# *Unit 6 Rational Functions (6)

Content Area: **Mathematics**
Course(s): **Integrated Modern Algebra**
Time Period: **March**
Length: **6 blocks**
Status: **Published**

## Transfer Skills

In this unit students will be introduced to rational functions. Students will gain a basic understanding of a rational graph identifying the key characteristics including the domain, range, vertical and horizontal asymptotes. Students will be introduced to the concept of end behavior.

## Enduring Understandings

Mastering a procedure is not the same as understanding the concept.

Analyizing a function provides key characterstics of a graph.

Understanding Domain allows for the creation of a graph of a rational function.

## Essential Questions

How do we determine the key characteristics of a graph?

When are asymptotes used to graph rational functions?

## Content

**Vocabulary**

Rational Functions

Asymptotes

Points of Discontinuity

End Behavior

Transformations

Domain

Range

## Skills

Identify Domain and Range from a graph using both interval notation and inequality notation.

Sketch a possible graph of a function given the domain and range.

Identify the Domain of a Rational Expression by inspection and with the use of the graphing calculator.

Identify the key characteristcs of a rational function from a graph, including the domain, range, vertical and horizontal asymptotes.

Graph the function f(x) = 1/x, identifying the domain, range, vertical and horizontal asymptotes.

Identify the End Behavior of a Rational Function.

Analyze graphs of rational functions identifying key characteristics including zeros, x- and y-intercepts, vertical and horizontal asymptotes, points of discontinuity and end behavior.

## Resources

KhanAcademy

YouTube Website for ALL Math Concepts from Arithmetic to Calculus

www.youtube.com/user/bullcleo1

Math IXL

## Standards

**NJSLS 2016**

**Algebra**

**Arithmetic with Polynomials and Rational Expressions**

**A-APR D. Rewrite rational expressions**
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

**Functions**

**Interpreting Functions**

**HSF-IF C. Analyze functions using different representations**
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

**Building Functions**

**HSF-BF B. Build new functions from existing functions**
3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
4. Find inverse functions.
a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠1.

**Mathematics l Standards for Mathematical Practice**

**1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

**2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

**4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y) 2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

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|---|---|
| MA.F-BF | Building Functions |
| MA.F-BF.B | Build new functions from existing functions |
| MA.F-BF.B.3 | Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. |
| MA.F-BF.B.4 | Find inverse functions. |
| MA.F-BF.B.4a | Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse. |
| MA.F-IF | Interpreting Functions |
| MA.F-IF.C | Analyze functions using different representations |
| MA.F-IF.C.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. |
| MA.K-12.1 | Make sense of problems and persevere in solving them. |
| MA.K-12.2 | Reason abstractly and quantitatively. |
| MA.K-12.4 | Model with mathematics. |
| MA.K-12.7 | Look for and make use of structure. |
| MA.A-APR | Arithmetic with Polynomials and Rational Expressions |
| MA.A-APR.D | Rewrite rational expressions |
| MA.A-APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. |
| | Mathematically proficient students can apply the mathematics they know to solve |

problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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