

Unit 8 Circular Motion & Kepler's Laws

Content Area: **Science**
Course(s):
Time Period: **April**
Length: **10 Blocks**
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Topic Outline

Motion Characteristics for Circular Motion

- Speed / Velocity / Acceleration / The Centripetal Force Requirement
- The Forbidden F-Word (Centrifugal) / Mathematics of Circular Motion

Applications of Circular Motion

- Newton's Second Law - Revisited / Amusement Park Physics / Athletics

Universal Gravitation

- Gravity / The Apple, the Moon, and the Inverse Square Law
- Newton's Law of Universal Gravitation / Cavendish and the Value of G
- The Value of g

Planetary and Satellite Motion

- Kepler's Three Laws / Circular Motion Principles for Satellites
- Mathematics of Satellite Motion

Weightlessness in Orbit / Energy Relationships for Satellites

Unit Summary

How was it possible for NASA to intentionally fly into Comet Tempel 1?

In this unit of study, students use mathematical and computational thinking to examine the processes governing the workings of the solar system and universe. The crosscutting concepts of scale, proportion, and quantity are called out as organizing concepts for these disciplinary core ideas. Students are expected to demonstrate proficiency in using mathematical and computational thinking and to use this practice to demonstrate understanding of core ideas.

updated from 11.19.15

Enduring Understandings

- Scientific laws can be used to make predictions about outcomes.
- Non-contact forces, which act at distance, are explained by fields.
- Objects orbiting the same massive object share properties that can be used to make predictions about their paths in which they orbit.
- Kepler's laws describe common features of the motions of orbiting objects, including their elliptical paths around the sun.
- Orbits may change due to the gravitational effects from, or collisions with, other objects in the solar system.
- The gravitational force between two bodies is inversely proportional to the square of the distance between the two bodies.

Essential Questions

- What is elliptical motion and eccentricity?
- How can one explain and predict interactions between objects and within systems of objects?
- Why is the inverse-square law seen in many different topics in physics?
- How does gravity affect objects in space?
- How does the separation of two bodies affect the gravitational force and motion between those bodies?
- How do observations in our universe lead to revolutionary explanations?

Student Learning Objectives (PE, SEP, DCI, CCC) & Aligned Standards

- Students will use equations with added support.
- Students will be introduced to Kepler's 3rd Law and they will discuss how the Universal Law of Gravitation is supported by Kepler's 3rd Law.

Performance Expectations

Use mathematical or computational representations to predict the motion of orbiting objects in the solar system. *[Clarification Statement: Emphasis is on Newtonian gravitational laws governing orbital motions, which apply to human-made satellites as well as planets and moons.] [Assessment Boundary: Mathematical representations for the gravitational attraction of bodies and Kepler's Laws of orbital motions should not deal with more than two bodies, nor involve calculus.]* (HS-ESS1-4)

Science and Engineering Practices

Using Mathematical and Computational Thinking

- Use mathematical or computational representations of phenomena to describe explanations. (HS-ESS1-4)

Disciplinary Core Ideas

ESS1.B: Earth and the Solar System

- Kepler's laws describe common features of the motions of orbiting objects, including their elliptical paths around the sun. Orbits may change due to the gravitational effects from, or collisions with, other objects in the solar system. (HS-ESS1-4)

Crosscutting Concepts

Scale, Proportion, and Quantity

- Algebraic thinking is used to examine scientific data and predict the effect of a change in one variable on another (e.g., linear growth vs. exponential growth). (HS-ESS1-4)

Connection to Engineering, Technology, and Applications of Science

Interdependence of Science, Engineering, and Technology

- Science and engineering complement each other in the cycle known as research and development (R&D). Many R&D projects may involve scientists, engineers, and others with wide ranges of

expertise. (HS-ESS1-4)

SCI.9-12.1.2	Classifications or explanations used at one scale may fail or need revision when information from smaller or larger scales is introduced; thus requiring improved investigations and experiments.
SCI.9-12.1.3	Patterns of performance of designed systems can be analyzed and interpreted to reengineer and improve the system.
SCI.9-12.1.4	Mathematical representations are needed to identify some patterns.
SCI.9-12.1.5	Empirical evidence is needed to identify patterns.
SCI.9-12.2.4	Changes in systems may have various causes that may not have equal effects.
SCI.9-12.3.2	Some systems can only be studied indirectly as they are too small, too large, too fast, or too slow to observe directly.
SCI.9-12.3.4	Using the concept of orders of magnitude allows one to understand how a model at one scale relates to a model at another scale.
SCI.9-12.3.5	Algebraic thinking is used to examine scientific data and predict the effect of a change in one variable on another (e.g., linear growth vs. exponential growth).
SCI.9-12.4.3	Models (e.g., physical, mathematical, computer models) can be used to simulate systems and interactions—including energy, matter, and information flows—within and between systems at different scales.
SCI.9-12.5.1.12.A.1	Refine interrelationships among concepts and patterns of evidence found in different central scientific explanations.
SCI.9-12.5.1.12.A.2	Develop and use mathematical, physical, and computational tools to build evidence-based models and to pose theories.
SCI.9-12.5.1.12.A.a	Mathematical, physical, and computational tools are used to search for and explain core scientific concepts and principles.
SCI.9-12.5.1.12.B.3	Revise predictions and explanations using evidence, and connect explanations/arguments to established scientific knowledge, models, and theories.
SCI.9-12.5.1.12.B.d	Scientific reasoning is used to evaluate and interpret data patterns and scientific conclusions.
SCI.9-12.5.1.12.C.1	Reflect on and revise understandings as new evidence emerges.
SCI.9-12.5.1.12.C.2	Use data representations and new models to revise predictions and explanations.
SCI.9-12.5.1.12.C.b	Data and refined models are used to revise predictions and explanations.
SCI.9-12.5.1.12.C.c	Science is a practice in which an established body of knowledge is continually revised, refined, and extended as new evidence emerges.
SCI.9-12.5.1.12.D.1	Engage in multiple forms of discussion in order to process, make sense of, and learn from others' ideas, observations, and experiences.
SCI.9-12.5.1.12.D.2	Represent ideas using literal representations, such as graphs, tables, journals, concept maps, and diagrams.
SCI.9-12.5.1.12.D.a	Science involves practicing productive social interactions with peers, such as partner talk, whole-group discussions, and small-group work.
SCI.9-12.5.1.12.D.b	Science involves using language, both oral and written, as a tool for making thinking public.
SCI.9-12.7.2	Change and rates of change can be quantified and modeled over very short or very long periods of time. Some system changes are irreversible.
SCI.9-12.CCC.1.1	students observe patterns in systems at different scales and cite patterns as empirical evidence for causality in supporting their explanations of phenomena. They recognize

classifications or explanations used at one scale may not be useful or need revision using a different scale; thus requiring improved investigations and experiments. They use mathematical representations to identify certain patterns and analyze patterns of performance in order to reengineer and improve a designed system.

