*Unit 1 Constructions and Transformations

Content Area: **Mathematics**
Course(s): **Geometry CP, Geometry Honors**
Time Period: **September**
Length: **10 blocks**
Status: **Published**

Transfer Skills

Previous coursework: students have informally defined many geometric terms, drawn figures using a ruler and a protractor given defined lengths and angle measures, explored the concept of congruence through transformations, informally proved relationships between angles associated with parallel lines and interior/exterior angles of triangles, worked with rotations, reflections, and translations to define congruency and informally with dilations

At the end of this unit: Students should be able to solve problems using constructions, explain the reasoning behind why a construction works, and formally define vocabulary words introduced. Students should formally define transformations on the coordinate plane as function rules; given an input, what happens to the coordinates to obtain the output. Given a rule, students should be able describe whether segment length and angle measure is preserved. Students should also be able to map a figures onto other figures, as well as map a figure onto itself.

Instructional Notes:

- Reading and writing using correct notation is important! Use it whenever possible and make sure students do the same.
- Definitions should be based in the undefined terms and "if-then" or "if-and-only-if" statements.
- Constructions should not be prescriptive in nature. Introduce constructions using application problems and discovery.
- Try to have the students accomplish the constructions in a variety of ways- using compass and straightedge (which seems to be the main focus of PARCC, geometry software (like Geogebra), and patty paper. Make connections between the mediums- how do you create congruence using patty paper vs. compass?
- The main focus behind the constructions is "Why do they work?", not "How do I do them?"
- The advanced constructions (inscribed polygons) and triangle centers (incenter, orthocenter) are a good capstone to this unit. Students can implement their basic construction skills to figure out how to create the advanced constructions, and then explain the properties of the triangle centers using geometric concepts.
- Use a variety of mediums for transformations: tracing paper, graph paper, geometry software.
- Allow students to explore transformation properties, such as corresponding congruent parts given different transformations.
- Provide students with a pre-image and an image and have them figure out different ways to map one onto the other.
- Transformations should be defined in terms of parallel lines.
- Rotations move along a circular arc given a specified angle.
- Reflections are drawn using perpendicular bisectors over the mirror line.
- **(+) = denotes Honors only skill**

Enduring Understandings

Circles are a collection of points equidistant from a given point- using this equidistance can help you create congruent figures.

Using a compass and straightedge to construct a figure is more accurate than a sketch because it does not rely on measurements, which are inherently estimates.

Isometries are transformations that preserve length and distance, while dilations change lengths and preserve angle measure.

Transformations can be represented on the coordinate plane by plotting points, off the coordinate plane using constructions, and as function mapping diagram.

Essential Questions

What is the importance of circles when creating constructions?

Why is constructing figures preferred over a sketch?

How can you change a figure without changing its shape?

What are different ways to represent transformations?

Content

Definitions, Notation, and Diagrams

Basic constructions

Angle/Segment Addition/Bisectors

Advanced Constructions

Properties of basic isometries and dilations

Symmetry

Compositions of isometries and dilations

Skills

Define applicable geometric terms using precise words, diagrams and correct notation (this will be ongoing throughout the course)

Use geometric concepts to explain the reasoning for the steps/procedures used in performing a constructions.

Use a compass and straightedge to create the following constructions:

- copy a given line segment and a given angle
- bisect a given line segment and a given angle
- construct a line perpendicular to a given line through a point on that line.
- construct a line perpendicular to a given line through a point not on that line.
- construct the perpendicular bisector of a line segment.
- construct a line parallel to a given line through a point not on the line.
- an equilateral triangle
- a square
- a regular hexagon inscribed in a circle
- **a line tangent to a circle (+)**

Use constructions to solve basic application problems (Bisecting and Addition with Algebra embedded)

Use constructions to solve advanced application problems (+)

Describe transformations as functions that take inputs and give other points as outputs.

Identify an isometric transformation by describing whether the image and pre-image have congruent lengths and angles

Define reflections, rotations, and translations in terms of angles, circles, perpendicular line, parallel line, and line segment

Reflect, rotate, and translate figures in the coordinate plane, including compositions

Describe the properties of dilation:

- Angles map to congruent angles
- Sides map to parallel segments that are larger or smaller (dependent on scale factor)

Dilate when given a center of dilation and a scale factor.

Determine the center of dilation and the scale factor from a diagram.

Determine the transformations that will map a quadrilateral onto itself.

Construct the image of a figure given the figures and its transformation NOT on the coordinate plane (+)

Given a pre-image and image, describe transformations that will map one figure onto another

Prove two figures are congruent if there is a sequence of rigid motions that map one figure to another.

Prove that two figures are congruent if and only if they have the same shape and size.

Use composite transformations to map on figure onto another.

Explain the effects of rigid motion on orientation and location of a figure.

Use the definition of congruence in terms of transformations as a test to see if two figures are congruent.

Resources

Pearson Resources:

1-2, 1-3, 1-4, 1-6, 3-1, CB 3-2, 3-6, 4-4, CB 4-5, 4-5 5-2, CB 6-9, CB 7-5, 10-3, 10-6, all of Ch. 9

Opportunities for Algebra Review:

- Slope (translations move along parallel lines, mirror line is perpendicular to reflection and its image)
- Functions and Function Notation (A transformation is a one to one correspondence between the points of the pre-image and the points of the image.)

Online Resources:

Dividing a town into Pizza delivery regions: http://illuminations.nctm.org/Lesson.aspx?id=2688

Security Cameras: http://illuminations.nctm.org/Lesson.aspx?id=2788

Placing a Fire Hydrant: https://www.illustrativemathematics.org/illustrations/508

Pop-Up Box design: http://mrpiccmath.weebly.com/blog/3-acts-pop-box-design

Transversals, Tape, and Stickies: http://mr-stadel.blogspot.com/2012/10/transversals-tape-and-stickies.html

Best Midpoint: http://threeacts.mrmeyer.com/bestmidpoint/

https://www.illustrativemathematics.org/HSG-CO.A

G.CO.A.1: http://www.shmoop.com/common-core-standards/ccss-hs-g-co-1.html

G.CO.D.12: http://www.shmoop.com/common-core-standards/ccss-hs-g-co-12.html

G.CO.D.13: http://www.shmoop.com/common-core-standards/ccss-hs-g-co-13.html

Construction directions:

http://www.mathsisfun.com/geometry/constructions.html

http://www.mathopenref.com/constructions.html

http://www.onlinemathlearning.com/geometry-construction.html

Transformations:

http://www.shmoop.com/common-core-standards/math-geometry-congruence.html

http://jdevarona.wordpress.com/2012/07/13/let-the-random-problem-ideas-begin/

https://www.illustrativemathematics.org/illustrations/1545

https://www.illustrativemathematics.org/illustrations/1546

http://emergentmath.com/2012/01/07/can-we-make-an-even-edgier-brownie-pan-what-about-the-perfect-brownie-pan/

http://illuminations.nctm.org/Lesson.aspx?id=2626

http://illuminations.nctm.org/Lesson.aspx?id=1540

http://illuminations.nctm.org/Lesson.aspx?id=3141

Standards

NJSLS 2016

Geometry

CONGRUENCE

A. Experiment with transformations in the plane

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines,

parallel lines, and line segments.

5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

B. Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

D. Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle

Mathematics | Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a

school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MA.G-CO.A.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
MA.G-CO.A.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
MA.G-CO.A.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
MA.G-CO.A.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MA.G-CO.A.5

Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MA.G-CO.B.6

Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

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MA.G-CO.D.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
MA.G-CO.D.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.