# *Unit 7- Sequences and Series

Content Area: **Mathematics**
Course(s): **Algebra 2 Honors**
Time Period: **April**
Length: **11 blocks**
Status: **Published**

## Transfer Skills

In this unit students will extend their prior study of algebraic patterns to include series. Students will make connections between patterns and graphs to prepare for graph types to be encountered in future units and courses.

**Instructional Notes**

Students will extend upon their study of patterns from Algebra 1. In this unit, the teacher will provide opportunities for students to review linear and exponential functions. Additional prerequisite skills should be included as necessary which will most likely include solving multi-step equations among other concepts.

Students should be fluent in working with linear equations and all forms of linear equations. Teacher should integrate concepts as necessary.

## Enduring Understandings

Patterns emerge from data.

Patterns show different ways of solving the same problem.

Patterns are used to make predictions.

Patterns are represented in different ways.

## Essential Questions

How can you use a pattern to predict outcomes?

What kinds of iteration rules yield different sequences?

What makes a series infinite?

What can a graph tell us about a pattern?

## Content

**Vocabulary:**

Explicit

Recursive

Arithmetic

Geometric

Sequence

Series

Summation

Infinite

Finite

## Skills

**Sequences**

Write the explicit and recursive rules for arithmetic and geometric sequences.

Examine arithmetic and geometric sequences to construct linear and exponential functions and graphs of such.

Write the explicit rule for a sequence given recursively and vice versa.

Write the recursive and explicit rules if possible for non-arithmetic or non-geometric sequences including squares, cubes, and Fibonacci.

**Series**

Write a series with summation notation.

Evaluate the sum of a series in summation notation.

Calculate the sum of finite geometric series.

Derive the formula for the sum of a finite geometric series (when the common ratio is not 1).

Use the formula for the sum of a finite geometric series to solve problems.

Estimate the rate of change from an explicit or recursive rule, graph or table

**Applications of Sequences and Series**

Solve real world applications using sequence and series formuals

## Resources

**Teacher Resources by Standard**

www.illustrativemathematics.org

katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf

illuminations.nctm.org/

www.pbslearningmedia.org/

**Online Teaching Websites**

www.khanacademy.org

www.ixl.com

**Algebra 2 Common Core Textbook**

Chapter 9 pg. 561

## Standards

**NJSLS 2016**

**Algebra**

**Creating Equations**

**A -CED A. Create equations that describe numbers or relationships**

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

**Seeing Structure in Expressions**

**A-SSE B. Write expressions in equilvalent fomrs to solve problems.**

4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

**Functions**

**Building Functions**

**F-BF Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model

situations, and translate between the two forms.

**Interpreting Functions**

**F-IF A. Understand the concept of a function and use function notation**

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

**B. Interpret functions that arise in applications in terms of the context**

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph

**Linear and Exponential Models**

**F-LE A. Construct and compare linear and exponential models and solve problems**

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

**B. Interpret expressions for functions in terms of the situation they model**

5. Interpret the parameters in a linear or exponential function in terms of a context.

**Mathematical Practices**

**1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger

students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

**7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y) 2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

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| MA.F-IF | Interpreting Functions |
| MA.A-SSE | Seeing Structure in Expressions |
| MA.K-12.1 | Make sense of problems and persevere in solving them. |
| | Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. |
| MA.F-IF.A.3 | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. |
| | For example, the Fibonacci sequence is defined recursively by 𝑓(0) = 𝑓(1) = 1, 𝑓(𝑛 + 1) = 𝑓(𝑛) + 𝑓(𝑛 − 1) for 𝑛 ≥ 1. |
| MA.A-SSE.B | Write expressions in equivalent forms to solve problems |
| | For example, if the function 𝑛(𝑠) gives the number of person-hours it takes to assemble 𝑠 engines in a factory, then the positive integers would be an appropriate domain for the function. |
| MA.F-IF.B.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. |
| MA.K-12.7 | Look for and make use of structure. |
| | Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as |

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| | seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression □² + 9□ + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(□ – □)² as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers □ and □. |
| MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.<br><br>For example, calculate mortgage payments. |
| MA.F-BF | Building Functions |
| MA.F-BF.A | Build a function that models a relationship between two quantities |
| MA.F-BF.A.1 | Write a function that describes a relationship between two quantities. |
| MA.F-BF.A.1a | Determine an explicit expression, a recursive process, or steps for calculation from a context. |
| MA.A-CED | Creating Equations |
| MA.A-CED.A | Create equations that describe numbers or relationships |
| MA.A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems. |
| MA.A-CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. |
| MA.F-BF.A.2 | Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. |
| MA.A-CED.A.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. |
| MA.F-LE | Linear and Exponential Models |
| MA.F-LE.A.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions. |
| MA.F-LE.A.1a | Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. |
| MA.F-LE.A.1b | Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. |
| MA.F-LE.A.1c | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. |
| MA.F-LE.A.2 | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). |
| MA.F-LE.A.3 | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. |