*Unit 6- Trigonometric Ratios and Functions

Content Area: **Mathematics**
Course(s): **Algebra 2 Honors**
Time Period: **February**
Length: **11 blocks**
Status: **Published**

Transfer Skills

Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena. Students will work with the unit circle and use radian measure for angles.

Instructional Notes

Students will be required to go beyond the standards to include solving trig equations, transformations of trig functions, and identifying resulting trig functions as even and odd trig functions after transformations. Teacher should pre-assess work with right triangles.

Enduring Understandings

Many real world applications involve the use of trigonometry.

There are fixed relationships between the angles and sides of a triangle.

Trigonometric functions are periodic functions.

Graphs of the trigonometric functions may be transformed and translated.

Essential Questions

How does the unit circle relate to the trigonometric functions?

In what way can knowledge of triangles, trigonometry, and ratios help you solve real life problems?

Why does any trigonometric value of any angle of a specific triangle remain the same regardless of the unit of measure?

Content

Vocabulary

Terminal side

Reciprocal

Tangent

Sine

Cosine

Radian

Reference angle

Sector

Arc

Even

Odd

Skills

Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

Write angles in both degrees and radians.

Construct the unit circle from special right triangles.

Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Sketch an angle given in radians and degrees. Determine an angle measure in radians and degrees from a picture.

Use the relationship of the six trigonometric functions to a central angle of the unit circle to determine the exact trigonometric ratio of angles on the unit circle. (0º to 360º, 0 to 2pi)

Identify the angle that produce an exact value of a trigonometric function.

Prove the trigonometric identity $\sin^2 x + \cos^2 x = 1$ and use it to calculate trigonometric ratios given one ratio and a quadrant.

Recognize periodic phenomena and determine key characteristics of such phenomena.

Represent trigonometric functions using tables, graphs, verbal statements, and equations. Translate among these representations.

Determine key characteristics of trigonometric functions and their graphs.

Identify the domain and the range of a sine, or cosine equation, both basic equations and transformed equations.

Graph transformations of sine, and cosine with all transformations

Identify even and odd trigonometric functions.

Estimate the rate of change over a specific interval

Resources

Teacher Resources by Standard

www.illustrativemathematics.org

katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf

illuminations.nctm.org/

www.pbslearningmedia.org/

Online Teaching Websites

www.khanacademy.org

www.ixl.com

Algebra 2 Common Core Textbook

Chapter 14 pg. 964

Standards

NJSLS 2016

Functions

Interpreting Functions

F-IF B. Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph

C. Analyze functions using different representations

7. Graph functions expressed symbollically and show key features of a the graph, by hand in simply cases and using technology for more complicated cases.

Building Functions

F-BF B. Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Trigonometric Functions

F-TF A.Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for π□x, $\pi+x$, and $2\pi–x$ in terms of their values for x, where x is any real number.

4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

B. Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

C. Prove and apply trigonometric identities

8. Prove the Pythagorean identity sin2 (θ) + cos2 (θ) = 1 and use it to find sin(θ), cos(θ),or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.

9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MA.F-BF	Building Functions
MA.F-BF.B	Build new functions from existing functions
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.F-IF	Interpreting Functions
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-TF	Trigonometric Functions
MA.F-TF.A	Extend the domain of trigonometric functions using the unit circle
MA.F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
MA.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
MA.F-TF.A.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
MA.F-TF.B	Model periodic phenomena with trigonometric functions
MA.F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
MA.F-TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
MA.F-TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
MA.F-TF.C	Prove and apply trigonometric identities
MA.F-TF.C.8	Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ given $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle.
MA.F-TF.C.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
MA.K-12.1	Make sense of problems and persevere in solving them.

MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.4	Model with mathematics.

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