Unit 3: Transformations on the Coordinate Plane

Content Area: **Mathematics**
Course(s): **Geometry Honors 8**
Time Period: **October**
Length: **3 weeks**
Status: **Published**

Transfer

Previous coursework: students have informally defined many geometric terms, drawn figures using a ruler and a protractor given defined lengths and angle measures, explored the concept of congruence through transformations, informally proved relationships between angles associated with parallel lines and interior/exterior angles of triangles, worked with rotations, reflections, and translations to define congruency and informally with dilations

At the end of this unit: . Students should formally define transformations on the coordinate plane as function rules; given an input, what happens to the coordinates to obtain the output. Given a rule, students should be able describe whether segment length and angle measure is preserved. Students should also be able to map a figures onto other figures, as well as map a figure onto itself.

Instructional Notes:

- Use a variety of mediums for transformations: tracing paper, graph paper, geometry software.
- Allow students to explore transformation properties, such as corresponding congruent parts given different transformations.
- Provide students with a pre-image and an image and have them figure out different ways to map one onto the other.
- Transformations should be defined in terms of parallel lines.
- Rotations move along a circular arc given a specified angle.
- Reflections are drawn using perpendicular bisectors over the mirror line.
- (+) = denotes Honors only skill not on PARCC

Essential Questions

What is coordinate geometry?

How can distance and slope connect geometry and algebra?

How can distance be used to create figures on the coordinate plane?

Enduring Understandings

Coordinate geometry is a tool for discovering and verifying properties of geometric shapes.

Calculations of slope and length can be used to determine the type of shape based on its attributes.

Circle and parabola equations are based on properties of equidistance between points and lines.

Standards in Mathematics

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.G-CO.A	Experiment with transformations in the plane
	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
MA.K-12.2	Reason abstractly and quantitatively.
	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
MA.G-CO.A.2	Represent transformations in the plane using, e.g., transparencies and geometry software;

	describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
MA.G-CO.A.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
MA.G-CO.A.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
MA.G-CO.B	Understand congruence in terms of rigid motions
MA.K-12.5	Use appropriate tools strategically.
MA.G-CO.B.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
MA.G-CO.B.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
MA.G-CO.B.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
MA.G-SRT.A	Understand similarity in terms of similarity transformations
MA.G-SRT.A.1	Verify experimentally the properties of dilations given by a center and a scale factor:
MA.G-SRT.A.1a	A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
MA.G-SRT.A.1b	The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
MA.G-SRT.A.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
MA.G-SRT.A.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Critical Knowledge and Skills

Vocabulary

Writing Equations Parallel and Perpendicular Lines

Distance Formula

Classifying Quadrilaterals

Partitioning a Line Segment

Midpoint Formula

Perimeter and Area of Shapes on the coordinate plane

Definition of a Conic

Equation of a Circle

Completing the Square

Equation of a Parabola

Learning Objectives

Prove that parallel lines have congruent slopes and its converse.

Write a coordinate proof that parallel lines have congruent slopes (+)

Prove that perpendicular lines have negative reciprocal slopes and its converse.

Write a coordinate proof that perpendicular lines have negative reciprocal slopes (+)

Determine the slope of a line on the coordinate plane

Write the equation of a line given two points

Determine the equation of a line parallel or perpendicular to a line through a given point

Derive the distance formula using the Pythagorean Theorem (+)

Determine the distance between to points on the coordinate plane

Use slope and distance formulas to classify parallelograms and triangles on the coordinate plane

Determine the points on a line that partitions the segment in a given ratio (1:3 and 3:1)

Use partitioning techniques to derive the midpoint formula (+)

Use the midpoint formula to determine the coordinates for the midpoint of a line

Use coordinates to determine the perimeter and area of a figure on the coordinate plane

*Derive the equation of a circle.

*Determine the center and radius of a circle when given an equation of a circle in vertex form.

*Complete the square to transform the standard equation of a circle into vertex form

*Explain how to construct a parabola.

*Construct a parabola. (+)

*Derive the equation of a parabola from the distance formula given the focus and the directrix

*Graph a parabola given the equation. (+)

*Determine the focus and directrix of a parabola based on its equation.

Resources

Pearson Resources:

12-5, CB 12-5, 6-9, 3-8, 3-7, 7-4, 1-3, 1-7, 6-7, 10-1

Resources:

- http://pamjwilson.wordpress.com/2012/08/27/distance-midpoint-on-a-map/
- https://www.illustrativemathematics.org/illustrations/1302
- http://map.mathshell.org/materials/lessons.php?taskid=226&subpage=concept
- https://www.illustrativemathematics.org/illustrations/1302
- https://www.illustrativemathematics.org/illustrations/1687
- https://www.illustrativemathematics.org/illustrations/605
- https://www.illustrativemathematics.org/illustrations/1880
- https://www.illustrativemathematics.org/illustrations/1876
- http://map.mathshell.org/materials/lessons.php?taskid=406&subpage=concept
- http://map.mathshell.org/materials/lessons.php?taskid=425&subpage=concept
- http://www.geometrycommoncore.com/content/unit4/ggpe2/teachernotes1.html
- https://www.illustrativemathematics.org/HSG-GPE.A (there's 3 in this one)