**Subject**

**Probability and Statistics**

**Curriculum Guide**

**­­\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**LINDEN PUBLIC SCHOOLS**

**LINDEN, NEW JERSEY**

**DR. MARNIE HAZELTON**

**SUPERINTENDENT**

**DENISE CLEARY**

**ASSISTANT SUPERINTENDENT**

**RICHARD MOLINARO**

**DIRECTOR OF MATHEMATICS, VOCATIONAL, & TECHNICAL SUBJECTS**

**The Linden Board of Education adopted the Curriculum Guide on:**

|  |  |  |
| --- | --- | --- |
| **July 28, 2022** |  | **Education Report #22** |
|  |  |  |
|  | | |
| **Rationale** | | |

**EDUCATION EQUITY:** The Linden Public School District guarantees each student equal educational opportunity regardless of age, race, color, creed, religion, gender, language, affectional or sexual orientation, ancestry, national origin, marital or economic status. For Information, contact District Educational Equity Officer Kevin Thurston at **(**908) 486-2800 x 8307**.**

**NONDISCRIMATION:** The Linden Public School District does not discriminate against handicapped persons in admission or access to or treatment or employment in its programs, activities, and vocational opportunities. For information contact District Public 504 Officer Annabell Louis at (908) 486-2800 x 8025.

**Linden Public Schools Vision**

The Linden Public School District is committed to developing respect for diversity, excellence in education, and a commitment to service, in order to promote global citizenship and ensure personal success for all students

**Linden Public Schools Mission**

The mission of the Linden Public School District is to promote distinction through the infinite resource that is Linden’s diversity, combined with our profound commitment to instructional excellence, so that each and every student achieves their maximum potential in an engaging, inspiring, and challenging learning environment.

**Math Department Vision**

To equip students with the understanding and application of mathematical skills and processes to foster a drive for advanced mathematics and higher-level thinking.

**Math Department Mission Statement**

To develop a community of learners who construct and communicate meaning from the mathematical world around them. Students will experience mathematics that encourage them to think critically, discover and apply concepts to solve problems strategically. Students will be encouraged to solve equations with accuracy, efficiency, and flexibility. Furthermore, students will have a multitude of opportunities to apply mathematical tools and practice standards to solve real-world and multi-step problems.

**Math Department Goals**

* Provide opportunities for student to develop computation skills, conceptual understanding, and problem-solving skills
* Require students to explain, justify or prove their thinking through mathematical reasoning, modeling, and speaking

Course Description

This course is targeted for students who have completed their 15 credits of required mathematics and need some additional instruction to prepare them for their college entrance exam. This course incorporates the New Jersey Student Learning Standards for Mathematical Practices as well as the following New Jersey Student Learning Standards for Mathematical Content: Expressions and Equations, The Number System, Functions, Algebra, Geometry, Number and Quantity, Statistics and Probability, and the New Jersey Student Learning Standards for High School Modeling. The standards align with the Mathematics Postsecondary Readiness Competencies deemed necessary for entry-level college courses.

Course Instructional Materials

* LPS Adopted Textbooks and Programs
  + Savvas – Triola, Elementary Statistics, 13th Edition
* Khan Academy
* College Board

Standards and NJDOE Mandates Guiding Instruction

* 1. New Jersey Student Learning Standards

<https://www.state.nj.us/education/cccs/>

General Interdisciplinary Connections / Materials

Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, and liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other.

Diversity, Equity, and Inclusion

* Use students’ interests in conceptualized tasks
* Expose students to a diverse group of mathematicians
* Design assessments and assignments with a variety of response types
* Use systematic grading and participation methods
* Encourage students to embrace a growth mindset

**Career Ready Practices**

CRP2.   Apply appropriate academic and technical skills.

CRP4.   Communicate clearly and effectively and with reason.

CRP6.   Demonstrate creativity and innovation.

CRP8.   Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11.   Use technology to enhance productivity.

CRP12.   Work productively in teams while using cultural global competence.

Pacing Guide

Linden Public Schools

Pacing Guide

Probability and Statistics

2022-2023

**First Semester**

Marking Period 1:

September 6, 2022 to November 15, 2022

Chapter 1 – Introduction to Statistics

Chapter 2 – Exploring Data with Tables and Graphs

Chapter 3 – Describing, Exploring, and Comparing Data

Chapter 15 – Ethics in Statistics

Marking Period 2:

November 16, 2022 to January 31, 2023

Chapter 4– Probability

Chapter 5– Discrete Probability Distributions

Chapter 6– Normal Probability Distributions

Chapter 7 – Estimating Parameters and Determining Sample Sizes

Chapter 8 – Hypothesis Testing (if time permits)

\***Assessment days are built into each chapter**.

