

# Unit #4: Expected Value and Continuous Random Variables

Content Area: **Mathematics**  
Course(s): **Probability**  
Time Period: **Semester 1 & 2**  
Length: **6 weeks**  
Status: **Published**

## Standards

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MA.S-MD.A.1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
MA.S-MD.A.2	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
MA.S-MD.A.3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
MA.S-MD.A.4	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.
MA.S-MD.B.5	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
MA.S-MD.B.6	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
MA.S-MD.B.7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

## Enduring Understandings

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1. A **continuous probability distribution** is a *probability distribution* that has a [probability density function](#). Mathematicians also call such a distribution **absolutely continuous**, since its [cumulative distribution function](#) is [absolutely continuous](#) with respect to the [Lebesgue measure](#)  $\lambda$ . If the distribution of  $X$  is continuous, then  $X$  is called a **continuous random variable**.
2. In [probability theory](#), the expected value of a [random variable](#) is intuitively the long-run average value of repetitions of the experiment it represents. For example, the expected value of a die roll is 3.5 because, roughly speaking, the average of an extremely large number of dice rolls is practically always nearly equal to 3.5.
3. normal (or Gaussian) distribution is a very commonly occurring [continuous probability distribution](#)—a function that tells the probability that any real observation will fall between any two real limits or [real numbers](#), as the curve approaches zero on either side. Normal distributions are extremely important in [statistics](#) and are often used in the [natural](#) and [social sciences](#) for real-valued [random variables](#) whose distributions are not known.
4. Multinomial distribution is a generalization of the [binomial distribution](#). For  $n$  [independent](#) trials each of which leads to a success for exactly one of  $k$  categories, with each category having a given fixed success probability, the multinomial distribution gives the probability of any particular combination of

**numbers of successes for the various categories.**

### **Essential Questions**

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1. What is a continuous random variable, and how is it different than a discrete random variable?
2. How can we calculate the expected value of a random variable?
3. What is normal distribution and how can we use it to calculate probabilities?
4. How is multinomial distribution related to binomial distribution?

### **Knowledge and Skills**

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Students will be able to:

- Know what a continuous random variable is and how it can be used to calculate the probability of events within a random experiment
- Calculate expected value and use expected value to help determine the likely outcome of a random experiment
- Understand normal distribution and use a "Z-chart" to determine the probability of normally distributed events
- Use multinomial distribution as a generalization of the binomial distribution, and use it to calculate probabilities.

### **Resources**

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Online resources which include, but not limited to: Delta Math and Class Kick.