# *Unit 3 Dilations, Similarity, and Trig

Content Area: **Mathematics**
Course(s): **Geometry CP, Geometry Honors**
Time Period: **December**
Length: **15 blocks**
Status: **Published**

## Transfer Skills

Previous coursework: explained a proof of the Pythagorean theorem and its converse, applied the pythagorean theorem, extensive work with scale drawings, scale factors, and proportional reasoning

At the end of this unit: Students should think about defining similarity in terms of transformations. Dilations should be constructed on and off the coordinate plane, with the center of dilation occurring anywhere on the coordinate plane. Triangle similarity is addressed in proofs and applied using side splitter and angle bisector theorem- make sure to prove these concepts before solving problems using them. Students should understand what a trig ratio is; what does it mean when we say "sin $32^o$"? They should focus on patterns and relationships with basic co-functions and complementary angles and problems should be solved using any of the six trig functions. Because they will be graphing trig functions in Algebra 2, they should be able to derive the sine and cosine of a 30, 45, and 60 degree angle easily. Honors students extend this knowledge to oblique triangles using Law of Sines and Cosines and the unit circle. The honors student should be able to quickly calculate any trig value on the unit circle given an angle with a 30, 45, or 60 degree reference angle. They should be able to do this with both degree or radian measures less than one rotation (between 0 and 360/2pi).

Instructional strategies:

- This is a great unit to revisit properties of transformations, on and off the coordinate plane, and construction techniques.
- Use previous coursework skills as an introduction to the unit.
- Students have a hard time labeling side lengths- highlight the reference angle in some way to help.
- Honors: Derive the Law of Sines and Cosines using algebra and make sure to address the Law of Sines Ambiguous Case.
- (+) = denotes Honors only skill

## Enduring Understandings

Dilations are composed of both a vertical and horizontal stretch using the same scale factor.

Similar triangles can be used to find lengths and distances.

Trigonometry uses properties of similar right triangles to determine common ratios between side lengths and acute angle measures.

Trigonometry can find lengths and angles given limited information.

## Essential Questions

What are the qualities of a transformation the produces similar figures?

How are similar triangles used in real life?

How is trigonometry related to similar right triangles?

How can trigonometry be used in real life?

## Content

Review: Dilations

Similar Polygons

Triangle Similarity

Special Right Triangles

Finding Sides and Angles using Trigonometric Ratios

Applications of Trigonometry

Law of Sines and Cosines (+)

Unit Circle Trigonometry (+)

Opportunities for Algebra:

- Solving Equations with a fraction on one side
- Ratios and Proportions (embedded with multiplying polynomials)
- Simplifying Radicals
- Solving Equations with Radicals

## Skills

Construct a dilation. (+)

Describe the properties of dilation:

- Angles map to congruent angles
- Sides map to parallel segments that are larger or smaller (dependent on scale factor)

Dilate when given a center of dilation and a scale factor.

Determine the center of dilation and the scale factor from a diagram.

Identify corresponding angles and sides based on similarity statements.

Write a similarity statement for two similar polygons.

Determine if two triangles are similar based on their corresponding parts.

Describe a sequence of similarity transformations between two similar polygons on the coordinate plane

Prove two triangles to be similar using the minimum requirements of AA, SAS and SSS.

Use the properties of similarity transformations to establish the AA, SAS and SSS criterion for two triangles to be similar.

Prove (the side splitting theorem) that a line parallel to one side of a triangle divides the other two proportionally.

Prove (the angle bisector theorem) that an angle bisector of an angle of a triangle divides the opposite side in two segments that are proportional to the other two sides of the triangle.

Solve for side lengths in Special Right Triangles (focus on triangles where the hypotenuse = 1)

Label a triangle in relation to the reference angle (opposite, adjacent & hypotenuse).

Determine the most appropriate trigonometric ratio (sine, cosine, tangent) to use for a given problem based on the information provided.

Solve for sides and angles of right triangles using trigonometry.

Explain why similar triangles have the same trigonometric ratio values

Determine the exact value of the trigonometric ratios for 30, 45, and 60 degree angles

Use trigonometry to solve application problems, including angle of elevation and depression (focus on double triangle problems)

Explain and use the relationship between the sine and cosine of complementary angles

Derive the Law of Sines and Cosines (+)

Use the Law of Sines and Cosines to solve right and oblique triangles (+)

Derive the SAS area formula from the Law of Sines (+)

## Resources

Pearson Resources:

CB 9-6, 9-7, Ch. 7, 8-1, 8-2, 8-3, 8-4, CB 8-4 (both CP and Honors)

10-5, 8-5, 8-6 (Honors Only)

Online Resources:

http://emergentmath.com/2011/02/18/the-pizza-casbah-30-inch-pizza-challenge/

http://mrpiccmath.weebly.com/blog/3-acts-mmm-juice

http://map.mathshell.org/materials/lessons.php?taskid=452&subpage=concept

http://emergentmath.com/2012/11/08/more-math-food-blogging-i-may-need-some-help-from-my-southern-friends/

http://map.mathshell.org/materials/lessons.php?taskid=429&subpage=problem

http://illuminations.nctm.org/Lesson.aspx?id=3165

http://jdevarona.wordpress.com/2013/02/18/random-problem-idea-the-giant-bat/

http://map.mathshell.org/materials/lessons.php?taskid=222&subpage=problem

http://mrhonner.com/archives/6153

https://www.illustrativemathematics.org/illustrations/1635

https://www.illustrativemathematics.org/illustrations/1443

https://www.illustrativemathematics.org/HSG-SRT.C.8 (there's 6 in this link)

## Standards

**NJSLS 2016**

**Geometry**

**SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY**

**A. Understand similarity in terms of similarity transformations**

1. Verify experimentally the properties of dilations given by a center and a scale factor:

    a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

    b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

**B. Prove theorems involving similarity**

4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

**C. Define trigonometric ratios and solve problems involving right triangles**

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

7. Explain and use the relationship between the sine and cosine of complementary angles.

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

**D. Apply trigonometry to general triangles**

9. (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.

11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). Circles G-C

**Mathematics | Standards for Mathematical Practice**

**1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

**4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

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|---|---|
| MA.G-SRT.A.1 | Verify experimentally the properties of dilations given by a center and a scale factor: |
| MA.G-SRT.A.2 | Given two figures, use the definition of similarity in terms of similarity transformations to |

| | |
|---|---|
| | decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. |
| MA.G-SRT.A.3 | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. |
| MA.G-SRT.A.1a | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. |
| MA.G-SRT.A.1b | The dilation of a line segment is longer or shorter in the ratio given by the scale factor. |
| MA.G-SRT.B.4 | Prove theorems about triangles. |
| MA.G-SRT.B.5 | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. |

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