*Unit 1 - Quadratic Equations, Functions and Systems

Content Area: **Mathematics**
Course(s): **Algebra 2 Honors**
Time Period: **September**
Length: **11 blocks**
Status: **Published**

Enduring Understandings

There are several strategies to solve quadratic equations.

Simplifying expressions and solving equations allows us to take a complex situation and make it simple.

A graph offers more than just plotted points.

Essential Questions

How do you know if an equation is quadratic?

How do you know which method to use when solving quadratics?

What is the best method for graphing a quadratic function?

Content

Vocabulary

Complex Number
Quadratic
Factors
Zeros
Completing the Square
Quadratic Formula
Axis of Symmetry
Intercepts
Domain
Range
Increasing
Decreasing
Standard Form

Vertex Form
Intercept Form
Transformation
Even Function
Odd Function

Skills

Complex Numbers

Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.

Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

(+) Extend polynomial identities to the complex numbers. *For example, rewrite x^2+4 as (x+2i)(x-2i)*

(+) Apply the application of conjugates to divide complex numbers.

Solving Quadratics

Solve Quadratic Equations for real and complex solutions applying a variety of methods including square roots, completing the square, the quadratic formula, factoring and the zero product property. Write complex solutions as a ± bi for real numbers a and b.

Recognize when the quadratic formula gives complex solutions. (Which of the following equations has no real solutions?)

+ Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Graphing Quadratics

Identify key characteristics of quadratic graphs including the axis of symmetry, vertex, maximum/minimum values, x-intercepts, y-intercepts, domain, range and intervals of increasing and decreasing.

Graph Quadratic Equations from standard form, and vertex form

Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

For quadratic functions, identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Recognize and identify even and odd quadratic functions from their graphs and algebraic expressions for them.

Relate the domain of a quadratic function to its graph and, where applicable, to the quantitative relationship it describes.

Systems of Equations

Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions

Solve systems of linear equations exaclty and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables

Solve a system of 3 linear equations.

Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. (For example, find the points of intersection between the line y= -3x, and the circle x^2 + y^2 = 3

Explain why the x-coordinates of the points where graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equations f(x) = g(x); find the solutions approximatly, e.g., using technology to graph the functions, make tables of values, or find successive approximations.

Estimate the rate of change over a specific interval

Use the quadratic formula to solve real life application problems

+ Honors Only Skills

Resources

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Teacher Resources by Standard
www.illustrativemathematics.org
katm.org/wp/wp-content/uploads/flipbooks/High-School-CCSS-Flip-Book-USD-259-2012.pdf
illuminations.nctm.org/
www.pbslearningmedia.org/
Online Teaching Websites
www.khanacademy.org
www.ixl.com

Algebra 2 Common Core Textbook
Quadratics: Chapter 4 pg. 191

Systems: Chapter 3 pg. 131, Chapter 4 pg. 258

Standards

NJSLS 2016

Algebra

Creating Equations★

A -CED A. Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

Reasoning with Equations and Inequalities

A-REI A. Understand solving equations as a process of reasoning and explain the reasoning.

B. Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

4. Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

C. Solve systems of equations

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = –3x$ and the circle $x2 + y2 = 3$.

D. Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Seeing Structure in Expression

A-SSE A. Interpret the structure of expressions

2. Use the structure of an expression to identify ways to rewrite it. For example, see $x4 – y4$ as $(x2) 2 – (y2) 2$, thus recognizing it as a difference of squares that can be factored as $(x2 – y2)(x2 + y2)$.

B. Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity

represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Functions

Building Functions

F-BF B. Build new functions from existing functions

3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Interpreting Functions

F-IF A. Understand the concept of a function and use function notation

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

B. Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

C. Analyze functions using different representations

7. Graph functions expressed symbollically and show key features of a the graph, by hand in simply cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

8. Write a function defined by an expression in different but equlvalent forms to reveal and explain diferent properties of the function.

a. Use the process of factoring and completeing the square in a quadratic function to show zeros, extema values, and symmetry of the graph, and interpret these in terms of a context.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions)

Numbers and Quantity

The complex Number System

N-CN A. Perfomr arithmetic operations with complex numbers

1. Know there is a complex nymber *i* such that *i^2*=-1, and evry complex number has the form *a+bi* where *a* and *b* are real.

2. Use the relation *i^2*=-1 and the commutative, associative, and distributve properties to add, subtract, and multiply complex numbers.

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

C. Use complex numbers in polynomial identieis and equations

7. Solve quadratic equations with real coefficients that have complex solutions.

8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite x^2+4 ad (x+2i)(x-2i)*

9. (+) Know the Fundamental Theorem of Algebra; show that this is true for quadratic polynomials

Mathematical Practices

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to

decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MA.A-SSE	Seeing Structure in Expressions
MA.K-12.1	Make sense of problems and persevere in solving them.
	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to

	help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
MA.F-IF.A	Understand the concept of a function and use function notation
MA.K-12.2	Reason abstractly and quantitatively.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
MA.K-12.4	Model with mathematics.
MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.A-SSE.B	Write expressions in equivalent forms to solve problems
	Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.K-12.5	Use appropriate tools strategically.
	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results

	of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines.
MA.A-SSE.B.3b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.N-CN.A.1	Know there is a complex number i such that i^2 = -1, and every complex number has the form $a + bi$ with a and b real.
MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.N-CN.A.2	Use the relation i^2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
MA.N-CN.A.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
MA.F-IF.C.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MA.N-CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
MA.N-CN.C.8	Extend polynomial identities to the complex numbers.
MA.F-BF	Building Functions
MA.N-CN.C.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
MA.A-CED.A	Create equations that describe numbers or relationships
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MA.A-REI.B	Solve equations and inequalities in one variable
MA.A-REI.B.4	Solve quadratic equations in one variable.

MA.A-REI.B.4a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
MA.A-REI.B.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
MA.A-REI.C	Solve systems of equations
MA.A-REI.C.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
MA.A-REI.C.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
MA.A-REI.C.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
MA.A-REI.D	Represent and solve equations and inequalities graphically
MA.A-REI.D.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
MA.A-REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.