*Unit 1 Expressions, Equations, and Function Families (10)

Content Area: **Mathematics**
Course(s): **Integrated Modern Algebra**
Time Period: **September**
Length: **10 blocks**
Status: **Published**

Enduring Understandings

In this unit students will build upon the introduction of functions presented in Algebra 1. Students will work with a variety of function types at a basic level to prepare for a more in depth focus in future units on linear, exponential and quadratic functions. Students who are able to identify functions, evaluate with function notation, and are able to relate the domain of a function to the quantitative relationship it describes will be prepared for future study of functions.

All of the facts of arithmetic and algebra follow from certain properties.

Variables in place of numbers allow the statement of relationship among numbers that are unknown or unspecified.

Multiple methods exist when solving equations.

Functions are a mathematical way to describe relationships between two quantities.

Functions can be represented in a variety of ways, such as graphs, tables, equations, or words. Each representation is particularly useful in certain situations.

Functions are used to analyze change in various contexts and model real-world phenomena.

Essential Questions

How do I determine the best numerical representation (pictorial, symbolic objects) for a given situation?

Can equations that appear to be different be equivalent?

How can two expressions that appear different actually be equivalent?

What is a function?

What are the different methods for representing functions?

How can functions be used to model real world situations?

Content

Essential Vocabulary

- equivalent expressions
- like terms
- distributive property
- equations
- terms
- factors
- coefficients
- relation
- function
- domain
- range
- mapping
- vertical line test
- function notation
- linear function
- exponential function
- quadratic function
- square root function
- cube root function
- piecewise function
- step function

- absolute value function

Skills

Apply the order of operations to simplify expressions involving rational numbers.

Interpret parts of an expression, such as terms, factors, and coefficients.

Simplify expressions using the commutative, associative and distributive properties.

Classify polynomials by degree and number of terms.

Add, subtract and multiply polynomials.

Factor monomials and trinomials.

Identify special polynomials including difference of squares and perfect square trinomials.

Use a graphing calculator to compare equivalent expressions.

Solve linear equations with one solution, no solution and infinitely many solutions.

Use a graphing calculator to solve an equation in one variable by graphing both sides of the equation and finding the point of intersection.

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. (If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x.)

Use the vertical line test to determine whether a graph represents a function.

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Identify families of functions considering linear, exponential, and quadratic functions.

Identify key features for linear, exponential and quadratic functions.

Create graphs for linear, exponential and quadratic functions by generating a table of values.

Create graphs for linear, exponential and quadratic functions using key features of the function type.

Introduce the square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Resources

KhanAcademy

YouTube Website for ALL Math Concepts from Arithmetic to Calculus
www.youtube.com/user/bullcleo1

Math IXL

Standards

NJSLS 2016

Number and Quantity

Quantities★

N-Q A. Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Algebra

Seeing Structure in Expressions

A-SSE A. Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.★
a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r) n as the product of P and a factor not depending on P

Creating Equations★

A-CED A. Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

Reasoning with Equations and Inequalities

A-REI A. Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A-REI B. Solve equations and inequalities in one variable
3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A-REI D. Represent and solve equations and inequalities graphically
10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Functions

Interpreting Functions

HSF-IF A. Understand the concept of a function and use function notation
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

HSF-IF B. Interpret functions that arise in applications in terms of the context
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★

HSF-IF C. Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Mathematics l Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures

about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account New Jersey Student Learning Standards for Mathematics 4 the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are

sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y) 2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MA.F-IF	Interpreting Functions
MA.K-12.1	Make sense of problems and persevere in solving them.
	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their

	answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
MA.F-IF.A	Understand the concept of a function and use function notation
MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MA.A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.K-12.4	Model with mathematics.
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
	Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
MA.K-12.5	Use appropriate tools strategically.
	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with

	tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.K-12.6	Attend to precision.
MA.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
MA.K-12.7	Look for and make use of structure.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
	Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
MA.F-IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MA.A-CED	Creating Equations
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

MA.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
MA.A-REI	Reasoning with Equations and Inequalities
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MA.A-REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
MA.A-REI.D	Represent and solve equations and inequalities graphically
MA.A-REI.D.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).