

# \*Unit 5 Trigonometric Functions

Content Area: **Mathematics**  
Course(s): **Trigonometry and Analytical Geometry**  
Time Period: **February**  
Length: **20 Blocks**  
Status: **Published**

## **Transfer Skills**

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Trigonometry involves patterns that can be applied throughout all aspects of trigonometry. These patterns can be applied to the future learning of mathematics.

## **Enduring Understandings**

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Right triangle trigonometry allows for indirect measurement of triangles with less given and variable information than previous geometric studies.

There are fixed relationships between the sides and angles of a right triangle.

There are multiple ways of solving right triangles. The methods used will be based upon the information provided.

Solving right triangles have many practical applications in the real world.

When you are given two sides and the included angle for a triangle you can use the law of cosines to solve the triangle.

When you are given two sides and the opposite angles for an oblique triangle you can use the law of sines to solve the triangle.

The law of cosines and the law of sines can be used in conjunction with each other to solve an oblique triangle.

Use properties of the law of the sines and cosines to find the area of an oblique triangle.

## **Essential Questions**

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Why is radian measure useful?

What happens to the length of an arc intercepted by a given central angle of a circle if the radius of the circle is doubled? Why?

How are trigonometric functions evaluated using the unit circle?

How can trigonometry be used in triangles that are "oblique"?

How do we address the ambiguous case of the law of sines?

How do we determine the number of solutions in the ambiguous case?

How is the Law of Sines used to model and solve real life problems?

## **Content**

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### **Vocabulary:**

Sine

Cosine

Tangent

Cosecant

Secant

Cotangent

Adjacent

Hypotenuse

Radian

Degree

Co-terminal

Central Angle

Arc Length

Sector

Standard Position

Terminal Side

Law of Sines

Law of Cosines

Ambiguous

Oblique

Heron's formula

### **Red Hot Topics:**

Rationalizing the denominator

### **Skills**

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Convert radian and degree measure

Describe angles

Find co-terminals and reference angles

Find the length of an arc, given the measure of the central angle

Find the central angle, given the measure of the arc and radius

Find the radius using the arc and central angle

Find the area of a sector

Find the exact values of the six trigonometric functions of an angle in standard position given a point on its terminal side

Find exact values for the six trigonometric functions of special angles and their multiples

Solve right triangles using trigonometry

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Determine whether a triangle has zero, one, or two solutions

Solve triangles by using the law of sines

Solve triangles by using the laws of cosines

Find the area of triangles

## Resources

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Content Vocabulary

Practices quizzes

Teacher website

[www.KhanAcademy.org](http://www.KhanAcademy.org)

[www.Desmos.com](http://www.Desmos.com)

## Standards

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### Trigonometric Functions F-TF

#### A. Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosines, and tangent for  $\pi \pm x$ ,  $\pi + x$ , and  $2\pi - x$  in terms of their values for  $x$ , where  $x$  is any real number.

### Similarity, Right Triangles, and Trigonometry G-SRT

#### C. Define trigonometric ratios and solve problems involving right triangles

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. □

#### D. Apply trigonometry to general triangles

10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.

11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

## **Circles G-C**

### **B. Find arc lengths and areas of sectors of circles**

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

### **MP1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### **MP4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

### **MP6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in

discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

### **MP8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

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| MA.G-SRT      | Similarity, Right Triangles, and Trigonometry  |
| MA.G-SRT.C    | Define trigonometric ratios and solve problems involving right triangles   |
| MA.G-SRT.C.8  | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.   |
| MA.G-SRT.D    | Apply trigonometry to general triangles  |
| MA.G-SRT.D.10 | Prove the Laws of Sines and Cosines and use them to solve problems.  |
| MA.G-SRT.D.11 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).   |
| MA.G-C        | Circles  |
| MA.G-C.B      | Find arc lengths and areas of sectors of circles   |
| MA.G-C.B.5    | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.   |
| MA.F-TF       | Trigonometric Functions  |
| MA.F-TF.A     | Extend the domain of trigonometric functions using the unit circle   |
| MA.F-TF.A.1   | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.  |
| MA.F-TF.A.2   | Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.  |
| MA.F-TF.A.3   | Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$ , $\pi/4$ and $\pi/6$ , and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$ , $\pi + x$ , and $2\pi - x$ in terms of their values for $x$ , where $x$ is any real number. |