**Second Semester**

Marking Period 3:

February 1, 2023 April 5, 2023

Chapter 1 – Introduction to Statistics

Chapter 2 – Exploring Data with Tables and Graphs

Chapter 3 – Describing, Exploring, and Comparing Data

Chapter 15 – Ethics in Statistics

Marking Period 4:

April 17, 2023 to June 22, 2023

Chapter 4– Probability

Chapter 5– Discrete Probability Distributions

Chapter 6– Normal Probability Distributions

Chapter 7 – Estimating Parameters and Determining Sample Sizes

Chapter 8 – Hypothesis Testing (if time permits)

\***Assessment days are built into each chapter**.

Vertical Integration – Program Mapping

N/A – Elective Course

Accommodations, Modifications, and Teacher Strategies

(specific recommendations are made in each unit)

|  |  |  |  |
| --- | --- | --- | --- |
| **Instructional Strategies**   * Teacher Presentation * Student Presentation * Class Discussion * Reading for Meaning * Inquiry Design Model * Interactive Lecture * Interactive Notetaking * Compare and Contrast * Research Based * Problem Based * Project Based   **504 Plans**  Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:   * walk, breathe, eat, or sleep * communicate, see, hear, or speak * read, concentrate, think, or learn * stand, bend, lift, or work   Examples of accommodations in 504 plans include:   * preferential seating * extended time on tests and assignments * reduced homework or classwork * verbal, visual, or technology aids * modified textbooks or audio-video materials * behavior management support * adjusted class schedules or grading * verbal testing * excused lateness, absence, or missed classwork * pre-approved nurse's office visits and accompaniment to visits occupational or physical therapy | **Gifted and Talent Accommodations and Modifications**   * Increase the level of complexity * Decrease scaffolding * Variety of finished products * Allow for greater independence * Learning stations, interest groups * Varied texts and supplementary materials * Use of technology * Flexibility in assignments * Varied questioning strategies * Encourage research * Strategy and flexible groups based on formative assessment or student choice * Acceleration within a unit of study * Exposure to more advanced or complex concepts, abstractions, and materials * Encourage students to move through content areas at their own pace * After mastery of a unit, provide students with more advanced learning activities, not more of the same activity * Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas | **Special Education and At-Risk Accommodations and Modifications**   * Remove unnecessary material, words, etc., that can distract from the content * Use of off-grade level materials * Provide appropriate scaffolding * Limit the number of steps required for completion * Time allowed * Level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Varied homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Ability to work at their own pace * Present ideas using auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment * Differentiated checklists and rubrics, if available and appropriate | **English Language Learners Accommodations and Modifications**   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials, including visuals * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Allow students to work at their own pace * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Role play * Provide graphic organizers, highlighted materials * Strategy and flexible groups based on formative assessment |

|  |
| --- |
| **Unit 1** |
| **Overview:**   In unit 1, the following topic will be covered: Statistics  Time Period: **First Marking Period (Semester 1 Students) / Third Marking Period (Semester 2 Students)** Length:  **10 Weeks** |

