

*Unit 7-Graph Theory

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 4**
Length: **14 Blocks**
Status: **Published**

Enduring Understandings

Graph Theory concerns the relationship among lines and points.

Graph theory can help deduce the shortest and perhaps least expensive route between major cities.

Graph Theory applies to many fields including operational research and chemistry to genetics and linguistics, and from electrical engineering and geography to sociology and architecture.

Essential Questions

How can a company minimize traveling cost between major destinations?

What other fields use graph theory?

How does Graph Theory relate to mathematics?

Content

Graphs, paths, circuits, and loops

Euler paths and circuits

Fleury's Algorithm

Hamilton paths and circuits

Trees

Kruskal's Algorithm

Skills

- Represent the Koenigsberg bridge problem and others using a graph.
- Determine the degree of a graph.
- Determine if a vertex is even or odd.
- Determine if a graph is connected or disconnected.
- Given a graph, determine if it is an Euler path or circuit.
- Use Euler's theorem to determine if a Euler path or circuit exists.
- Determine if a path is traceable.
- Determine if a path or circuit is a Hamilton path or circuit.
- Determine if a graph is a complete graph.
- Determine the number of unique Hamilton circuits in a complete graph.
- Determine how many different solutions exist for a given graph.
- Determine different spanning trees for a given graph.
- Determine the minimum cost spanning tree for a graph.
- Use Kruskal's Algorithm to construct the minimum cost spanning tree from a weighted graph.

Resources

Text: *A Survey of Mathematics with Applications, Pearson 2005*

Each skill is aligned to the text as a reference.

- Represent the Koenigsberg bridge problem and others using a graph. (14.1)
- Determine the degree of a graph (14.1)
- Determine if a vertex is even or odd. (14.1)
- Determine if a graph is connected or disconnected. (14.1)
- Given a graph, determine if it is an Euler path or circuit. (14.2)
- Use Euler's theorem to determine if a Euler path or circuit exists. (14.2)
- Determine if a path is traceable. (14.2)
- Determine if a path or circuit is a Hamilton path or circuit. (14.3)

- Determine if a graph is a complete graph. (14.3)
- Determine the number of unique Hamilton circuits in a complete graph. (14.3)
- Determine how many different solutions exist for a given graph. (14.3)
- Determine different spanning trees for a given graph. (14.4)
- Determine the minimum cost spanning tree for a graph. (14.4)
- Use Kruskal's Algorithm to construct the minimum cost spanning tree from a weighted graph. (14.4)

<https://www.math.ku.edu/~jmartin/courses/math105-F11/Lectures/chapter5-part2.pdf>

<https://www.youtube.com/watch?v=REfC1-igKHQ>

<http://www.cut-the-knot.org/Curriculum/Combinatorics/FleuryAlgorithm.shtml>

<http://jwilson.coe.uga.edu/EMAT6680Fa2012/Faircloth/InstructionalUnit/MapColoring.html>

<http://jwilson.coe.uga.edu/emat6680/yamaguchi/emat6690/essay1/gt.html>

Standards

NJSLS 2016

Math Analysis

MA.K-12.1

Make sense of problems and persevere in solving them.

MA.K-12.2

Reason abstractly and quantitatively.

MA.K-12.4

Model with mathematics.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social,

and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.