06- Inference for Categorical Data: Proportions

Content Area: **Mathematics**
Course(s): **AP Statistics**
Time Period: **February**
Length: **11-12 blocks**
Status: **Published**

Transfer

Exam Weighting: 12-15%

Previous Coursework: NA

Developing Understanding
This unit introduces statistical inference, which will continue through the end of the course. Students will analyze categorical data to make inferences about binomial population proportions. Provided conditions are met, students will use statistical inference to construct and interpret confidence intervals to estimate population proportions and perform significance tests to evaluate claims about population proportions. Students begin by learning inference procedures for one proportion and then examine inference methods for a difference between two proportions. They will also interpret the two types of errors that can be made in a significance test, their probabilities, and possible consequences in context.

Building Course Skills
Unit 6 is a critical transition point in the course, as students begin learning skills that will be applied repeatedly in subsequent units. Students need to familiarize themselves with these procedures so they can build proficiency over time. Applying different inference methods requires fluency with verifying conditions. Students often check conditions superficially (e.g., just listing "SRS") without explicitly connecting them to the problem. Teachers can make sure students practice verifying conditions in context by providing numerical calculations and explaining how each condition is met. Precision of language is key. Students often interpret confidence intervals and confidence levels incorrectly. Providing students with sentence starters or templates can help them learn to generate appropriate responses (e.g., Confidence interval: "We are 95% confident that the interval from — to — captures the [parameter in context]."). For decisions based on a hypothesis test, students may incorrectly claim that "we can accept" or "have proven" the null. Teachers can reinforce early and often that statistical tests do not provide evidence for what can be accepted or proved; they only provide evidence for "rejecting" or "failing to reject" the null.

Preparing for the AP Exam
When using statistical inference to construct confidence intervals or perform significance tests, students should identify the appropriate inference method by name or formula. For inference with population proportions, students should verify that the following conditions are met: (I) random sample and (2) large sample (e.g., 210 and n(l — i) 210). When sampling without replacement, students should also verify that the sample size is at most 10% of the population. Verification should be simple and specific. Next, students should present calculations and then interpret results in the context of the problem. Students often find it beneficial to use language provided in the question. In 2017 FRQ 2, for example, the response might read "We can be 95% confident that the proportion of all customers who, having asked for a cup of water

when placing an order, will fill the cup with a soft drink is between 0.1883 and 0.3867."

Enduring Understandings

Given that variation may be random or not, conclusions are uncertain.

An interval of values should be used to estimate parameters, in order to account for uncertainty.

The normal distribution may be used to model variation.

Significance testing allows us to make decisions about hypotheses within a particular context.

Probabilities of Type I and Type II errors influence inference.

Essential Questions

When can we use a normal distribution to perform inference calculations involving population proportions?

How can we narrow the width of a confidence interval?

If the proportion of subjects who experience serious side effects when taking a new drug is smaller than the proportion of subjects who experience serious side effects when taking a placebo, how can we determine if the difference is statistically significant?

Student Learning Objectives

TEXT Section 8.1 Confidence Intervals: The Basics

- Identify an appropriate point estimator and calculate the value of a point estimate.
- Interpret a confidence interval in context.
- Determine the point estimate and margin of error from a confidence interval.
- Use a confidence interval to make a decision about the value of a parameter.
- Interpret a confidence level in context.
- Describe how the sample size and confidence level affect the margin of error.
- Explain how practical issues like nonresponse, undercoverage, and response bias can affect the interpretation of a confidence interval.

TEXT Section 8.2 Estimating a Population Proportion

- State and check the Random, 10%, and Large Counts conditions for
- constructing a confidence interval for a population proportion.
- Determine the critical value for calculating a C% confidence interval for a population proportion using a table or technology.
- Construct and interpret a confidence interval for a population proportion.
- Determine the sample size required to obtain a C% confidence interval for a population proportion with a specified margin of error.

TEXT Section 8.3 Estimating a Difference in Proportions

- Determine whether the condition are met for constructing a confidence interval about a difference between two proportions.
- Construct and interpret a confidence interval for the difference between two proportions.

