94/30.pdf](https://catalog.mcgraw-hill.com/repository/private_data/DOC/50001167/94/30.pdf)

Mastery and infusion of **21st Century Skills & Technology** and their Alignment to the core content areas is essential to student learning. The core content areas include:

- English Language Arts;
- Mathematics;
- Science and Scientific Inquiry (Next Generation);
- Social Studies and Economics;
- World languages;
- Technology;
- Visual and Performing Arts.

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.
TECH.8.1.12.A.3	Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.
TECH.8.1.12.F.CS1	Identify and define authentic problems and significant questions for investigation.

## **21st Century Skills/Interdisciplinary Themes**

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Glencoe -McGrawHill Resources:

Chapter Projects - Map Your Town, Scientific Method,Function of Lines in Construction, Classifying Triangles, Architecture:Triangular Design, Game Time, Picture This Photography &Tesselations, Graduation Planning, Time or Rebound

- Communication and Collaboration

- Creativity and Innovation
- Critical thinking and Problem Solving
- ICT (Information, Communications and Technology) Literacy
- Information Literacy
- Life and Career Skills
- Media Literacy

## **21st Century Skills**

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- Civic Literacy
- Environmental Literacy
- Financial, Economic, Business and Entrepreneurial Literacy
- Global Awareness
- Health Literacy

## **Differentiation**

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Glencoe-McGrawHill Resources:

*Chapter Openers Animation*

*Student Anticipation Guide*

*Student Anticipation Guide(Spanish)*

*Teaching with Foldables*

*Math Triumphs: Foundations for Geometry*

*Interactive Student Guide*

*Personal Tutor*

*Personal Tutor (Spanish)*

Kutasoftware Geometry

**Differentiations:**

- Small group instruction
- Small group assignments
- Extra time to complete assignments
- Pairing oral instruction with visuals
- Repeat directions
- Use manipulatives
- Center-based instruction
- Token economy
- Study guides
- Teacher reads assessments allowed
- Scheduled breaks
- Rephrase written directions
- Multisensory approaches
- Additional time
- Preview vocabulary
- Preview content & concepts
- Story guides
- Behavior management plan
- Highlight text
- Student(s) work with assigned partner
- Visual presentation
- Assistive technology
- Auditory presentations
- Large print edition
- Dictation to scribe
- Small group setting

**Hi-Prep Differentiations:**

- Alternative formative and summative assessments
- Choice boards
- Games and tournaments
- Group investigations
- Guided Reading
- Independent research and projects
- Interest groups
- Learning contracts
- Leveled rubrics
- Literature circles
- Multiple intelligence options
- Multiple texts
- Personal agendas
- Project-based learning
- Problem-based learning
- Stations/centers
- Think-Tac-Toes
- Tiered activities/assignments
- Tiered products

- Varying organizers for instructions

#### **Lo-Prep Differentiations**

- Choice of books or activities
- Cubing activities
- Exploration by interest
- Flexible grouping
- Goal setting with students
- Jigsaw
- Mini workshops to re-teach or extend skills
- Open-ended activities
- Think-Pair-Share
- Reading buddies
- Varied journal prompts
- Varied supplemental materials

## **Special Education Learning (IEP's & 504's)**

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Glencoe-McGrawHill Resources:

*Chapter Openers Animation*

*Student Anticipation Guide*

*Teaching with Foldables*

*Math Triumphs: Foundations for Geometry*

*Personal Tutor*

## Interactive Student Notebooks

- printed copy of board work/notes provided
- additional time for skill mastery
- assistive technology
- behavior management plan
- Center-Based Instruction
- check work frequently for understanding
- computer or electronic device utilizes
- extended time on tests/ quizzes
- have student repeat directions to check for understanding
- highlighted text visual presentation
- modified assignment format
- modified test content
- modified test format
- modified test length
- multi-sensory presentation
- multiple test sessions
- preferential seating
- preview of content, concepts, and vocabulary
- Provide modifications as dictated in the student's IEP/504 plan
- reduced/shortened reading assignments
- Reduced/shortened written assignments
- secure attention before giving instruction/directions
- shortened assignments
- student working with an assigned partner
- teacher initiated weekly assignment sheet
- Use open book, study guides, test prototypes

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## English Language Learning (ELL)

Glencoe-McGrawHill Resources:

*Chapter Openers Animation*



*Student Anticipation Guide(English & Spanish)*

*Teaching with Foldables*

*Math Triumphs: Foundations for Geometry*

*Interactive Student Guide*

*Personal Tutor (English and Spanish)*

Kutasoftware Geometry

Khan Academy - Geometria Spanish Website

<https://es.khanacademy.org/math/geometry-home>

- teaching key aspects of a topic. Eliminate nonessential information
- using videos, illustrations, pictures, and drawings to explain or clarify
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning;
- allowing students to correct errors (looking for understanding)
- allowing the use of note cards or open-book during testing
- decreasing the amount of work presented or required
- having peers take notes or providing a copy of the teacher's notes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using computer word processing spell check and grammar check features
- using true/false, matching, or fill in the blank tests in lieu of essay tests

## At Risk

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Glencoe-McGrawHill Resources:

*Chapter Projects*

*Chapter Openers Animation*

*Student Anticipation Guide*

*Student Anticipation Guide(Spanish)*

*Teaching with Foldables*

*Math Triumphs: Foundations for Geometry*

*Interactive Student Guide*

*Personal Tutor*

*Personal Tutor (Spanish)*

Kutasoftware Geometry

Khan Academy Geometry Lessons

- allowing students to correct errors (looking for understanding)
- teaching key aspects of a topic. Eliminate nonessential information
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning
- allowing students to select from given choices
- allowing the use of note cards or open-book during testing
- collaborating (general education teacher and specialist) to modify vocabulary, omit or modify items to reflect objectives for the student, eliminate sections of the test, and determine how the grade will be determined prior to giving the test.

- decreasing the amount of work presented or required
- having peers take notes or providing a copy of the teacher's notes
- marking students' correct and acceptable work, not the mistakes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using authentic assessments with real-life problem-solving
- using true/false, matching, or fill in the blank tests in lieu of essay tests
- using videos, illustrations, pictures, and drawings to explain or clarify

## **Talented and Gifted Learning (T&G)**

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Glencoe-McGrawHill Resources:

*Chapter Openers Animation*

*Student Anticipation Guide*

*Student Anticipation Guide(Spanish)*

*Teaching with Foldables*

*Interactive Student Guide*

*Personal Tutor*

*Personal Tutor (Spanish)*

*Math Forum*

Kutasoftware Geometry

Khan Academy Lessons

SAT/ACT practice problems

- Above grade level placement option for qualified students
- Advanced problem-solving
- Allow students to work at a faster pace
- Cluster grouping
- Complete activities aligned with above grade level text using Benchmark results
- Create a blog or social media page about their unit
- Create a plan to solve an issue presented in the class or in a text
- Debate issues with research to support arguments

- Flexible skill grouping within a class or across grade level for rigor
- Higher order, critical & creative thinking skills, and discovery
- Multi-disciplinary unit and/or project
- Teacher-selected instructional strategies that are focused to provide challenge, engagement, and growth opportunities
- Utilize exploratory connections to higher-grade concepts
- Utilize project-based learning for greater depth of knowledge

## Sample Lesson

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Using the template below, please develop a **Sample Lesson** for the first unit only.

Unit Name:

NJSLS:

Interdisciplinary Connection:

Statement of Objective:

Anticipatory Set/Do Now:

Learning Activity:

Student Assessment/CFU's:

Materials:

21st Century Themes and Skills:

Differentiation/Modifications:

Integration of Technology: