Unit 1 - Number Theory and the Real Number System

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 1**
Length: **20 blocks**
Status: **Published**

Course Description & Instructional Notes

Course Description: This course is designed to help students in grade 12 develop and strengthen basic mathematical skills, problem solving techniques and the application of such skills. The course is intended to further develop the mathematical skills necessary for the college entrance exam.

In this unit, students will build upon concepts taught in Algebra 1 that include operations with integers, exposure to number systems, operations with expressions, and solving equations and inequalities. An emphasis will be placed on essential academic vocabulary and college placement test skills.

Prior Knowledge: The following skills are a part of the Algebra 1 standards, they should be pre-assessed and remediated: Classifying Numbers, Integer Operations, Simplifying Expressions, Solving Equations/Inequalities

Instructional Notes: Students will need extended time with non-calculator skills. Daily warm-ups should emphasize the use of no calculator to solve problems.

Graphing Calculator Integration: Teacher will model the usage of the graphing calculator throughout the unit to check their work. Students will become comfortable with navigating and using the graphing calculator to solve a variety of problems efficiently. In this unit, the teacher should spend time getting students accustomed to the TI 84 and the basic essential skills [i.e. Executing integer operations (proper usage of parentheses, performing roots, powers) Solving equations on the graphing calculator].

Technology Integration: Students will use Khan Academy as a supplemental resource.

Enduring Understandings

- A quantity can be represented numerically in various ways.
- Computational fluency includes understanding the meaning and the appropriate use of numerical operations.

- Numeric fluency includes both the understanding of and the ability to appropriately use numbers.

Essential Questions

- What makes a computational strategy both effective and efficient?
- How can we compare and contrast numbers?
- How do mathematical ideas interconnect and build on one another to produce a coherent whole?

Student Learning Objectives

- Perform arithmetic operations with real numbers (integers and rationals).
- Simplify fractions.
- Convert between fractions, decimals, and percents.

- Simplify ratios, rates, and unit rates.
- Solve proportions
- Solve problems involving percentages.

- Apply the Order of Operations on integers.
- Evaluate an expression.
- Translate words into algebraic expressions
- Solve multi-step equations.

- Simplify an expression using the properties of exponents.
- Simplify radicals without a calculator.

Vocabulary & Learning Experiences

Essential Academic Vocabulary

absolute value, associative property, commutative property, difference, equations, evaluate, exponent, integer, irrational, justify, natural numbers, product, quotient, radical, rational, real, refute, scientific notation, simplify, solution, solve, sum, terms, whole numbers

Planned Learning Experiences

Scavenger Hunts

Communicator Practice

Online Test Prep

Resources

- Math IXL
- Khan Academy
- Barron's Accuplacer Math

Assessments

Formative Assessments

Quiz - Order of Operations & Absolute Value

Quiz - Properties of Exponents

Quiz - Decimals & Percentages

Quiz - Ratios, Proportions, Radicals

Summative Assessments

Unit 1 Non-Calculator Assessment

NJSLS Standards - Mathematics

NJSLS Standards in Mathematics Copied and Pasted as well as linked.

NJSLS Standards - Mathematics

Number and Quantity

The Real Number System N -RN

A. Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

B. Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities N -Q

A. Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Algebra

Seeing Structure in Expressions A-SSE

A. Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.

Reasoning with Equations and Inequalities A -REI

A. Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Mathematical Practices

1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account New Jersey Student Learning Standards for Mathematics 4 the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional

reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y) 2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MA.N-RN.A	Extend the properties of exponents to rational exponents.
MA.N-RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
MA.N-RN.B	Use properties of rational and irrational numbers.
MA.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
MA.N-Q.A	Reason quantitatively and use units to solve problems.
MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Additional NJSLS Standards

NJSLS Standards Copied and Pasted as well as linked.

Interdisciplinary Connections

NJSLS Companion Standards Grades 9-12 (Reading & Writing in Science & Technical Subjects)

RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.

RST.11-12.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.

Techonolgy (8.1 & 8.2)

8.1 Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

A. Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems and operations.

8.1.12.A.CS1 Understand and use technology systems.

8.1.12.A.CS2 Select and use applications effectively and productively.

21st Century Life and Careers (9.1 & 9.2)

CRP2. Apply appropriate academic and technical skills.

CRP4. Communicate clearly and effectively and with reason.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

9.3.ST.6 Demonstrate technical skills needed in a chosen STEM field.

Modifications/Accommodations