Unit 3 - Quadratic Functions

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 2**
Length: **18 blocks**
Status: **Published**

Course Description & Instructional Notes

Course Description: This course is designed to help students in grade 12 develop and strengthen basic mathematical skills, problem solving techniques and the application of such skills. The course is intended to further develop the mathematical skills necessary for the college entrance exam.

In this unit, students will build upon concepts taught in Algebra 1 that include factoring, exposure to quadratic functions, and choosing the best method to solve a quadratic equation. An emphasis will be placed on essential academic vocabulary and college placement test skills.

Prior Knowledge: The following skills are a part of the Algebra 1 standards, they should be pre-assessed and remediated: Factoring, Solving Equations, Domain/Range

Instructional Notes: Students will need extended time with non-calculator skills. Daily warm-ups should emphasize the use of no calculator to solve problems.

Graphing Calculator Integration: Teacher will model the usage of the graphing calculator throughout the unit. Students will become comfortable with navigating and using the graphing calculator to solve a variety of problems efficiently (i.e. different ways to find the zeros, identifying key features of a quadratic function, solving quadratics, etc).

Technology Integration: Students will use Khan Academy as a supplemental resource.

Enduring Understandings

- Different quadratic forms reveal different characteristics of the function.
- Many real world situations can be modeled with a quadratic function.
- There are several strategies to solve quadratic equations.
- Simplifying expressions and solving equations allows us to take a complex situation and make it simple.
- A graph offers more than just plotted points.

Essential Questions

- How do you know if an equation is quadratic?
- How can you solve quadratic equations using concrete models, tables, graphs, and algebraic method?
- What are the characteristics of quadratic functions?
- How can you use functions to model real-world situations?
- How do you know which method to use when solving quadratics?

Student Learning Objectives

Solving Quadratics

- Solve quadratic equations by taking square roots, the quadratic formula and factoring, as appropriate to the initial form of the equation.
- Relate the value of the discriminant to the type of root to expect for the graph of a quadratic function. (one real root: 1 x-intercept, two real roots: 2 x-intercepts, no real roots: no x-intercepts)
- Understand the relationship between zeroes and factors.
- Solve Quadratic Equations for real and complex solutions applying a variety of methods including square roots, completing the square, the quadratic formula, factoring and the zero product property. Write complex solutions as $a \pm bi$ for real numbers a and b.
- Recognize when the quadratic formula gives complex solutions. (Which of the following equations has no real solutions?)

Graphing Quadratics

- Identify key characteristics of quadratic graphs including the axis of symmetry, vertex, maximum/minimum values, x-intercepts, y-intercepts, domain, range and intervals of increasing and decreasing.
- Graph Quadratic Equations from standard form, and vertex form
- For quadratic functions, identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.
- Identify domain, intercepts, and intervals where the function is increasing or decreasing and positive or negative

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Complex Numbers

- Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.
- Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

- Perform arithmetic operations with complex numbers

Vocabulary & Learning Experiences

Essential Academic Vocabulary

axis of symmetry, binomial, completing the square, complex number, decreasing, degree, difference of squares, discriminant, domain, factors, imaginary number, increasing, intercepts, monomial, perfect square trinomial, polynomial, quadratic, quadratic formula, range, roots, solutions, standard form. trinomial, vertex form, zeros

Planned Learning Experiences

Scavenger Hunts

Communicator Practice

Delta Math Practice

Think-Pair-Share

Resources

Khan Academy: https://www.khanacademy.org/math/algebra-basics

Math IXL

Assessments

Formative Assessments

Quiz: Classifying Polynomials

Quiz: Factoring

Quiz: Solving Quadratics

Quiz: Graphing Quadratics

Quiz: Complex Numbers

Summative Assessments

Unit 2 Non-Calculator Assessment

Unit 2 Calculator Assessment

NJSLS Standards - Mathematics

NJSLS Standards in Mathematics Copied and Pasted as well as linked.

NJSLS Standards - Mathematics

Arithmetic with Polynomials and Rational Expressions A -APR

A. Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials

Seeing Structure in Expressions A-SSE

A. Interpret the structure of expressions

2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.

B. Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★

a. Factor a quadratic expression to reveal the zeros of the function it defines.

Reasoning with Equations and Inequalities A -REI

B. Solve equations and inequalities in one variable

4. Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p) 2 = q that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

Interpreting Functions F-IF

A. Understand the concept of a function and use function notation

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

B. Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

C. Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Mathematical Practices

1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account New Jersey Student Learning Standards for Mathematics 4 the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making

assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y) 2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-REI.B.4	Solve quadratic equations in one variable.
MA.A-REI.B.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Additional NJSLS Standards

NJSLS Standards Copied and Pasted as well as linked.

Interdisciplinary Connections

NJSLS Companion Standards Grades 9-12 (Reading & Writing in Science & Technical Subjects)

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

Techonolgy (8.1 & 8.2)

8.1 Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

A. Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems and operations.

8.1.12.A.CS1 Understand and use technology systems.

8.1.12.A.CS2 Select and use applications effectively and productively.

21st Century Life and Careers (9.1 & 9.2)

CRP2. Apply appropriate academic and technical skills.

CRP4. Communicate clearly and effectively and with reason.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them

9.3.ST.2 Use technology to acquire, manipulate, analyze and report data.

Modifications/Accommodations