*Unit 3- Probability

Content Area: **Mathematics**
Course(s):
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Enduring Understandings

Probability is the approximate proportion of outcomes that we would expect to occur over a large number of trials.

The basic rules of probability and using area under approximately normal distributions can be used to calculate most probabilities.

Given certain conditions are met, sampling distributions can be used to estimate the likelihood of a sample's summary statistics occuring in a population.

Essential Questions

What is probability?

How do we use normal distributions and other techniques to calculate probability in a variety of situations?

How do we calculate likelihood of getting a specific sample in a population?

Content

Randomness, Probability, and Simulation

Probability Rules

Conditional Probability and Independence

Discrete and Continuous Random Variables

Transforming and Combining Random Variables

Binomial and Geometric Random Variables

What is a Sampling Distribution?

Sampling Distributions of Sample Proportions

Sampling Distributions of Sample Means

Skills

Randomness, Probability, and Simulation:

Interpret probability as a long-run relative frequency.

Use simulation to model chance behavior.

Determine a probability model for a chance process.

Probability Rules:

Use basic probability rules, including the complement rule and the addition rule for mutually exclusive events.

Use a two-way table or Venn diagram to model a chance process and calculate probabilities involving two events.

Use the general addition rule to calculate probabilities.

Conditional Probability and Independence:

Calculate and interpret conditional probabilities.

Use the general multiplication rule to calculate probabilities.

Use tree diagrams to model a chance process and calculate probabilities involving two or more events.

Determine whether two events are independent.

When appropriate, use the multiplication rule for independent events to compute probabilities.

Discrete and Continuous Random Variables:

Compute probabilities using the probability distribution of a discrete random variable.

Calculate and interpret the mean (expected value) of a discrete random variable.

Calculate and interpret the standard deviation of a discrete random variable.

Compute probabilities using the probability distribution of certain continuous random variables.

Transforming and Combining Random Variables:

Describe the effects of transforming a random variable by adding or subtracting a constant and multiplying or

dividing by a constant.

Find the mean and standard deviation of the sum or difference of independent random variables.

Find probabilities involving the sum or difference of independent Normal random variables.

Binomial and Geometric Random Variables:

Determine whether the conditions for using a binomial random variable are met.

Compute and interpret probabilities involving binomial distributions.

Calculate the mean and standard deviation of a binomial random variable. Interpret these values in context.

Find probabilities involving geometric random variables.

When appropriate, use the Normal approximation to the binomial distribution to calculate probabilities.

What is a samping distribution:

Distinguish between a parameter and a statistic. (Review)

Use the sampling distribution of a statistic to evaluate a claim about a parameter.

Distinguish among the distribution of a population, the distribution of a sample, and the sampling distribution of a statistic.

Determine whether or not a statistic is an unbiased estimator of a population parameter.

Describe the relationship between sample size and the variability of a statistic.

Sampling Distributions of Sample Proportions:

Find the mean and standard deviation of the sampling distribution of a sample proportion.

Check the 10% condition before calculating the standard deviation of the sampling distribution of sample proportion.

Determine if the sampling distribution of sample proportion is approximately Normal.

If appropriate, use a Normal distribution to calculate probabilities involving sample proportion.

Sampling Distributions of Sample Means:

Find the mean and standard deviation of the sampling distribution of a sample mean.

Check the 10% condition before calculating the standard deviation of the sampling distribution of a sample

mean.

Explain how the shape of the sampling distribution of x is affected by the shape of the population distribution and the sample size.

If appropriate, use a Normal distribution to calculate probabilities involving sample mean.

Resources

Rossman-Chance Applet Collection

StatsMonkey

Rice Virtual Lab in Statistics

Khan Academy Mission: AP Statistics

Standards

NJSLS 2016

Statistics and Probability

CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY

A. Understand independence and conditional probability and use them to interpret data

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

5. Recognize and explain the concepts of conditional probability and independence in everyday language and

everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

B. Use the rules of probability to compute probabilities of compound events in a uniform probability model

6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.

USING PROBABILITY TO MAKE DECISIONS

A. Calculate expected values and use them to solve problems

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.

4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.

B. Use probability to evaluate outcomes of decisions

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast food restaurant.

b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.

6. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing,

pulling a hockey goalie at the end of a game).

MAKING INFERENCES AND JUSTIFYING CONCLUSIONS

A. Understand and evaluate random processes underlying statistical experiments

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population

Mathematics | Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose,

including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.S-CP.A	Understand independence and conditional probability and use them to interpret data
MA.S-CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
MA.S-CP.A.2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
MA.S-CP.A.3	Understand the conditional probability of A given B as $P(A\ and\ B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
MA.S-CP.A.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
MA.S-CP.A.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
MA.S-CP.B.6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
MA.S-CP.B.7	Apply the Addition Rule, $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$, and interpret the answer in terms of the model.
MA.S-CP.B.8	Apply the general Multiplication Rule in a uniform probability model, $P(A\ and\ B) = [P(A)] \times [P(B\|A)] = [P(B)] \times [P(A\|B)]$, and interpret the answer in terms of the model.
MA.S-IC.A.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
MA.S-MD.A	Calculate expected values and use them to solve problems
MA.S-MD.A.1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
MA.S-MD.A.2	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
MA.S-MD.A.3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
MA.S-MD.A.4	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.
MA.S-MD.B	Use probability to evaluate outcomes of decisions
MA.S-MD.B.5	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and

	finding expected values.
MA.S-MD.B.6	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
MA.S-MD.B.7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).
MA.S-MD.B.5a	Find the expected payoff for a game of chance.
MA.S-MD.B.5b	Evaluate and compare strategies on the basis of expected values.