TEXT Section 9.1 Significance Tests: The Basics

- State appropriate hypotheses for a significance test about a population parameter.
- Interpret a P-value in context.
- Make an appropriate conclusion for a significance test.
- Interpret a Type I and a Type II error in context. Give a consequence of each error in a given setting.

TEXT Section 9.2 Tests about a Population Proportion

- State and check the Random, 10%, and Large Counts conditions for performing a significance test about a population proportion.
- Calculate the standardized test statistic and P-value for a test about a population proportion.
- Perform a significance test about a population proportion.
- Interpret the power of a significance test and describe what factors affect the power of a test.

TEXT Section 9.3 Tests about a Difference in Proportions

- State appropriate hypotheses for a significance test about the difference between two proportions.
- Determine whether the conditions are met for performing a test about a difference between two proportions.
- Calculate the standardized test statistics and P-value for a test about the difference between two proportions.
- Perform a significance test about the difference between two proportions.

Vocabulary and Planned Learning Experiences

Vocabulary: point estimator, point estimate, confidence interval, confidence level, margin of error, Random, 10%, Large Counts Condition, standard error, critical value, on sample z interval for p, two sample z-interval for p_1-p_2, significance test, null hypotheses, alternative hypotheses, one-sided, two-sided, p-value, significance level, Type I Error, Type II Error, standardized test statistic, power, one sample z test for p, two sample z test

for p_1-p_2

Planned Learning Experiences:

Error Analysis: Give student pairs a worksheet with 20 sets of hypotheses (including hypotheses for a population proportion and for the difference of two proportions), each with a common student mistake. Have students circle the incorrect part, write why the circled component is incorrect, and then write the correct hypotheses. Include errors such as using statistics instead of parameters, and interchanging the = and > in the two hypotheses.

Sentence Starters: For a given question, provide students with a set of hypotheses, p-value, significance level, and context. Have them compare the p-value to the significance level to determine whether or not to reject the null hypothesis. Using a given sentence starter with blanks to fill in, have students write a sentence in context explaining if they have enough evidence to "reject Ho", or if they will "fail to reject Ho." Make sure students avoid the common mistake of implying that evidence supports an "accept Ho" conclusion or a "reject H a" conclusion.

The Scribe and the Calculator: Have students work with a partner to construct and interpret a confidence interval for a population proportion. Only one partner is allowed to use the calculator, and only the other partner is allowed to write. When a calculation needs to be made, the scribe can only describe to the calculator operator which buttons to push; when writing needs to be done, the calculator operator can only describe to the scribe what needs to be written. Have students switch roles when constructing and interpreting a confidence interval for the difference of two population proportions.

Resources

TEXT: The Practice of Statistics, 6th Edition

AP Classroom and the APCD 2019 Course Description

Rossman-Chance Applets

Stats Medic

Assessments

Reading Quiz: Confidence Intervals: The Basics (Introductory Level)

Reading Quiz: Estimating a Population Proportion (Introductory Level)

Reading Quiz: Estimating a Difference in Proportions (Introductory Level)

TEST: Estimating Proportions with Confidence (AP Level)

Reading Quiz: Significance Tests: The Basics (Introductory Level)

Reading Quiz: Tests about a Population Proportion (Introductory Level)

Reading Quiz: Tests about a Difference in Proportions (Introductory Level)

TEST: Testing Claims about Proportions (AP Level)

Standards

MA.S-IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

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Modifications (ELL, Special Education, At-Risk Students, Gifted & Talented, & 504 Plans)

ELL:

- Use visuals
- Introduce key vocabulary before lesson
- Provide peer tutoring
- Guided notes and/or scaffold outline for written assignments

Supports for Students With IEPs:

- Allow extra time to complete assignments or tests
- Guided notes and/or scaffold outline for written assignments
- Work in a small group
- Follow all IEP modifications

At-Risk Students:

- Guided notes and/or scaffold outline for written assignments
- Introduce key vocabulary before lesson

- Work in a small group
- Lesson taught again using a differentiated approach
- Use visuals / Anchor Charts

Gifted and Talented:

- Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles)
- Organize and offer flexible small group learning activities
- Teach cognitive and methodological skills
- Organize integrated problem-solving simulations
- Propose interest-based extension activities

Supports for Students With 504 Plans:

- Follow all the 504 plan modifications
- Text to speech/audio recorded selections