Unit 10 Polynomial Approximations and Series

Content Area: **Mathematics**
Course(s): **AP Calculus BC**
Time Period: **March**
Length: **Aproximately 15 blocks**
Status: **Published**

Enduring Understandings

Series can be helpful in approximating many kinds of functions.

Approximating a function with a series allows for evaluating their derivative and integral and furthering the study of the function's behavior.

Essential Questions

How are the concepts of series and sequence related to derivatives and antderivatives?

How can a Taylor series or Maclaurin series be used to approximate a function, integral, or derivative?

How did mathematicians discover the calculus of infinite series?

Content

Vocabulary:
Sequence, convergent, divergent, limit of a sequence, Limit Laws for sequences, bounded above, bounded below, bounded sequence, monotonic sequence, monotonic sequence theorem, infinite series, series, sum of the series, geometric series, harmonic series, test for divergence, the integral test, p-series, remainder estimate for the integral test, comparison test, the limit comparison test, alternating series, alternating series estimation theorem, absolutely convergent, conditionally convergent, ratio test, root test, power series, radius of convergence, interval of convergence, Taylor series, Maclaurin series, binomial series

Red Hot Topics:

*Writing rules for sequences

*Geometric sequences and series

*Finite vs. Infinite

*Sigma Notation

Skills

List the terms of a sequence.

Determine whether a sequence converges or diverges.

Apply the geometric series test to determine if a geometric series converges.

Calculate the sum of a geometric series.

Apply the nth term test for convergence.

Apply the integral test to determine if a series converges or diverges.

Apply p-series test to determine if series converges or diverges.

Apply direct comparison test and limit comparison test to determine if a series converges of diverges.

Apply alternating series test to determine if a series converges or diverges.

Determine if a series is absolutely convergent or conditionally convergent.

Use alternating series remainder to determine the error of an approximation.

Apply the ratio test to determine if a series converges or diverges.

Approximate a function using Taylor polynomials.

Use derivatives to find a Maclaurin Series or Taylor series generated by a differentiable function.

Use the ratio test to determine the radius and interval of convergence of a power series.

Differentiate, integrate, or substitute into a known power series in order to find additional power series representations.

Find Taylor/Maclaurin series for given functions.

Calculate Lagrange Error Bound.

Resources

Single Variable Calculus with Vector Functions by James Stewart Chapter 11

AP Calculus BC AP Central at collegeboard.com

Khan Academy: www.khanacademy.org

Standards

Mathematical Practice For AP Calculus 1: Reasoning with Definitions and Threorems

- Use definitions and theorems to build arguments,
- Justify conclusions or answers, and prove results;
- Confirm that hypotheses have been satisfied in order to apply the conclusion of a theorem;
- Apply definitions and theorems in the process of solving a problem; interpret quantifiers in definitions

and theorems;
- Develop conjectures based on exploration with technology;
- Produce examples and counterexamples to clarify understanding of definitions, to investigate whether converses of theorems are true or false, or to test conjectures.

Mathematical Practice For AP Calculus 2: Connecting Concepts

- Relate the concept of a limit to all aspects of calculus;
- Use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process antidifferentiation) to solve problems;
- Connect concepts to their visual representations with and without technology;
- Identify a common underlying structure in problems involving different contextual situations.

Mathematical Practice For AP Calculus 3: Implementing algebraic/computational processes

- Select appropriate mathematical strategies;
- Sequence algebraic/computational procedures logically;
- Complete algebraic/computational processes correctly;
- Apply technology strategically to solve problems; attend to precision graphically, numerically, analytically, and verbally and specify units of measure;
- Connect the results of algebraic/computational processes to the question asked.

Mathematical Practice For AP Calculus 4: Building notational fluency

- Know and use a variety of notations (e.g., f '(x), y', dy/dx);
- Connect notation to definitions (e.g., relating the notation for the definite integral to that of the limit of a Riemann sum);
- Connect notation to different representations (graphical, numerical, analytical, and verbal);
- Assign meaning to notation, accurately interpreting the notation in a given problem and across different contexts.

Mathematical Practice For AP Calculus 5: Connecting Multiple Representations

- Associate tables, graphs, and symbolic representations of functions;
- Develop concepts using graphical, symbolical, or numerical representations with and without technology;
- identify how mathematical characteristics of functions are related in different representations;
- Extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);
- Construct one representational form from another (e.g., a table from a graph or a graph from given information);
- Consider multiple representations of a function to select or construct a useful representation for solving a problem.

Mathematical Practice For AP Calculus 6: Communicating

- Clearly present methods, reasoning, justifications, and conclusions;
- Use accurate and precise language and notation;
- Explain the meaning of expressions, notation, and results in terms of a context (including units);
- Explain the connections among concepts;
- Critically interpret and accurately report information provided by technology;

- Analyze, evaluate, and compare the reasoning of others

MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.