*Unit 3: Analysis of Rational Functions

Content Area: **Mathematics**
Course(s): **Pre-Calculus Honors**
Time Period: **December**
Length: **8 blocks**
Status: **Published**

Transfer Skills

The unit continues with the study of rational functions and their graphs. Students will find the domain, explore limits, identify vertical and horizontal asymptotes, and use transformations to graph a rational function. Finally, students will learn the basic techniques for solving polynomial inequalities and use those techniques to solve inequalities involving rational functions.

Enduring Understandings

Functions can be written in many ways- some forms can be helpful under different circumstances.

The vocabulary is just as important as the math.

Functions families have common characteristics.

We can use algebra to help graph functions.

Technology is a resource for discoveries and investigations.

There is a difference between using technology to find the answer compared to using technology to aid in supporting the answer.

Essential Questions

How do the characteristics of graphs relate to their corresponding equations?

How can algebra help us get information about a graph from an equation?

How can the graph of a function be used to determine the essential characteristics of the function?

How are horizontal asymptoptes connected to end behavior?

How can technology assist in the understanding of graphing functions?

Content

Vocabulary

End behavior, zeros, multiplicity, factored form, intercepts, long division, synthetic division,real and complex zeros, vertical asymptotes, horizontal asymptotes, oblique asymptotes.

Instructional Note: Students have seen much of this material in Algebra 2

Red Hot Topics:

- Factoring polynomials of higher degree is a necessary prerequisite skill for this unit.
- The following are review skills that might require a refresher.
    - Divide polynomials with long division and synthetic division.
    - Use the remainder and factor theorems to find remainders and factors.
    - Use the rational zeros theorem to list the potential rational zeros of a polynomial function.

Skills

Solve rational equations and inequalities.

Discover graphical and analytical connections including restricted values, domain, and intercepts for rational functions.

Discover properties of rational functions analytically including point discontinuities and vertical asymptotes.

Determine equations of horizontal and slant asymptotes of rational functions.

Develop graphs of rational functions.

Understand the numerical behavior of rational functions at discontinuities.

Resources

Pearson Pre-Calculus Textbook

Content Vocabulary

Practices quizzes

Teacher website

www.KhanAcademy.org,

www.kutasoftware site

www.njctl.org/courses/math/precalculus

Standards

Arithmetic with Polynomials and Rational Expressions **A -APR**

D. Rewrite rational expressions

6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Reasoning with Equations and Inequalities **A -REI**

A. Understand solving equations as a process of reasoning and explain the reasoning

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

D. Represent and solve equations and inequalities graphically

11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Interpreting Functions **F-IF**

C. Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing

symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account

the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such

as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y) 2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

MA.K-12.1	Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the

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MA.K-12.3 Construct viable arguments and critique the reasoning of others.

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MA.K-12.4 Model with mathematics.

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MA.K-12.5 Use appropriate tools strategically.

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MA.K-12.6	Attend to precision.
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MA.K-12.7	Look for and make use of structure.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
	Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $□^2 + 9□ + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(□ - □)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers □ and □.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.A-APR	Arithmetic with Polynomials and Rational Expressions
	Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(□ - 2)/(□ - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(□ - 1)(□ + 1)$, $(□ - 1)(□^2 + □ + 1)$, and $(□ - 1)(□^3 + □^2 + □ + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.
MA.F-IF.C.7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations

	are available, and showing end behavior.
MA.A-APR.D	Rewrite rational expressions
MA.A-APR.D.6	Rewrite simple rational expressions in different forms; write $\square(\square)/\square(\square)$ in the form $\square(\square) + \square(\square)/\square(\square)$, where $\square(\square)$, $\square(\square)$, $\square(\square)$, and $\square(\square)$ are polynomials with the degree of $\square(\square)$ less than the degree of $\square(\square)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.A-REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
MA.A-REI.D	Represent and solve equations and inequalities graphically
MA.A-REI.D.11	Explain why the $\square$-coordinates of the points where the graphs of the equations $\square = \square(\square)$ and $\square = \square(\square)$ intersect are the solutions of the equation$\square(\square) = \square(\square)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $\square(\square)$ and/or $\square(\square)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.