*Math Unit 4 - Additional Topics

Content Area: **Mathematics**
Course(s): **ACT/SAT Prep**
Time Period: **December**
Length: **5 Blocks**
Status: **Published**

Enduring Understandings

Congruent triangles have corresponding parts that are congruent, which can help us prove other properties about lines and special triangles.

Similar triangles can be used to find lengths and distances.

Trigonometry uses properties of similar right triangles to determine common ratios between side lengths and acute angle measures. Trigonometry can find lengths and angles given limited information.

The circle has many specialized properties and relationships and the knowledge of them help us to determine more relationships in the circle.

Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Essential Questions

What information is needed to prove two triangles congruent?

How are similar triangles used in real life?

How can trigonometry be used in real life?

Why are circle properties helpful?

Content

Geometry Concepts

- Proofs Involving Parallel Lines and Angle Pairs
- Proving Triangles Congruent
- Dilations and their Properties
- Similar Polygons
- Triangle Similarity
- Special Right Triangles
- Finding Sides and Angles using Trigonometric Ratios
- Applications of Trigonometry
- Circle Properties
- Radians, Arc Length, Area

Skills

Solve real-world and mathematical problems about a geometric figure or an object that can be modeled by a geometric figure using given information such as length, area, surface area, or volume

Use concepts and theorems relating to congruence and similarity of triangles to solve problems

Solve problems in a variety of contexts using the Pythagorean theorem, right triangle trigonometry, and the properties of special right triangles

Use definitions, properties, and theorems relating to circles and parts of circles, such as radii, diameters, tangents, angles, arcs, arc lengths, and sector areas, to solve problems

Convert between angle measures in degrees and radians.

Complete the square in an equation representing a circle to determine properties of the circle when it is graphed in the xy-plane, and use the distance formula in problems related to circles

Apply knowledge and understanding of the complex number system to add, subtract, multiply, and divide with complex numbers and solve problems.

SAT Emphasized Skills

- Making area and volume calculations in context
- Investigating lines, angles, triangles, and circles theorems
- Working with trigonometric functions

Resources

https://www.khanacademy.org/test-prep/sat/sat-math-practice#new-sat-additional-topics-math

Standards

The Complex Number System N -CN

A. Perform arithmetic operations with complex numbers.

2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Congruence G-CO

B. Understand congruence in terms of rigid motions

8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Similarity, Right Triangles, and Trigonometry G-SRT

B. Prove theorems involving similarity

5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in

geometric figures.

C. Define trigonometric ratios and solve problems involving right triangles

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Circles G-C

A. Understand and apply theorems about circles

2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Geometric Measurement and Dimension G-GMD

A. Explain volume formulas and use them to solve problems

3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the

symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MA.G-CO.B.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
MA.N-CN.A.1	Know there is a complex number □ such that □² = -1, and every complex number has the

	form □ + □□ with □ and □ real.
MA.G-SRT.B.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
MA.G-SRT.C.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
MA.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
MA.G-C.A.2	Identify and describe relationships among inscribed angles, radii, and chords.
MA.G-GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.