*Unit 6 Quadratic Equations and Functions

Content Area: **Mathematics**
Course(s): **Algebra 1 CP**
Time Period: **February**
Length: **15 blocks**
Status: **Published**

Transfer Skills

In this unit students will problem solve to choose the best method to solve a quadratic equations. A connection will be made between the graph of a quadratic function and the solution to the quadratic equation. Students will continue work with domain and range and modeling of functions.

Instructional Notes:

Prior Knowledge: Factoring is covered in Unit 1 and should be reviewed prior to starting the Unit. In 8th grade math, students began exploring quadratic relationships from tables, graphs, equations and real life models. Students were introduced to features of the quadratic function.

Graphing Calculator Integration: Teacher will model the usage of the graphing calculator throughout the unit. Students will become comfortable with navigating and using the graphing calculator to solve a variety of problems efficiently.

i.e. Different ways to find zeroes, identifying features of quadratic functions, and solving quadratic equations using the graphing calculator,illustrate the effects of transformations

Enduring Understandings

A single quantity may be represented by many equivalent but different expressions.

Different quadratic forms reveal different characteristics of the function.

Many real world situations can be modeled with a quadratic function.

Essential Questions

How can you solve quadratic equations using concrete models, tables, graphs, and algebraic method?

What are the characteristics of quadratic functions?

How can you use functions to model real-world situations?

Content

Vocabulary

quadratic equation, discriminant, quadratic formula, zero product property, complete the square, quadratic formula, square root method, factoring method, radical, complex number, irrational, rational, parabola, standard form, vertex form, factored form, intercept form, roots, zeros, vertex, maximum, minimum, x-intercept, y-intercept, concave up, concave down, continuous, interval, domain, range, transformation, parent function, even function, odd function

Skills

Note: Not all content should be emphasized equally. Skills that are starred are Major Standards and should be prioritized. However, the supporting and additonal clusters are necessary to be taught and should be embedded in instruction

★Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation.

Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

★Relate the value of the discriminant to the type of root to expect for the graph of a quadratic function. (one real root: 1 x-intercept, two real roots: 2 x-intercepts, no real roots: no x-intercepts)

★Understand the relationship between zeroes and factors.

Identify zeros of cubic functions when suitable factorizations are available and use the zeros to construct a rough graph of the function. (*cubic functions are presented as the product of a linear and a quadratic factor)

★ Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative) for both linear and quadratic functions; find the value of k given the graphs.

★Recognize transformations of the parent $f(x) = x^2$ as vertical $f(x) = x^2 + k$, horizontal f(x + k), stretch or reflections.

★Graph quadratic functions symbolically and more complicated cases using technology.

★Identify domain, intercepts, and intervals where the function is increasing or decreasing and positive or negative

Recognize that different forms of quadratic functions reveal different key features of its graph. (Standard Form: y-intercept, Vertex Form: Vertex & Max/Min Value, Factored Form: x-intercepts)

Factor a quadratic expression for the purpose of revealing the zeros of a function.

Complete the square for the purpose of revealing the maximum or minimum of a function.

★Create quadratic equations in one variable.

★Interpret models of quadratic functions given as equations or graphs.

★interpret maximum/minimum and intercepts of quadratic functions from graphs and tables in the context of the problem.

★Use the quadratic formula to solve real life application problems

★Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Determine the practical domain of a function.

★Compare properties of two quadratic functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum

★Calculate and interpret the average rate of change of a quadratic function presented symbolically or as a table.

Resources

Quizlet.com : Flashcard practice for Academic Vocabulary

NJSLS - New Jersey Student Learning Standards

Teacher Resources by Standard

https://www.illustrativemathematics.org/

https://illuminations.nctm.org/

https://nj.pbslearningmedia.org/

Online Teaching Websites

Khan Academy

MATH IXL

Standards

NJSLS 2016

Creating Equations

A -CED A. Create equations that describe numbers or relationships

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Reasoning with Equations and Inequalities

A -REI B. Solve equations and inequalities in one variable

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p) ^2 = q that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

Seeing Structure in Expressions

A-SSE B. Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Interpreting Functions

F-IF C. Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme

values, and symmetry of the graph, and interpret these in terms of a context.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Building Functions

F-BF B. Build new functions from existing functions

3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Mathematics | Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to

contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y) 2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.K-12.4	Model with mathematics.
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.A-SSE.B	Write expressions in equivalent forms to solve problems
MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines.
MA.K-12.6	Attend to precision.
MA.A-SSE.B.3b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.K-12.7	Look for and make use of structure.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MA.F-IF.C.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.F-BF	Building Functions
MA.A-CED	Creating Equations
MA.A-CED.A	Create equations that describe numbers or relationships
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.A-REI.B	Solve equations and inequalities in one variable
MA.A-REI.B.4	Solve quadratic equations in one variable.

MA.A-REI.B.4a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
MA.A-REI.B.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
MA.F-LE.A.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.