Unit 1 Limits

Content Area: **Mathematics**
Course(s): **Calculus Honors**
Time Period: **Marking Period 1**
Length: **10 Blocks**
Status: **Published**

Transfer Skills

Limits: Taking it to the Limits

In this unit, students will evaluate limits numerically, graphically, and analytically as a means of transitions from Pre-Calculus procedures to the Calculus process of differentiation.

Enduring Understandings

The concept of limits is one of the foundations of calculus.

The concept of limits can be applied throughout mathematics.

Essential Questions

What does the slope of a line represent?

What is calculus?

What is a limit?

When does a limit not exist?

What role do limits play in calculus?

How are limits and continuity related?

Content

Notes:

* Begin with describing the tangent line and rates of change problems as the big ideas for this unit

* A conceptual look at limits using graphs is helpful before your begin calculating them

* Properties of limits are sum/difference, product/quotient, scalar multiplication, limits raised to an exponent, limit of a constant

* Trig limits are generally a weak point here

Vocabulary

average rate of change

secant line

instantaneous rate of change

tangent line

limits

continutity

Intermediate Value Theorem

Skills

Find the average rate of change of a function over an interval

Determine the slope of a secant line through a curve

Define the instantaneous rate of change of a function at a given point

Write the equation of a tangent line

Find the slope of a curve at a point using a table

Determine the value of one and two sided limits using a table and a graph

Use properties of limits to evaluate limits

Evaluate limits algebraically using cancelation, rationalization, and special trigonometric limits

Evaluate limits approaching infinity

Relate limits to continutity

Use the Intermediate Value Theorem to verify function values

Resources

Text:

James Stewart Calculus Eighth Edition

Graphing Calculator

Online Resources:

Khan Academy

Geogebra

Desmos

Standards

NJSLS 2016

Arithmetic with Polynomials and Rational Expressions

A -APR A. Perform arithmetic operations on polynomials

B. Understand the relationship between zeros and factors of polynomials

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Seeing Structure in Expressions

A-SSE B. Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★

a. Factor a quadratic expression to reveal the zeros of the function it defines.

Interpreting Functions

F-IF A. Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

B. Interpret functions that arise in applications in terms of the context

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★

Mathematics | Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MA.A-APR	Arithmetic with Polynomials and Rational Expressions
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.A-APR.B	Understand the relationship between zeros and factors of polynomials
CCSS.Math.Content.HSA-APR	Arithmetic with Polynomials and Rational Expressions
CCSS.Math.Content.HSA-APR.B	Understand the relationship between zeros and factors of polynomials
CCSS.Math.Content.HSA-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros

	to construct a rough graph of the function defined by the polynomial.
CCSS.Math.Content.HSA-SSE	Seeing Structure in Expressions
CCSS.Math.Content.HSA-SSE.B	Write expressions in equivalent forms to solve problems
CCSS.Math.Content.HSA-SSE.B.3.a	Factor a quadratic expression to reveal the zeros of the function it defines.
CCSS.Math.Content.HSF-IF	Interpreting Functions
CCSS.Math.Content.HSF-IF.A	Understand the concept of a function and use function notation
CCSS.Math.Content.HSF-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
CCSS.Math.Content.HSF-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
CCSS.Math.Content.HSF-IF.B	Interpret functions that arise in applications in terms of the context
CCSS.Math.Content.HSF-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
CCSS.Math.Practice.MP1	Make sense of problems and persevere in solving them.
CCSS.Math.Practice.MP2	Reason abstractly and quantitatively.
CCSS.Math.Practice.MP5	Use appropriate tools strategically.
CCSS.Math.Practice.MP6	Attend to precision.
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