*Unit 5- Rational Equations and Functions

Content Area: **Mathematics**
Course(s): **Algebra 2 Honors**
Time Period: **February**
Length: **11 blocks**
Status: **Published**

Transfer Skills

In this unit students will be introduced to rational functions. Students will continue work with creating equivalent expressions to identify different properties. Students will gain a basic understanding of a rational graph and use inverses to aid in finding Domain and Range.

Instructional Notes

Teacher is encouraged to use this unit to continue work with factoring. Students should have a strong understanding of the graph of $f(x) = 1/x$ and be able to work with transformations of the graph and be able to generate table of values to graph rational functions. Honors students should go in much more depth as to the graphing of the functions.

Enduring Understandings

Mastering a procedure is not the same as understanding the concept.

Simplified expressions are essential in being able to solve equations.

Domain affects graphing and solving of rational functions.

Essential Questions

How do we decide which method is most appropriate when solving rational equations?

When are asymptotes used to graph rational functions?

Content

Vocabulary

Rational Expressions

Rational Functions

Asymptotes

Points of Discontinuity

End Behavior

Transformations

Inverse

Domain

Range

Variation

Skills

Operations with Rational Functions

+ Multiply and Divide rational expressions and identify the excluded values.

+ Add and subtract rational expressions and identify the excluded values.

+ Simplify complex fractions and identify the excluded values.

Rewrite simple rational expressions in different forms using inspection, long division, or, for the more complicated examples, a computer algebra system. (See standard HSA-APR.D.6. above)

Find the inverse of a rational function and identify the domain and range for the function and its inverse.

Solve simple rational equations in one variable and use them to solve problems, justify each step in the process and the solution.

Show how extraneous solutions may arise when solving a rational equation.

Graphing Rationals (HONORS ONLY)

+ Graph Rational functions identifying key characteristics including zeros, x- and y-intercepts, vertical and horizontal asymptotes, points of discontinuity and end behavior.

+ Interpret key features of graphs and table in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (Basic Level of Graph)

+ Identify the effect on the graph of a rational function by replacing f(x) by

f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

+ Honors Only

Resources

Teacher Resources by Standard

www.illustrativemathematics.org

katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf

illuminations.nctm.org/

www.pbslearningmedia.org/

Online Teaching Websites

www.khanacademy.org

www.ixl.com

Algebra 2 Common Core Textbook

Chapter 8 pg. 495

Standards

NJSLS 2016

Algebra

Arithmetic with Polynomials and Rational Expressions

A -APR D. Rewrite rational expressions

6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations

A -CED A. Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Reasoning with Equations and Inequalities

A -REI A. Understand solving equations as a process of reasoning and explain the reasoning

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Functions

Building Functions

F-BF B. Build new functions from existing functions

3. (+) Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

4. Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠1.

Interpreting Functions

F-IF B. Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph

C. Analyze functions using different representations

7. (+) Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Mathematical Practices

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use

technological tools to explore and deepen their understanding of concepts.

MA.F-IF	Interpreting Functions
MA.K-12.1	Make sense of problems and persevere in solving them.
	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.K-12.5	Use appropriate tools strategically.
	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
	For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.A-APR	Arithmetic with Polynomials and Rational Expressions

MA.F-IF.C.7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MA.F-BF	Building Functions
MA.A-APR.D	Rewrite rational expressions
MA.A-APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x)$ + $r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.A-CED	Creating Equations
MA.A-CED.A	Create equations that describe numbers or relationships
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.F-BF.B	Build new functions from existing functions
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.F-BF.B.4	Find inverse functions.
MA.F-BF.B.4a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
MA.A-REI	Reasoning with Equations and Inequalities
MA.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MA.A-REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.