- SCI.9-12.CCC.2.1 students understand that empirical evidence is required to differentiate between cause and correlation and to make claims about specific causes and effects. They suggest cause and effect relationships to explain and predict behaviors in complex natural and designed systems. They also propose causal relationships by examining what is known about smaller scale mechanisms within the system. They recognize changes in systems may have various causes that may not have equal effects.
- SCI.9-12.CCC.3.1 students understand the significance of a phenomenon is dependent on the scale, proportion, and quantity at which it occurs. They recognize patterns observable at one scale may not be observable or exist at other scales, and some systems can only be studied indirectly as they are too small, too large, too fast, or too slow to observe directly. Students use orders of magnitude to understand how a model at one scale relates to a model at another scale. They use algebraic thinking to examine scientific data and predict the effect of a change in one variable on another (e.g., linear growth vs. exponential growth).
- SCI.9-12.CCC.5.1 students learn that the total amount of energy and matter in closed systems is conserved. They can describe changes of energy and matter in a system in terms of energy and matter flows into, out of, and within that system. They also learn that energy cannot be created or destroyed. It only moves between one place and another place, between objects and/or fields, or between systems. Energy drives the cycling of matter within and between systems. In nuclear processes, atoms are not conserved, but the total number of protons plus neutrons is conserved.
- SCI.9-12.CCC.6.1 students investigate systems by examining the properties of different materials, the structures of different components, and their interconnections to reveal the system's function and/or solve a problem. They infer the functions and properties of natural and designed objects and systems from their overall structure, the way their components are shaped and used, and the molecular substructures of their various materials.
- SCI.9-12.CCC.7.1 students understand much of science deals with constructing explanations of how things change and how they remain stable. They quantify and model changes in systems over very short or very long periods of time. They see some changes are irreversible, and negative feedback can stabilize a system, while positive feedback can destabilize it. They recognize systems can be designed for greater or lesser stability.
- SCI.9-12.SEP.1.a Ask questions
- SCI.9-12.SEP.1.a.1 that arise from careful observation of phenomena, or unexpected results, to clarify and/or seek additional information.
- SCI.9-12.SEP.1.a.2 that arise from examining models or a theory, to clarify and/or seek additional information and relationships.
- SCI.9-12.SEP.1.a.4 to clarify and refine a model, an explanation, or an engineering problem.
- SCI.9-12.SEP.1.d Ask and/or evaluate questions that challenge the premise(s) of an argument, the interpretation of a data set, or the suitability of a design.
- SCI.9-12.SEP.2.a Evaluate merits and limitations of two different models of the same proposed tool, process, mechanism or system in order to select or revise a model that best fits the evidence or design criteria.
- SCI.9-12.SEP.3.e Make directional hypotheses that specify what happens to a dependent variable when an independent variable is manipulated.
- SCI.9-12.SEP.3.f Manipulate variables and collect data about a complex model of a proposed process or system to identify failure points or improve performance relative to criteria for success or other variables.

SCI.9-12.SEP.4.a	Analyze data using tools, technologies, and/or models (e.g., computational, mathematical) in order to make valid and reliable scientific claims or determine an optimal design solution.
SCI.9-12.SEP.4.b	Apply concepts of statistics and probability (including determining function fits to data, slope, intercept, and correlation coefficient for linear fits) to scientific and engineering questions and problems, using digital tools when feasible.
SCI.9-12.SEP.7.c	Respectfully provide and/or receive critiques on scientific arguments by probing reasoning and evidence, challenging ideas and conclusions, responding thoughtfully to diverse perspectives, and determining additional information required to resolve contradictions.
SCI.9-12.SEP.8.a	Critically read scientific literature adapted for classroom use to determine the central ideas or conclusions and/or to obtain scientific and/or technical information to summarize complex evidence, concepts, processes, or information presented in a text by paraphrasing them in simpler but still accurate terms.
SCI.9-12.HS-ESS1-4	Use mathematical or computational representations to predict the motion of orbiting objects in the solar system.
SCI.9-12.HS-PS2-4	Use mathematical representations of Newton's Law of Gravitation and Coulomb's Law to describe and predict the gravitational and electrostatic forces between objects.

Concepts & Formative Assessment

Part A: How was it possible for NASA to intentionally fly into Comet Tempel 1?

Concepts

- Kepler's laws describe common features of the motions of orbiting objects, including their elliptical paths around the sun. Orbits may change due to the gravitational effects from, or collisions with, other objects in the solar system.
- Algebraic thinking is used to examine scientific data and predict the effect of a change in one variable on another. (e.g., linear growth vs. exponential growth).

Formative Assessment

Students who understand the concepts are able to:

- Use mathematical or computational representations to predict the motion of orbiting objects in the solar system.
- Use mathematical and computational representations of Newtonian gravitational laws governing orbital motion that apply to moons and humanmade satellites.
- Use algebraic thinking to examine scientific data and predict the motion of orbiting objects in the solar system.

Resources

- [Orbital Motion Interactive](#) - The purpose of this Physics Classroom activity is to investigate the nature of an elliptical orbit of a planet or other satellite about the Sun or some central body.
- [Physics Classroom webpage and Worksheet](#) - Kepler's Laws and Planetary Motion

- [Circular and Satellite Motion Interactives](#) - the physics classroom list of virtual activities
- [Planetary Orbits Lab](#) - Understanding and utilizing Kepler's laws of motion plus the effects of velocity and force on a satellites' orbit.
- [Gravity and Orbits](#) - In this activity students will be exploring Kepler's Laws and Orbital Motion using the "Gravity and Orbits" PhET simulation.
- [Gravity Force Lab](#) - Students will use the Gravity Force Lab PhET Simulation to investigate what the gravitational force between two objects depends on and experimentally determine the Universal Gravitational constant, G. [Lab Sheet](#)
- [Period of Jupiter's moons](#) - Students use a series of 31 images of Jupiter's 4 Galilean moons to find their orbit periods and orbit radii. They compare their results with known data for those moons. Finally they test various mathematical expressions to find a "constant" relationship between orbit period (T) and orbit radius (R) to arrive at Kepler's 3rd Law. ([All activities Kepler's NASA](#))
- [Periodic Planetary Orbits](#) - This activity will show how to calculate the period of the orbit (length of the year) for planets in the Solar System.
- [Curtate of Planetary Orbits](#) - Calculate and plot orbits of Planets in Solar System
- [Exploring Kepler's Laws and the Universal Law of Gravitation](#) - Using Interactive Physics to explore Kepler's laws of planetary motion and the universal law of gravitation.
- [Basic Kepler Activity](#) - This activity will discuss the properties of ellipses and Kepler's laws of orbital motion.

Assessments

- [Orbital Motion Interactive](#) - The purpose of this Physics Classroom activity is to investigate the nature of an elliptical orbit of a planet or other satellite about the Sun or some central body.
- Students will be given planetary data to draw the elliptical paths of different planets.
- Linking Newton's Law of Universal Gravitation to Kepler's 3rd Law.

The Science Classroom

In this unit, students will develop an understanding of Kepler's laws, which describe common features of the motions of orbiting objects, including their elliptical paths around the sun. They will also learn how orbits may change due to the gravitational effect from, or collisions with, other objects in the solar system. They will also use algebraic thinking and mathematical and computational representations to examine data and predict the motion of orbiting objects, including moons in our solar system and human-made satellites.

Regarding Kepler's first law, students must have experience in creating an ellipse with two foci in order to

appreciate that the sun and the center of the solar system's mass are the two foci around which the Earth orbits. Having students actually create ellipses with tacks, cardboard, and string will provide a concrete example of Kepler's first law. Students should also use a mathematical model to explain the motion of orbiting objects in the solar system, identifying any important quantities and relationships and using units when appropriate.

Regarding Kepler's second law, students must understand that a line joining a planet and the sun sweeps out equal areas during equal intervals of time. Diagrams should be used to facilitate understanding of this concept. For example, students can represent the ellipse from the previous exercise on graph paper. The ellipse can then be divided into equal arc lengths representing time intervals. Next, the area of each wedge can be approximated by finding the area of each approximate triangle. Students should keep accuracy and limitations of measurement in mind while modeling the motion of orbiting objects. Using a pizza that isn't cut symmetrically as an example, ask students where planets are moving fastest and slowest. Ask where areas of greatest centripetal force and acceleration are located.

Students must be able to perform mathematical computations with using Kepler's third law.

$$\text{Kepler's third law: } r_A^3 / T_A^2 = r_B^3 / T_B^2$$

Kepler observed in the law of harmonies that this ratio is the same for every planet in our solar system. Students should understand the value of one astronomical unit (AU) and the distance from the Earth to the sun (149,597,870.700 kilometers) in order to facilitate calculations for astronomical bodies orbiting our sun. Time can be measured in Earth days or Earth years.

Students must also be able to combine Newton's law of universal gravitation with Kepler's third law to obtain Newton's version of Kepler's third law. This can then be used to describe planetary motion in our solar system with no more than two bodies at a time. Students must be able to predict the motion of human-made satellites as well as planets and moons. Students should be able to describe, for example, why any geosynchronous satellite must always maintain a specific orbit.

Students should apply Kepler's and Newton's laws to astronomical data in order to determine the validity of the laws. They might be given astronomical data in the form of numerical tables showing periods and radii. Examples should also include pictorial data of the shapes of orbits of planets in our solar system.

It might be useful to reinforce prior learning of Newton's laws ($F=ma$, law of inertia) while showing how orbits may change due to the gravitational effects from, or collisions with, other objects in the solar system. Students must be able to explain why planetary orbits may change (e.g., the Kessler Effect, perturbations, wobble, etc.).

Students should appreciate how astronomers find extrasolar planets. They should also be able to explain how observations about an orbiting planet can yield information about the mass and location of the star it orbits.

Students should be able analyze data in which variables such as force, mass, period, and radius of orbit are changed in order to visualize the relationships between a central force and an orbiting body within the context of Kepler's laws as well as the law of universal gravitation. For example, lab data or planetary data may be fed into a computer simulation (PhET), and the resulting orbital behavior analyzed for its compliance with Kepler's laws and universal gravitation

Connecting Mathematics

Connections to Mathematics-

- Represent the motion of orbiting objects in the solar system symbolically, and manipulate the representing symbols. Make sense of quantities and relationships about the motion of orbiting objects in the solar system symbolically and manipulate the representing symbols.
- Use a mathematical model to explain the motion of orbiting objects in the solar system. Identify important quantities in the motion of orbiting objects in the solar system and map their relationships using tools. Analyze those relationships mathematically to draw conclusions, reflecting on the results and improving the model if it has not served its purpose.
- Use units as a way to understand the motion of orbiting objects in the solar system and to guide the solution of multistep problems; choose and interpret units representing the motion of orbiting objects in the solar system consistently in formulas; chose and interpret the scale and the origin in graphs and data displays representing the motion of orbiting objects in the solar system.
- Define appropriate quantities for the purpose of descriptive modeling of the motion of orbiting objects in the solar system.
- Choose a level of accuracy appropriate to limitations on measurement when reporting quantities representing the motion of orbiting objects in the solar system.
- Interpret expressions that represent the motion of orbiting objects in the solar system.
- Create equations in two or more variables to represent relationships between quantities representing the motion of orbiting objects in the solar system; graph equations representing the motion of orbiting objects in the solar system on coordinate axes with labels and scales.
- Rearrange formulas representing the motion of orbiting objects in the solar system to highlight a quantity of interest, using the same reasoning as in solving equations.

Modifications

Teacher Note: Teachers identify the modifications that they will use in the unit. The unneeded modifications

can then be deleted from the list.

- Restructure lesson using UDL principals (http://www.cast.org/our-work/about-udl.html#.VXmoXcfD_UA)
- Structure lessons around questions that are authentic, relate to students' interests, social/family background and knowledge of their community.
- Provide students with multiple choices for how they can represent their understandings (e.g. multisensory techniques-auditory/visual aids; pictures, illustrations, graphs, charts, data tables, multimedia, modeling).
- Provide opportunities for students to connect with people of similar backgrounds (e.g. conversations via digital tool such as SKYPE, experts from the community helping with a project, journal articles, and biographies).
- Provide multiple grouping opportunities for students to share their ideas and to encourage work among various backgrounds and cultures (e.g. multiple representation and multimodal experiences).
- Engage students with a variety of Science and Engineering practices to provide students with multiple entry points and multiple ways to demonstrate their understandings.
- Use project-based science learning to connect science with observable phenomena.
- Structure the learning around explaining or solving a social or community-based issue.
- Provide ELL students with multiple literacy strategies.
- Collaborate with after-school programs or clubs to extend learning opportunities.

Research on Student Learning

Research suggests teaching the concepts of spherical Earth, space, and gravity in close connection to each other. Students typically do not understand gravity as a force and misconceptions about the causes of gravity persist after traditional high-school physics instruction. These misconceptions about the causes of gravity can be overcome by specially designed instruction. Students of all ages may hold misconceptions about the magnitude of the earth's gravitational force. Even after a physics course, many high-school students believe that gravity increases with height above the earth's surface.

High-school students also have difficulty in conceptualizing gravitational forces as interactions between two objects. In particular, they have difficulty in understanding that the magnitudes of the gravitational forces that two objects of different mass exert on each other are equal therefore resulting in a deeper understanding of the relationships between the object as per of measurable phenomenon. The difficulties persist even after specially designed instruction ([NSDL, 2015](#)).

