*Unit 2 Introduction to Proof through Congruency

Content Area: **Mathematics**
Course(s): **Geometry CP, Geometry Honors**
Time Period: **October**
Length: **10 blocks**
Status: **Published**

Transfer Skills

Previous coursework: Used facts about complementary, supplementary, vertical, and adjacent angles to solve problems, informally defined congruency through transformations

By the end of this unit: Student should be comfortable with proofs about segments, parallel lines, angles pairs, and triangle congruency. The triangle sum theorem stems from the parallel line proofs and is a great application of those concepts. Constructions also play a part here- you can use constructions such as "copy a segment" and "copy an angle" as reasons in the proofs you write.

Instructional strategies:

- Although there is no explicit attention paid to conditional statements, thinking about proofs as "If statement 1, then statement 2, because reason 2" can help students feel the logical flow of the proof.
- Students generally get overwhelmed with the number of possible theorems, postulates, and corollaries in proofs- creating a "Reason Bank" has been helpful to give them a reference until they are familiar with the commonly used reasons.
- Another helpful option is to note which reasons generally occur together. For example, definition of vertical angles/ vertical angles are congruent/ definition of congruence occur together in many proofs.
- To begin the proofs, start with a lot of assistance and then as the students become more confident, challenge them little by little. For example, filling in reasons first, then statements, then giving each line of a proof out of order, followed by each statement out of order and no reasons.
- Also, encourage students to come up with a plan before they write even one line of the proof. Just have them informally write their ideas down to figure out the flow (maybe as a flow diagram) can really help organize their thoughts before the formal proof.
- Be sure to address different types of proofs- students generally gravitate toward flow or two-column formats, but maybe provide a paragraph proof for them to analyze so they are familiar with how to break it down into a format they are more comfortable with.
- (+) = denotes Honors only skill

Enduring Understandings

Transformations that do not have a change in scale produce congruent figures.

There are five ways to prove triangles congruent without proving all six parts congruent: SSS, ASA, SAS, AAS, and HL.

Essential Questions

What are characteristics of a transformation that produces congruent figures?

What information is needed to prove two triangles congruent?

Content

Proofs using properties of Equality and Congruence

Proofs Involving Parallel Lines and Angle Pairs

Proving Triangles Congruent

Using Congruent Triangles in Proofs (CPCTC)

Skills

Write proofs involving solving algebraic equations.

Write basic proofs involving angles and segments.

Write advanced proofs involving segments and angles. (+)

Write proofs involving angle pairs associated with parallel lines.

Prove and apply that the sum of the interior angles of a triangle is 180°.

Identify corresponding angles and sides based on congruence statements.

Write congruence statements for two congruent triangles.

Determine if two triangles are congruent based on their corresponding parts.

Explain and apply the criteria of SSS, SAS, ASA, AAS, and HL to prove triangles congruent.

Explain why AA and SSA don't prove triangle congruency.

Prove two triangles congruent (basic)

Prove two triangles congruent (advanced) (+)

CPCTC Proofs

- Prove that all points on a perpendicular bisector of a segment are equidistant from the segment endpoints.
- Prove that the base angles of an isosceles triangle are congruent.
- Prove constructions from Unit 1
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Resources

Pearson Resources:

2-6, 3-2, 5-2, 3-5, 4-5, 5-1, Ch. 9 (use transformations to explain congruency)

Opportunities for Algebra Review:

- Solving Equations (using congruent and supplementary angles, congruent figures)

Online Resources:

https://www.illustrativemathematics.org/HSG-CO.A

http://mr-stadel.blogspot.com/2012/10/transversals-tape-and-stickies.html

http://fivetriangles.blogspot.com/2014/04/155-three-angle-sum.html

http://fivetriangles.blogspot.com/2014/03/150-angle-sliver.html

http://map.mathshell.org/materials/lessons.php?taskid=212&subpage=concept

http://map.mathshell.org/materials/lessons.php?taskid=452&subpage=concept

Standards

NJSLS 2016

Geometry

CONGRUENCE

B. Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

C. Prove geometric theorems

9. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

10. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Mathematics | Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y) 2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MA.K-12.1	Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.5	Use appropriate tools strategically.
MA.G-CO.B.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
MA.G-CO.B.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
MA.G-CO.B.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
MA.G-CO.C.9	Prove theorems about lines and angles.
MA.K-12.7	Look for and make use of structure.
MA.G-CO.C.10	Prove theorems about triangles.