# \*Unit 2 Systems of Equations and Inequalities

Content Area:MathematicsCourse(s):Trigonometry and Analytical GeometryTime Period:OctoberLength:10 BlocksStatus:Published

# **Enduring Understandings**

A variety of families of functions and methods can be used to model and solve real-world situations.

Creating a graph is not the same as interpreting the information displayed.

# **Essential Questions**

What methods can be used to solve systems of equations?

How are systems of linear equations useful?

How can systems of linear equations be used to represent real-world situations?

What is the best way to solve a particular system of equations?

How many solutions can a system of linear equations have?

When is using a matrix an efficient tool?

What is a determinant and how is it calculated?

Why do we need the determinant?

Content	
Vocabulary	
Substitution	
Elimination	
Matrix	

# Determinant

Inverse

# Skills Solve systems of equations graphically and algebraically Solve systems of inequalities Add, subtract, and multiply matrices Evaluate determinants Find inverses of matrices Solve systems of equations by using inverses Use a graphing calculator to perform operations on matrices

# Resources

Content Vocabulary

Practices quizzes

Teacher website

www.KhanAcademy.org

www.Desmos.com

# Standards

Vector and Matrix Quantities N -VM

C. Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

8. (+) Add, subtract, and multiply matrices of appropriate dimensions.

9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

12. (+) Work with  $2 \times 2$  matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.

# Seeing Structure in Expressions A-SSE

# B. Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.  $\star$ 

# Creating Equations★ A -CED

# A. Create equations that describe numbers or relationships

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

# **Reasoning with Equations and Inequalities A -REI**

#### C. Solve systems of equations

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. New Jersey Student Learning Standards for Mathematics 69

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

# D. Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

12. Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

#### MP1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

# MP6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

#### MP8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y - 2)/(x - 1) = 3. Noticing the regularity in the way terms cancel when expanding (x - 1)(x + 1), (x - 1)(x2 + x + 1), and (x - 1)(x3 + x2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students

maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

MA.A-SSE	Seeing Structure in Expressions
MA.A-SSE.B	Write expressions in equivalent forms to solve problems
MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MA.N-VM	Vector and Matrix Quantities
MA.A-CED	Creating Equations
MA.A-CED.A	Create equations that describe numbers or relationships
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-REI	Reasoning with Equations and Inequalities
MA.N-VM.C	Perform operations on matrices and use matrices in applications.
MA.N-VM.C.6	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
MA.N-VM.C.7	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
MA.N-VM.C.8	Add, subtract, and multiply matrices of appropriate dimensions.
MA.N-VM.C.9	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
MA.N-VM.C.10	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
MA.N-VM.C.11	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
MA.A-REI.C	Solve systems of equations
MA.N-VM.C.12	Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.
MA.A-REI.C.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
MA.A-REI.C.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
MA.A-REI.D	Represent and solve equations and inequalities graphically
MA.A-REI.D.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
MA.A-REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
MA.A-REI.D.12	Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.