Prior Learning

Physical science-

- For any pair of interacting objects, the force exerted by the first object on the second object is equal in strength to the force that the second object exerts on the first, but in the opposite direction (Newton's third law).
- The motion of an object is determined by the sum of the forces acting on it; if the total force on the object is not zero, its motion will change. The greater the mass of the object, the greater the force needed to achieve the same change in motion. For any given object, a larger force causes a larger change in motion.
- All positions of objects and the directions of forces and motions must be described in an arbitrarily chosen reference frame and arbitrarily chosen units of size. In order to share information with other people, these choices must also be shared.
- Electric and magnetic (electromagnetic) forces can be attractive or repulsive, and their sizes depend on the magnitudes of the charges, currents, or magnetic strengths involved and on the distances between the interacting objects.
- Gravitational forces are always attractive. There is a gravitational force between any two masses, but it is very small except when one or both of the objects have large mass—e.g., Earth and the sun.
- Forces that act at a distance (electric, magnetic, and gravitational) can be explained by fields that extend through space and can be mapped by their effect on a test object (a charged object or a ball, respectively).

Earth and space sciences-

- Patterns of the apparent motion of the sun, the moon, and the stars in the sky can be observed, described, predicted, and explained with models.
- Earth and its solar system are part of the Milky Way Galaxy, which is one of many galaxies in the universe.
- The solar system consists of the sun and a collection of objects, including planets, their moons, and asteroids that are held in orbit around the sun by its gravitational pull on them.
- This model of the solar system can explain eclipses of the sun and the moon. Earth's spin axis is fixed in direction over the short term but tilted relative to its orbit around the sun. The seasons are a result of that tilt and are caused by the differential intensity of sunlight on different areas of Earth across the year.
- The solar system appears to have formed from a disk of dust and gas, drawn together by gravity.

Connections to Other Courses

Physical Science

- Newton’s law of universal gravitation and Coulomb’s law provide the mathematical models to describe and predict the effects of gravitational and electrostatic forces between distant objects.
- Forces at a distance are explained by fields (gravitational, electric, and magnetic) permeating space that can transfer energy through space. Magnets or electric currents cause magnetic fields; electric charges or changing magnetic fields cause electric fields.
- Attraction and repulsion between electric charges at the atomic scale explain the structure, properties, and transformations of matter, as well as the contact forces between material objects.

References

Adapted from the New Jersey NGSS Science Model Curriculum

Connections to NJSL

English Language Arts/Literacy

Assess the extent to which the reasoning and evidence in a text support the author’s claim or a recommendation for solving a scientific or technical problem. (HS-PS4-3), (HS-PS4-4),(HS-PS4-2) **RST.9-10.8**

Cite specific textual evidence to support analysis of science and technical texts, attending to important distinctions the author makes and to any gaps or inconsistencies in the account. (HS-PS4-3), (HS-PS4-4),(HS-PS4-2) **RST.11-12.1**

Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem. (HS-PS4-4),(HS-ETS1-1),(HS-ETS1-3) **RST.11-12.7**

Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information. (HS-PS4-3), (HS-

ETS1-1), (HS-ETS1-3),(HS-PS4-2) **RST.11-12.8**

Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible. (HS-ETS1-1), (HS-ETS1-3) **RST.11-12.9**

Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes. (HS-PS4-5) **WHST.11-12.2**

Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the strengths and limitations of each source in terms of the specific task, purpose, and audience; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and overreliance on any one source and following a standard format for citation. (HS-PS4-4) **WHST.11-12.8**

Mathematics

Reason abstractly and quantitatively. (HS-ESS1-4) **MP.2**

Model with mathematics. (HS-ESS1-4) **MP.4**

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (HS-ESS1-4) **HSN-Q.A.1**

Define appropriate quantities for the purpose of descriptive modeling. (HS-ESS1-4) **HSN-Q.A.2**

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. (HS-ESS1-4) **HSN-Q.A.3**

Interpret expressions that represent a quantity in terms of its context. (HS-ESS1-4) **HSA-SSE.A.1**

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (HS-ESS1-4) **HSA-CED.A.2**

Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (HS-ESS1-4) **HSA-CED.A.4**