07- Inference for Quantitative Data: Means

Content Area: **Mathematics**
Course(s): **AP Statistics**
Time Period: **February**
Length: **11-12 blocks**
Status: **Published**

Transfer

Exam Weighting: 10-18%

Previous Course Knowledge: NA

Developing Understanding

In this unit, students will analyze quantitative data to make inferences about population means. Students should understand that t* and t-tests are used for inference with means when the population standard deviation, o, is not known. Using s for in the formula for z gives a slightly different value, t, whose distribution, which depends on sample size, has more area in the tails than a normal distribution. The boundaries for rejecting a null hypothesis using a t-distribution tend to be further from the mean than for a normal distribution. Students should understand how and why conditions for inference with proportions and means are similar and different.

Building Course Skills

Unit 7 focuses on means, which has many similarities to the conditions and procedures for proportions. Since students sometimes confuse t-tests with z-tests, it will help to review the underlying rationales each time conditions come up. This will help students develop understanding through repeated practice in new situations. Teachers can encourage students to be mindful of notation and use the formula sheet as a reference. Teachers can reinforce that inference testing requires careful selection of a procedure based on specific conditions for a given problem. Common errors include mislabeling conditions (e.g., incorrectly associating the large sample condition with independence), relying upon vague references to the normal distribution, or applying an inappropriate large sample condition. The null and alternative hypotheses must be clearly stated in terms of population parameters, not sample statistics. A formal decision compares the p-value to the level of significance. Students should also practive providing numerical reference to support their claim (e.g., "Because $p < 0.05$, we reject the null hypothesis.") and interpreting findings within the context of the question.

Preparing for the AP Exam

It is critical for students to recognize that free-response questions asking whether data provide convincing evidence of some finding are asking for a significance test, not just a descriptive analysis. When using statistical inference for significance tests, students should identify the correct parameter and hypotheses, identify an appropriate test procedure and check conditions, calculate a test statistic and p-value, and provide a conclusion in context, along with a justification based on linkage between the p-value and the conclusion. For inference with means, the appropriate test will often be a t-test, but if is known, a z-test would be appropriate (see 2018 FRQ 6(a)). For a t-test, conditions are (1) random sample and (2) large sample (e.g., $n > 30$). When

sampling without replacement, students should also verify that the sample size is at most 10% of the population.

Enduring Understandings

Given that variation may be random or not, conclusions are uncertain.

The t-distribution may be used to model variation.

An interval of values should be used to estimate parameters, in order to account for uncertainty.

Significance testing allows us to make decisions about hypotheses within a particular context.

Essential Questions

How do we know whether to use a t-test or a z-test for inference with means?
How can we make sure that samples are independent?
Why is it inappropriate to accept a hypothesis as true based on the results of statistical inference testing?

Student Learning Objectives

TEXT Section 10.1 Estimating a Population Mean

- Determine the critical value for calculating a C% confidence interval for a population mean using a table or technology.
- State and check the Random, 10%, and Normal/Large Sample conditions for constructing a confidence interval for a population mean.
- Construct and interpret a confidence interval for a population mean.

TEXT Section 10.2 Estimating a Difference in Means

- Determine whether the conditions are met for constructing a confidence interval for the difference between two means.
- Construct and interpret a confidence interval for a difference between two means.
- Analyze the distribution of differences in a paired data set using graphs and summary statistics.
- Construct and interpret a confidence interval for a mean difference.

TEXT Section 11.1 Tests about a Population Mean

- State and check the Random, 10%, and Normal/Large Sample conditions for performing a significance test about a population mean.

- Calculate the standardized test statistic and P-value for a test about a population mean.
- Perform a significance test about a population mean.
- Use a confidence interval to make a conclusion for a two-sided test about a population parameter.

TEXT Seciton 11.2 Tests About a Difference in Means

- State appropriate hypotheses for a significance test about a difference in means.
- Determine whether the conditions are met for performing a test about a difference between two means.
- Calculate the standardized test statistic and P-value for a test about a difference between two means.
- Perform a significance test about a difference between two means.
- Perform a significance test about a mean difference.
- Determine when it is appropriate to use paired t procedures versus two-sample t procedures.

Vocabulary and Planned Learning Experiences

Vocabulary: standard error of sample mean, margin of error, one sample t interval for a population mean, two-sample t-interval for a difference in population means, paired t interval for a mean difference, one-sample t test for a population mean, two sample t test for a difference in population means, paired test for a mean difference

Planned Learning Experiences:

Predict and Confirm: After introducing the confidence interval formula for a population mean when sigma is known, f ± z have students discuss in small groups what will happen if we substitute the sample standard deviation s into the formula when o is unknown. Have students use the Simulating Confidence Intervals for Population Parameter applet to test their conjectures (see link for Rossman/Chance Applets on page 210).

Team Challenge: Give each team of three to four students a copy of 2004 FRQ 6, which focuses on the connection between a one-sample t-interval, a one-sample t-test, and the unfamiliar concept of a one-sided confidence interval. Challenge teams to collaboratively produce a model solution in 30 minutes.

Discussion Groups: Ask each group of three to four students to identify the conditions for performing a test about a population mean. For each condition, have them explain why the condition is required and what would go wrong with the test if the condition were violated. Have groups pair up and compare answers.

Team FRQ: Give each team of four students copies of a free-response question that involves performing a two-sample t-test (e.g., 2011 FRQ 4). Have each team member take responsibility for writing one part of the model solution (hypotheses, procedure and

conditions, calculations, conclusion) with group input.

Graphic Organizer: Have students work in teams of two to three to develop a flowchart for determining which inference procedure from Units 6 and 7 to use in a given setting.

Resources

TEXT: The Practice of Statistics, 6th Edition

AP Classroom and the APCD 2019 Course Description

Rossman-Chance Applets

Stats Medic

Assessments

Reading Quiz: Estimating a Population Mean (Introductory Level)

Reading Quiz: Estimating a Difference in Means (Introductory Level)

TEST: Estimating Means with Confidence (AP Level)

Reading Quiz; Tests about a Population Mean (Introductory Level)

Reading Quiz: Tests about a Difference in Means (Introductory Level)

TEST: Testing Claims about Means (AP Level)

Standards

MA.S-IC.A.1: Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

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Modifications (ELL, Special Education, At-Risk Students, Gifted & Talented, & 504 Plans)

ELL:

- Use visuals
- Introduce key vocabulary before lesson
- Provide peer tutoring
- Guided notes and/or scaffold outline for written assignments

Supports for Students With IEPs:

- Allow extra time to complete assignments or tests
- Guided notes and/or scaffold outline for written assignments
- Work in a small group
- Follow all IEP modifications

At-Risk Students:

- Guided notes and/or scaffold outline for written assignments
- Introduce key vocabulary before lesson
- Work in a small group
- Lesson taught again using a differentiated approach
- Use visuals / Anchor Charts

Gifted and Talented:

- Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles)
- Organize and offer flexible small group learning activities
- Teach cognitive and methodological skills
- Organize integrated problem-solving simulations
- Propose interest-based extension activities

Supports for Students With 504 Plans:

- Follow all the 504 plan modifications
- Text to speech/audio recorded selections