*Unit 4 Exponents, Roots, and Radicals

Content Area: **Mathematics**
Course(s): **Algebra 2 CP**
Time Period: **Marking Period 2**
Length: **10 blocks**
Status: **Published**

Transfer Skills

In this unit, students extend their work by solving equations with exponents and radicals. A major emphasis will be placed on students ability to transform equations into equivalent forms.

Instructional Notes

- Knowledge of exponent rules and simplifying radicals are an Algebra 1 skill and should be reviewed using retention strategies and warm-ups.
- The use of a graphing calculator is encouraged for students to check their work when graphing exponential and radical equations.

BLUE* = 9/10 Only**

RED = 9/10 & 11/12

Enduring Understandings

Algebraic representation can be used to generalize patterns in mathematics.

Patterns and relationships can be represented graphically, numerically, symbolically, or verbally.

The arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Essential Questions

How do we apply mathematical principles?

What makes an algebraic algorithm both effective and efficient?

How do operations affect numbers?

Content

Vocabulary

- Rational Exponents
- Radicals
- Domain
- Range
- Growth
- Decay
- Inverse
- Average Rate of Change

Skills

Equivalent Forms

- Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
- Apply the properties of exponents to simplify expressions including positive and negative integers and fractional exponents.
- Rewrite radical expressions using rational exponents and vice versa.
- Rewrite expressions involving radicals and rational exponents using the properties of exponents.
- Evaluate nth roots of real numbers using both radical notation and rational exponent notation.
- Write equivalent expressions for exponential functions using the properties of exponents. (For example, rewrite exponential function $f(x)=3^{x}\cdot 2^{3x}$ in the form $f(x) = a(b^{x})$.
- Simplify radical expressions using absolute value symbols when appropriate.

Solving all types of Exponential Equations

- Solve equations with radicals and rational exponents.
- Solve equations with extraneous solutions.
- Solve exponential equations with like bases.
- Solve exponential equations with unlike bases using an equivalent form to rewrite with like bases (For example: rewrite $7^{3x} = 49^{x}$ as $7^{3x} = 7^{2x}$).

- Solve for the inverse of radical functions.

Graphing Exponential, and Radical Functions

- Find and interpret the domain of radical functions.
- Graph radical functions using a graphing calculator and table function.
- Identify domain and range of exponential and radical functions.
- Identify increasing and decreasing behavior of exponential and radical functions.
- Identify and write end behavior of exponential and radical functions in various notations.
- Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. (Compare to finding the average rate of change for a linear function.)
- State the domain and range of exponential function
- Sketch the graph of exponential function showing intercepts, key points, asymptotes, and end behavior.
- Identify the effect on the graph of exponentials by replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Applications

- Apply problem solving to applicational problems including exponential functions.
- Solve real-world problems including exponential growth and decay.
- Use graphing technology to relate key characteristics of an exponential graph to real-world problems.

Resources

NJGPA Practice Test

https://nj.mypearsonsupport.com/practice-tests/njgpa-math/

NJSLA Practice Test*

https://nj/mypearsonsupport.com/practice-tests/math/

Teacher Resources by Standard

www.illustrativemathematics.org

illuminations.nctm.org/

www.pbslearningmedia.org/

Online Teaching Websites

www.khanacademy.org

www.ixl.com

Algebra 2 Common Core Textbook

Roots and Radicals: Chapter 6 pg. 357

Exponents and Logs: Chapter 7 pg. 431

Assessments

Quiz

Formative: Other Evidence: Other: Quiz

Simplify Exponential Expressions Using Exponent Properties, Rewriting Expressions in Equivalent Forms

Quiz
Formative: Other Evidence: Other: Quiz

Solving and Graphing Exponential Equations, Radical Equations with Extraneous Solutions

Unit Test
Summative: Transfer Tasks: Test: Common

Exponents, Roots, & Radicals

Standards

NJSLS 2016

Number and Quantity

The Real Number System

N-RN A. Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 51/3 to be the cube root of 5 because we want (51/3) 3 = 5(1/3) 3 to hold, so (51/3) 3 must equal 5.

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Algebra

Seeing Structure in Expressions

A-SSE B. Write expressions in equivalent forms to solve problems

c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.151/12) 12t ≈1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Reasoning with Equations and Inequalities

A-REI A. Understand solving equations as a process of reasoning and explain the reasoning

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Functions

Interpreting Functions

F-IF B. Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

C. Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t , y = (0.97)t , y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay

Building Functions

F-BF B. Build new functions from existing functions

3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

4a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠1.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y) 2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MA.F-BF	Building Functions
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-BF.B	Build new functions from existing functions
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.F-BF.B.4a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
MA.F-IF	Interpreting Functions
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and

	explain different properties of the function.
MA.F-IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.F-IF.C.8b	Use the properties of exponents to interpret expressions for exponential functions.
MA.F-LE	Linear and Exponential Models
MA.F-LE.A.1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.4	Model with mathematics.
MA.K-12.7	Look for and make use of structure.
MA.N-RN	The Real Number System
MA.N-RN.A	Extend the properties of exponents to rational exponents.
MA.N-RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
MA.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MA.A-REI	Reasoning with Equations and Inequalities
MA.A-REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
MA.A-SSE	Seeing Structure in Expressions
MA.A-SSE.B	Write expressions in equivalent forms to solve problems
MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
	For example, $f(x) = 2x^3$ or $f(x) = (x + 1)/(x - 1)$ for $x \neq 1$.

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For example, identify percent rate of change in functions such as y = (1.02) to the t power, y = (0.97) to the t power, y = (1.01) to the 12t power, y = (1.2) to the t/10 power, and classify them as representing exponential growth or decay.

For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

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