Unit 4 Quadratic Functions

Content Area: **Mathematics**
Course(s): **Integrated Modern Algebra**
Time Period: **December**
Length: **4 weeks**
Status: **Published**

Transfer Skills

In this unit students will graph quadratic functions through a variety of methods.

Enduring Understandings

A single quantity may be represented by many equivalent but different expressions.

Different quadratic forms reveal different characteristics of the function.

Many real world situations can be modeled with a quadratic function.

Essential Questions

What are the characteristics of quadratic functions?

How can you use functions to model real-world situations?

Content

Vocabulary

quadratic equation

complete the square

quadratic formula

standard form

vertex form

factored form

roots

zeros

discriminant

transformation

Skills

Investigate the graph of quadratic functions through the use of the graphing calculator.

Recognize transformations of the parent $f(x) = x^2$ as vertical $f(x) = x^2 + k$, horizontal $f(x + k)$, stretch or reflections.

Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and

$f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Graph quadratic functions given in vertex form ($f(x) = a(x - h)2 + k$) through the process of generating points in function notation and apply the meaning of symmetry to plot points.

Recognize that different forms of quadratic functions reveal different key features of its graph. (Standard Form: y-intercept, Vertex Form: Vertex & Max/Min Value, Factored Form: x-intercepts)

Relate the value of the discriminant to the type of root to expect for the graph of a quadratic function. (one real root: 1 x-intercept, two real roots: 2 x-intercepts, no real roots: no x-intercepts)

Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Interpret models of quadratic functions given as equations or graphs.

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

Resources

Standards

CCSS.Math.Content.HSA-CED	Creating Equations
CCSS.Math.Content.HSA-CED.A	Create equations that describe numbers or relationships
CCSS.Math.Content.HSA-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
CCSS.Math.Content.HSA-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
CCSS.Math.Content.HSA-REI	Reasoning with Equations and Inequalities
CCSS.Math.Content.HSA-REI.B	Solve equations and inequalities in one variable
CCSS.Math.Content.HSA-REI.B.4	Solve quadratic equations in one variable.
CCSS.Math.Content.HSA-REI.B.4.a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
CCSS.Math.Content.HSA-REI.B.4.b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
CCSS.Math.Content.HSA-SSE	Seeing Structure in Expressions
CCSS.Math.Content.HSA-SSE.B	Write expressions in equivalent forms to solve problems
CCSS.Math.Content.HSA-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
CCSS.Math.Content.HSA-SSE.B.3.a	Factor a quadratic expression to reveal the zeros of the function it defines.
CCSS.Math.Content.HSA-SSE.B.3.b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
CCSS.Math.Content.HSF-BF	Building Functions
CCSS.Math.Content.HSF-BF.B	Build new functions from existing functions
CCSS.Math.Content.HSF-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
CCSS.Math.Content.HSF-IF	Interpreting Functions
CCSS.Math.Content.HSF-IF.B	Interpret functions that arise in applications in terms of the context

CCSS.Math.Content.HSF-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
CCSS.Math.Content.HSF-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
CCSS.Math.Content.HSF-IF.C	Analyze functions using different representations
CCSS.Math.Content.HSF-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
CCSS.Math.Content.HSF-IF.C.7.a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
CCSS.Math.Content.HSF-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
CCSS.Math.Content.HSF-IF.C.8.a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
CCSS.Math.Practice.MP1	Make sense of problems and persevere in solving them.
CCSS.Math.Practice.MP2	Reason abstractly and quantitatively.
CCSS.Math.Practice.MP4	Model with mathematics.
CCSS.Math.Practice.MP7	Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize-to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents-and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.