*Unit 5 Exponents & Exponential Functions (6)

Content Area: **Mathematics**
Course(s): **Integrated Modern Algebra**
Time Period: **December**
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Status: **Published**

Enduring Understandings

A single quantity may be represented by many equivalent but different expressions.

Exponential functions model real life situations.

Essential Questions

What characterizes exponential growth or decay?

How can one differentiate a linear model from an exponential model?

When is one exponential form more useful than another form?

Content

Vocabulary

exponent

rational exponent

irrational number

rational number

radicals

exponential function

growth

decay

geometric sequence

explicit rule

recursive rule

Skills

Apply the laws of exponents to simplify expressions.

Interpret complicated exponential expressions by viewing one or more of their parts as a single entity. (Growth or Decay factor)

Use the properties of exponents to transform expressions for exponential functions.

Distinguish between situations that can be modeled with linear functions and with exponential functions.

Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Recognize that geometric sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Write geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Graph geometric sequences.

Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table) and compare and contrast the results of each.

Resources

KhanAcademy

YouTube Website for ALL Math Concepts from Arithmetic to Calculus

www.youtube.com/user/bullcleo1

Math IXL

Standards

NJSLS 2016

Algebra

Seeing Structure in Expressions

A-SSE B. Write expressions in equivalent forms to solve problems
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.151/12) 12t ≈1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Creating Equations★

A-CED A. Create equations that describe numbers or relationships
1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising

from linear and quadratic functions, and simple rational and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales

Reasoning with Equations and Inequalities

A-REI D. Represent and solve equations and inequalities graphically
10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Functions

Interpreting Functions

HSF-IF A. Understand the concept of a function and use function notation
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

HSF-IF B. Interpret functions that arise in applications in terms of the context
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

HSF-IF C. Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Building Functions

HSF-BF A. Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.★

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Linear and Exponential Models★

HSF-LE A. Construct and compare linear and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

HSF-LE B. Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Mathematics l Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression x2

+ 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y) 2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), (x – 1)(x2 + x + 1), and (x – 1)(x3 + x2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

MA.F-BF	Building Functions
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.F-BF.A.1b	Combine standard function types using arithmetic operations.
MA.F-IF	Interpreting Functions
MA.F-IF.A	Understand the concept of a function and use function notation
MA.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

MA.F-LE	Linear and Exponential Models
MA.F-LE.A	Construct and compare linear and exponential models and solve problems
MA.F-LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.
MA.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MA.F-LE.A.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
MA.F-LE.A.1a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
MA.F-LE.A.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MA.F-LE.A.1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MA.F-LE.B	Interpret expressions for functions in terms of the situation they model
MA.F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.N-RN	The Real Number System
MA.N-RN.A	Extend the properties of exponents to rational exponents.
MA.N-RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
MA.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MA.N-RN.B	Use properties of rational and irrational numbers.
MA.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
MA.A-CED	Creating Equations
MA.A-CED.A	Create equations that describe numbers or relationships
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-REI	Reasoning with Equations and Inequalities
MA.A-REI.D.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
MA.A-SSE	Seeing Structure in Expressions

MA.A-SSE.A	Interpret the structure of expressions
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MA.A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MA.A-SSE.B	Write expressions in equivalent forms to solve problems
MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MA.A-SSE.B.3c	Use the properties of exponents to transform expressions for exponential functions.

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