Unit 4: Quadratic Functions and Radical Expressions

Content Area: **Mathematics**
Course(s): **Algebra 3CP**
Time Period: **Semester 1**
Length: **4 weeks**
Status: **Published**

Enduring Understanding

Students will understand that:

- Quadratic equations can be solved using multiple methods, each appropriate for different situations.
- Factoring reveals important structural relationships within quadratic expressions.
- Square roots and radical expressions represent quantities that can be simplified and manipulated according to established properties.
- Complex numbers extend the real number system and allow solutions to equations with no real solutions.
- The quadratic formula provides a universal method for solving quadratic equations.
- Quadratic functions can be represented algebraically, graphically, numerically, and verbally.
- Different forms of a quadratic equation reveal different characteristics of the graph.
- Technology can enhance mathematical understanding and support problem-solving.
- Quadratic functions effectively model many real-world phenomena involving maximums, minimums, and projectile motion.

Unit Description

This unit develops students' understanding of quadratic functions and equations through multiple solution methods, including factoring, square roots, and the quadratic formula. Students explore operations involving radicals and complex numbers while examining the structure and graphical representations of quadratic relationships. Emphasis is placed on conceptual understanding, mathematical modeling, technology integration, and real-world problem solving. Through visual representations, guided practice, and graphing technology, students learn to connect algebraic, numeric, and graphical representations of quadratic functions and interpret their significance in contextual situations.

Standards

MA.K-12.4	Model with mathematics.
MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given

	a verbal description of the relationship.
MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MA.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.A-APR.B	Understand the relationship between zeros and factors of polynomials
MA.A-APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
MA.F-IF.C.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.A-REI.B.4	Solve quadratic equations in one variable.
MA.A-REI.B.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
MA.A-REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Essential Questions

How can the structure of a quadratic expression help determine the most efficient solution method?

Why are multiple methods available for solving quadratic equations?

How do radical expressions relate to quadratic equations?

Why were complex numbers developed, and how do they expand the number system?

What information can be learned from the graph of a quadratic function?

How are algebraic and graphical representations of quadratics connected?

When is the quadratic formula the most appropriate method for solving an equation?

How can quadratic functions be used to model and solve real-world problems?

How does changing the form of a quadratic equation reveal different characteristics of the function?

How can technology support mathematical analysis and decision-making?

Knowledge and Skills

- Identify and label the vertex, x-intercepts, y-intercept, and axis of symmetry from a graph or a table.
- Convert and match quadratic equations between Standard, Vertex, and Intercept forms.
- Input quadratic equations into a Desmos graphing calculator to locate critical structural points.
- Translate descriptive word problems into quadratic functions and solve for peak values (vertex) or boundary points (intercepts) within context.

Transfer Goals

Students will independently apply their understanding of quadratic relationships, radical expressions, and complex numbers to solve unfamiliar mathematical and real-world problems. They will select appropriate solution strategies, interpret multiple representations of functions, utilize technology effectively, and communicate mathematical reasoning with precision. Students will transfer their knowledge to applications involving optimization, motion, engineering, finance, science, and other situations where quadratic relationships naturally occur. Through strategic problem solving, they will develop confidence in analyzing patterns, modeling quantitative relationships, and making informed decisions based on mathematical evidence.

Resources

Intermediate Algebra with Applications 5/6th ed by Aufmann/Barker/Lockwood

Online resources which include, but are not limited to: Desmos Graphing Calculator, Class Kick, Delta Math, Khan Academy, GeoGebra, and CK-12 Foundation's Algebra 2 with Trigonometry Concepts by CK-12/Gloag/ Rawley, last modified April 12, 2024