# Unit 1: A brief review of differential calculus of a single variable

Content Area: **Mathematics**
Course(s):
Time Period: **September**
Length: **8 Blocks**
Status: **Published**

## Enduring Understandings

Calculus is a collection of powerful ideas; not a set of rules, formulas and procedures. To learn calculus requires focus on the understanding of a few big ideas.

The extension of the limit allows for the calculation of the instantaneous rate of change for any given point on a continuous function.

There exist efficient approaches to determine the derivatives of functions.

The use of the derivative has many applications.

The calculation of the derivative allows for and efficient approach to the computation of possible maximum profit, minimum amount used, and other economic and engineering applications.

The derivative can provide useful information on the graph of a function.

## Essential Questions

What is Calculus and how is it a significant area of mathematical study?

How can calculus and the concepts of limit and continuity assist us in analyzing the rate of change for curves?

What is a derivative, how do we determine it, and how is it built from the limit?

How is it possible to find the slope of a tangent line?

What role does calculus play as a tool in science, business, and other areas of study?

How is the derivative used to solve problems involving area, velocity and acceleration?

What information can be determined from the derivative to help sketch the graph of a function?

## Content

**Goal: The student will demonstrate the ability to use a problem-solving approach to apply basic calculus concepts, including techniques for curve sketching, exponential an logarithmic functions and differentiation. The knowledge of the differential**

**calculus concepts are expected at a mastery level.  In AP Calculus BC hyperbolic trigonometric functions are not in the curriculum but are important to a pursuit of a STEM path.  This unit will largely focus on extending the ideas of differential calculus in exploration of the six hyperbolic trigonometric functions.**

**Instructor's Notes:  This unit will set the tone for the course.  It is a brief overview while exploring a particularly new type of function for the students.  What it is not intended to be is a reteaching of all of differential calculus.  The student needs to revisit differential calculus topics but at a more advanced level with a emphasis on applications and problem solving.  In particular students need to become comfortable with chain rule for differentiation as well as simplifying answers.  The AP exam allows non-simpified answers but pursuing higher levels of math will require advanced simplifying techniques.**

**Vocabulary:**

- Absolute minimum
- Acceleration
- Average rate of change
- Average velocity
- Chain Rule
- Concavity
- Continuous
- Critical point
- Derivative
- Differentiability
- Explicit equation
- Exponential function
- Extrema Extreme value theorem
- First derivative test
- Implicit differentiation
- Implicit equation
- Inflection Point
- Instantaneous rate of change
- Intermediate Value Theorem
- Inverse Trigonometric Function
- Local Maximum
- Local Minimum
- Logarithmic Function
- Mean Value Theorem
- Natural Logarithm
- Normal line
- Optimization
- Product Rule
- Quotient Rule
- Related Rates
- Second derivative Test
- Slope field
- Speed
- Tangent line
- Trigonometric Function
- Velocity
- Vertical asymptote

## Skills

Use a derivative as the instantaneous rate of change of a function to evaluate for the slope of function at a given value.

Apply the methods of rules of differentiation (chain rule, implicit differentiation etc.) to derive functions.

Determine maximum and minimum values, rates of change, accumulation over time, position, velocity, acceleration, work done on an object and various other real world situations utilizing the concepts and methods of single variable calculus.

Construct the definition of a derivative as the limit of the difference quotient of the function as the change in the independent variable approaches zero.

State the definitions of the six hyperbolic trigonometric functions

Sketch the graphs of the hyperbolic functions by applying such basic curve sketching techniques as asymptotes, concavity, odd or even, increasing and decreasing.

Identify the relationship between th hyperbolic functions and the trigonometric functions.

Identify the derivatives of the six hyperbolic functions.

## Resources

Text Book : Stewart: *Calculus (8th edition):* Chapters 1-3: Functions and Limits, Derivatives, and Application of Derivatves.

Online Websites:

**Kahn Academy:**

**https://www.khanacademy.org/math/differential-calculus**

**Multivariable Calculus Online by Jeff Knisley, Dept. of Math., East Tennessee State University:**

http://math.etsu.edu/MultiCalc/

**Online lecture notes with visuals for multivariable calculus by Paul Dawkins of Lamar University:**

[**http://tutorial.math.lamar.edu/Classes/CalcIII/CalcIII.aspx**](http://tutorial.math.lamar.edu/Classes/CalcIII/CalcIII.aspx)

**MIT online courseware: Calculus Revisited : Multivariable Calculus**

[**http://ocw.mit.edu/resources/res-18-007-calculus-revisited-multivariable-calculus-fall-2011/**](http://ocw.mit.edu/resources/res-18-007-calculus-revisited-multivariable-calculus-fall-2011/)

## Standards

**Standards for mathematical practices**

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account

the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y) 2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), (x – 1)(x2 + x + 1), and (x – 1)(x3 + x2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

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|---|---|
| MA.9-12.II | Derivatives |
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| MA.9-12.II.A | Concept of the derivative |
| MA.9-12.II.A.1 | Derivative presented graphically, numerically, and analytically |
| MA.9-12.II.A.2 | Derivative interpreted as an instantaneous rate of change |
| MA.9-12.II.A.3 | Derivative defined as the limit of the difference quotient |
| MA.9-12.II.A.4 | Relationship between differentiability and continuity |
| MA.9-12.II.B | Derivative at a point |
| MA.9-12.II.B.1 | Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents. |
| MA.9-12.II.B.2 | Tangent line to a curve at a point and local linear approximation |
| MA.9-12.II.B.3 | Instantaneous rate of change as the limit of average rate of change |

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| MA.9-12.II.B.4 | Approximate rate of change from graphs and tables of values |
| MA.9-12.II.C | Derivative as a function |
| MA.9-12.II.C.1 | Corresponding characteristics of graphs of f and f' |
| MA.9-12.II.C.2 | Relationship between the increasing and decreasing behavior of f and the sign of f' |
| MA.9-12.II.C.3 | The Mean Value Theorem and its geometric interpretation |
| MA.9-12.II.C.4 | Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa. |
| MA.9-12.II.D | Second derivatives |
| MA.9-12.II.D.1 | Corresponding characteristics of the graphs of f, f', and f" |
| MA.9-12.II.D.2 | Relationship between the concavity of f and the sign of f" |
| MA.9-12.II.D.3 | Points of inflection as places where concavity changes |
| MA.9-12.II.E | Applications of derivatives |
| MA.9-12.II.E.2 | Optimization, both absolute (global) and relative (local) extrema |
| MA.9-12.II.E.3 | Modeling rates of change, including related rates problems |
| MA.K-12.1 | Make sense of problems and persevere in solving them. |
| MA.K-12.2 | Reason abstractly and quantitatively. |
| MA.K-12.3 | Construct viable arguments and critique the reasoning of others. |
| MA.K-12.4 | Model with mathematics. |
| MA.K-12.5 | Use appropriate tools strategically. |
| MA.K-12.6 | Attend to precision. |
| MA.K-12.7 | Look for and make use of structure. |
| MA.K-12.8 | Look for and express regularity in repeated reasoning. |
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