*Unit 4 Quadrilaterals and Coordinate Geometry

Content Area: **Mathematics**
Course(s): **Geometry CP, Geometry Honors**
Time Period: **January**
Length: **10 blocks**
Status: **Published**

Transfer Skills

Previous coursework: experimentally proved the triangle sum theorem and the exterior angle theorem, used the pythagorean theorem to find the distance between two points in the coordinate plane, writing equations of lines- including parallel and perpendicular, graphs of functions are the solutions to the equation of a function plotted on a coordinate plane

By the end of this unit: Students should use congruent triangles and their corresponding parts frequently in this unit- they should become very good at recognizing how to use CPCTC to prove properties in a variety of scenarios. Each parallelogram's properties should be first addressed as a proof, and then reinforced withpractice involving algebra. The side and angle measure properties of each of the parallelograms should be stressed to prepare for coordinate geometry. Remember, the general focus of the high school geometry course to formalize concepts previously address informally.

Students should have a good grasp on using formulas and the coordinate plane to solve problems involving classification of geometric figures, finding area and perimeter, and parallel and perpendicular slopes. Students should understand that not all equations represent functions, but can describe a relationship about distances between points and lines on the coordinate plane. Equations of parabolas and circles are basically a function of using the distance formula to plot points given a set of restraints based on equidistance.

Instructional Strategies:

- This unit is all about applying the skills in the previous unit to new situations- encourage group work and analyzing peer's work.
- There is no mention of trapezoids and kites in the common core, but you can address them briefly, if time allows, to prove their properties.
- (+) = denotes Honors only skill
- The proof for the slope criteria for parallel and perpendicular lines easily relates to transformations and congruency- try and focus on side lengths and not coordinates here.
- Writing equations of lines is a review from Algebra 1 and 8th grade math.
- When classifying triangles and parallelograms, make sure to include four points that do not make a parallelogram at all and right triangles.
- For area on the coordinate plane, consider using negative space to calculate the area of irregular triangle and quadrilaterals: Sketch a rectangle around the shape, so that each side of the original space intersects a side of the rectangle, then calculate the area of the rectangle and subtract the right triangles surrounding the original shape. (Also called the box method)
- Circles and Parabolas are really all about the distance formula and understanding that a graph on the coordinate plane is all of the points that are solutions to a given equation.
- Start with the definition of a circle and a parabola and then using the distance formula, derive the equations. This unit is about DERIVING the equation, not knowing it.
- The circle and parabola are NOT on the PARCC PBA, and only on the EOY.
- * = denotes a skill not tested on the PBA, only tested on the EOY.

Enduring Understandings

Parallelograms have two sets of parallel sides and can be divided into triangles, these qualities lead to these special quadrilaterals having a plethora of other properties.

Coordinate geometry is a tool for discovering and verifying properties of geometric shapes.

Calculations of slope and length can be used to determine the type of shape based on its attributes.

Circle and parabola equations are based on properties of equidistance between points and lines.

Essential Questions

Why are congruent triangles useful in proofs?

Why are parallelograms important?

What is coordinate geometry?

How can distance and slope connect geometry and algebra?

How can distance be used to create figures on the coordinate plane?

Content

Proving Parallelogram Properties

Applying Parallelogram Properties

Writing Equations Parallel and Perpendicular Lines

Distance Formula

Classifying Quadrilaterals

Partitioning a Line Segment

Midpoint Formula

Perimeter and Area of Shapes on the coordinate plane

Algebra Review:

- Solving Equations (properties of quadrilaterals)
- Slope
- Writing Equation of Lines
- Evaluating Expressions
- Completing the Square

Skills

Prove properties of parallelograms and then apply them.

Prove that the medians of a triangle meet at a point, a point of concurrency. (+)

Prove the properties of rectangles and then apply them.

Prove the properties of rhombi and then apply them.

Prove the properties of squares and then apply them.

Classify a quadrilateral by its properties on and off the coordinate plane

Use slope and distance formulas to classify parallelograms and triangles on the coordinate plane.

Use the slope, distance, and midpoint formula to solve problems

Write the equation of a line given two points

Prove that parallel lines have congruent slopes and its converse using corresponding parts of similar triangles are proportional.

Prove that perpendicular lines have negative reciprocal slopes and its converse.

Solve systems of equations (+)

Determine the equation of a line parallel or perpendicular to a line through a given point

Use slope and distance formulas to classify parallelograms and triangles on the coordinate plane

Determine the points on a line that partitions the segment in a given ratio (1:3 and 3:1)

Use coordinates to determine the perimeter and area of a figure on the coordinate plane

Derive the equation of a circle.

Determine the center and radius of a circle when given an equation of a circle in vertex form.

Complete the square to transform the standard equation of a circle into vertex form

Derive the equation of a parabola given the focus and the directrix

Determine the focus and directrix of a parabola based on its equation.

Resources

Pearson Resources:

3-5, 4-2, 4-3, 4-4, 4-5, 4-6, 4-7, 5-1, 5-2, 5-4, 6-2, 6-3, 6-4, 6-5, 6-6, 12-5, CB 12-5, 6-9, 3-8, 3-7, 7-4, 1-3, 1-7, 6-7, 10-1

Online Resources:

http://illuminations.nctm.org/Lesson.aspx?id=2469

http://fivetriangles.blogspot.com/2012/04/paper-folding.html

http://map.mathshell.org/materials/lessons.php?taskid=212&subpage=concept

http://mrpiccmath.weebly.com/blog/3-acts-tv-space

http://fivetriangles.blogspot.com/2012/04/isosceles-triangles.html

http://pamjwilson.wordpress.com/2012/08/27/distance-midpoint-on-a-map/

https://www.illustrativemathematics.org/illustrations/1302

http://map.mathshell.org/materials/lessons.php?taskid=226&subpage=concept

https://www.illustrativemathematics.org/illustrations/1302

https://www.illustrativemathematics.org/illustrations/1687

https://www.illustrativemathematics.org/illustrations/605

https://www.illustrativemathematics.org/illustrations/1880

https://www.illustrativemathematics.org/illustrations/1876

http://map.mathshell.org/materials/lessons.php?taskid=406&subpage=concept

http://map.mathshell.org/materials/lessons.php?taskid=425&subpage=concept

http://www.geometrycommoncore.com/content/unit4/ggpe2/teachernotes1.html

https://www.illustrativemathematics.org/HSG-GPE.A (there's 3 in this one)

Standards

NJSLS 2016

Geometry

CONGRUENCE

C. Prove geometric theorems

10. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

11. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY

B. Prove theorems involving similarity

4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS

A. Translate between the geometric description and the equation for a conic section

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

2. Derive the equation of a parabola given a focus and directrix.

B. Use coordinates to prove simple geometric theorems algebraically

4. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).

5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

6. Find the point on a directed line segment between two given points that partitions the segment in a given

ratio.

7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Mathematics | Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments

can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MA.G-CO.C.10	Prove theorems about triangles.
MA.G-CO.C.11	Prove theorems about parallelograms.
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.G-GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
MA.G-GPE.A.2	Derive the equation of a parabola given a focus and directrix.
MA.G-GPE.B.4	Use coordinates to prove simple geometric theorems algebraically.

MA.G-GPE.B.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
MA.G-GPE.B.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
MA.G-SRT.B.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.