*Unit 4 Modeling with Exponential Functions

Content Area: **Mathematics**
Course(s): **Algebra 1 CP**
Time Period: **December**
Length: **14 blocks**
Status: **Published**

Transfer Skills

A main focus will be placed on modeling with exponential functions. In this unit students will focus on comparing functions, mainly linear and exponential. Comparing functions by finding the average rate of change will be emphasized.

Instructional Notes:

Prior Knowledge: Prior experience with Identifying Linear vs Exponential. Connect to prior work with linear function modeling and average rate of change.

Graphing Calculator Integration: Teacher will model the usage of the graphing calculator throughout the unit. Students will become comfortable with navigating and using the graphing calculator to solve a variety of problems efficiently.

i.e. Graphing Familiar and Unifamiliar Functions, Graphing Functions by hand using the Table Function of the calculator, Changing the Window of the Calculator, Using the ASK Table feature,

Enduring Understandings

Functions have distinct properties that help identify function types.

A variety of functions exist that can be used to model real life situations.

Essential Questions

What makes one function different from another function?

How does the average rate of change over an interval help one identify a function type?

How can functions be used to model real world situations?

Content

Vocabulary

base. exponent, expand, simplify, exponential, exponential growth/decay, growth/decay factor, model, average rate of change, interval, geometric sequence, explicit rule, recursive rule,

Skills

Note: Not all content should be emphasized equally. Skills that are starred are Major Standards and should be prioritized. However, the supporting and additonal clusters are necessary to be taught and should be embedded in instruction

★ Use the properties of exponents to simplify or expand exponential expressions, recognizing these are equivalent forms

Prove that linear functions grow by equal differences over equal intervals and exponential functions frow by equal factors over equal intervals.

Graph exponential functions from a table

★Identify key features of exponential functions including intercepts, intervals where the function is increasing, decreasing, positive, or negative. (Focus should be placed on exponential functions)

★Determine the practical domain of a function.

★Construct exponential functions, given a graph, a description of a relationship, a pattern or two input-output pairs and include reading these from a table.

★Interpret parts of an expression, such as terms, factors, and coefficients.

★Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)n$ as the product of P and a factor not depending on P.

★Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

★ Construct recursive and explicit formulas to represent geometric sequences.

★Calculate and interpret the average rate of change of a function (presented symbolically, as a table or on graph) over a specified interval in order to identify linear or exponential functions.

Compare two functions represented in two different ways.

Resources

Quizlet.com : Flashcard practice for Academic Vocabulary

NJSLS - New Jersey Student Learning Standards

Teacher Resources by Standard

https://www.illustrativemathematics.org/

https://illuminations.nctm.org/

https://nj.pbslearningmedia.org/

Online Teaching Websites

Khan Academy

MATH IXL

Standards

NJSLS 2016

Creating Equations★

A -CED A. Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

D. Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Building Functions

F-BF A. Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

B. Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a

table) over a specified interval. Estimate the rate of change from a graph.★

C. Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Linear and Exponential Models★

F-LE A. Construct and compare linear and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

B. Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Mathematics | Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of

interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y) 2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), (x – 1)(x2 + x + 1), and (x – 1)(x3 + x2 + x + 1) might lead them to the general formula for the sum of a geometric

series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

MA.F-IF	Interpreting Functions
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.F-IF.A	Understand the concept of a function and use function notation
MA.K-12.2	Reason abstractly and quantitatively.
MA.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.K-12.4	Model with mathematics.
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.K-12.5	Use appropriate tools strategically.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.K-12.7	Look for and make use of structure.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.F-BF	Building Functions
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-BF.A.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.
MA.F-BF.A.1b	Combine standard function types using arithmetic operations.
MA.A-CED	Creating Equations
MA.A-CED.A	Create equations that describe numbers or relationships
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.F-LE	Linear and Exponential Models

MA.F-LE.A	Construct and compare linear and exponential models and solve problems
MA.F-LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.
MA.F-LE.A.1a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
MA.F-LE.A.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MA.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MA.F-LE.B	Interpret expressions for functions in terms of the situation they model
MA.F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.
MA.A-REI.D	Represent and solve equations and inequalities graphically
MA.A-REI.D.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).