|  |
| --- |
| **STAGE 1 – Desired Results** |

|  |  |  |
| --- | --- | --- |
| **Educational Standards**  The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:  1. Make sense of problems and persevere in solving them.  2. Reason abstractly and quantitatively.  3. Construct viable arguments and critique the reasoning of others.  4. Model with mathematics.  5. Use appropriate tools strategically.  6. Attend to precision.  7. Look for and make use of structure.  8. Look for and express regularity in repeated reasoning.   |  | | --- | |  |  |  | | --- | | **Mathematics – High School**  **Statistics and Probability** |   Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.  Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.  Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.  Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.  Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.  Connections to Functions and Modeling. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.  **Interpreting Categorical and Quantitative Data S-ID**  **A. Summarize, represent, and interpret data on a single count or measurement variable**  1. Represent data with plots on the real number line (dot plots, histograms, and box plots). 2. Use statistics appropriate to the shape of the data distribution to compare center (median,  mean) and spread (interquartile range, standard deviation) of two or more different data  sets. 3. Interpret differences in shape, center, and spread in the context of the data sets, accounting  for possible effects of extreme data points (outliers). 4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to  estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.  **B. Summarize, represent, and interpret data on two categorical and quantitative variables**  5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.  6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data (including with the use of technology); use functions fitted to  data to solve problems in the context of the data. *Use given functions or choose a function*  *suggested by the context. Emphasize linear and exponential models.*  b. Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology.  c. Fit a linear function for a scatter plot that suggests a linear association.  **C. Interpret linear models**  7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.  8. Compute (using technology) and interpret the correlation coefficient of a linear fit. 9. Distinguish between correlation and causation.  **Making Inferences and Justifying Conclusions S-IC**  **A. Understand and evaluate random processes underlying statistical experiments**  1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.  2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*    **B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies**  3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.  4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.  5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.  6. Evaluate reports based on data.  **Essential Questions…**   * What exactly does the median do with the data? * What is the bottom line on the relationship between the mean and the median for skewed distributions?   **Enduring Understanding…**   * If it is impossible for two events to occur simultaneously, the events are mutually exclusive. * A population is the set containing all obejects whose properties are to be described and analyzed. * A sample set is a subset of the population. * As the spread of the data items increases, the standard deviation gets larger. * The normal distribution is a theoretical distribution for the entire population. * A plot of data points is called a scatter plot.   **Students will know...**   * Sampling methods * Quantitative data * Categorical data * Discrete data * Continuous data * Simple random sample * Frequency distribution * Histograms * Scatterplots * Mean * Median * Mode * Variation * Standard deviation * Range * Z score * Box plots   **Students will be able to...**   * Analyze sample relative to context, source, and sampling method. * I Understand the difference between statistical significance and practical significance. * Define and identify a voluntary response samples and know that statistical conclusions based on data from such a sample are generally not valid. * Distinguish between a parameter and a statistic. * Distinguish between quantitative data and categorical data. * Distinguish between discrete data and continuous data. * Determine whether basic statistical calculations are appropriate for a particular dataset. * Define and identify a simple random sample. * Understand the importance I’ve sound sampling methods and the importance of good design of experiments. * Develop an ability to summarize data in the format of a frequency distribution and the relative frequency distribution. * For a frequency distribution, identify values of class width, class midpoint, class limits, and class boundaries. * Develop the ability to picture the distribution of data in the format of a histogram or relative frequency histogram. * Examine histogram and identify common distributions, including a uniform distribution and a normal distribution. * Develop an ability to grab data using a dot plot, stem plot, timeseries graph, pie chart, and frequency polygon. * Determine when a graph is deceptive through the use of a nonzero axis or pictograph that uses an object of area or volume for one-dimensional data. * Develop an ability to construct the scatter plot of pair data. * Analyze a scatter plot to determine whether there appears to be correlation between two variables. * Developed the ability to measure the center of data by finding the mean, median, and mode. * Determine whether an outlier has a substantial effect on the mean and median. * Develop the ability to measure variation in a set of sample data by finding values of the range, variance, and standard deviation. * Develop the ability to interpret values of the standard deviation by applying the range rule of thumb to determine whether a particular value is significantly low were significantly high. * Develop the ability to compute a Z score. * Identify percentile values and quartile values from a set of data. * Develop the ability to construct a box plot from a set of data. |

|  |
| --- |
| **STAGE 2 – Evidence of Learning** |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Formative Assessment Suggestions**   |  | | --- | | • 3- Minute Pause | | • A-B-C Summaries | | • Analogy Prompt | | • Choral Response | | • Debriefing | | • Exit Card / Ticket | | • Hand Signals | | • Idea Spinner | | • Index Card Summaries | | • Inside-Outside Circle Discussion (Fishbowl) | | • Journal Entry | | • Misconception Check | | • Observation | | • One Minute Essay | | • One Word Summary | | • Portfolio Check | | • Questions & Answers | | • Quiz | | • Self-Assessment | | • Student Conference | | • Think-Pair-Share | | • Web or Concept Map |   **Authentic Assessment Suggestions**  Through the following authentic assessments, students will develop traits regarding thinking and reasoning, settings, mathematical tools and attitudes and dispositions:   * Performance Assessments * Short Investigations * Open Ended Response Questions * Self-Assessments   **Assessments**   * Chapter Tests |

|  |
| --- |
| **STAGE 3 – Learning Plan** |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Instructional Map**   |  | | --- | | **Modifications/Differentiation of Instruction** | | **Differentiation Strategies for Special Education Students**   * Remove unnecessary material, words, etc., that can distract from the content * Use of off-grade level materials * Provide appropriate scaffolding * Limit the number of steps required for completion * Time allowed * Level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Varied homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Ability to work at their own pace * Present ideas using auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment * Differentiated checklists and rubrics, if available and appropriate   **Differentiation Strategies for Gifted and Talented Students**   * Increase the level of complexity * Decrease scaffolding * Variety of finished products * Allow for greater independence * Learning stations, interest groups * Varied texts and supplementary materials * Use of technology * Flexibility in assignments * Varied questioning strategies * Encourage research * Strategy and flexible groups based on formative assessment or student choice * Acceleration within a unit of study * Exposure to more advanced or complex concepts, abstractions, and materials * Encourage students to move through content areas at their own pace * After mastery of a unit, provide students with more advanced learning activities, not more of the same activity * Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas   **Differentiated Strategies for ELL Students**   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials, including visuals * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Allow students to work at their own pace * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Role play * Provide graphic organizers, highlighted materials * Strategy and flexible groups based on formative assessment   **Differentiation Strategies for At Risk Students**   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessments   **504 Plans**  Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:   * walk, breathe, eat, or sleep * communicate, see, hear, or speak * read, concentrate, think, or learn * stand, bend, lift, or work   Examples of accommodations in 504 plans include:   * preferential seating * extended time on tests and assignments * reduced homework or classwork * verbal, visual, or technology aids * modified textbooks or audio-video materials * behavior management support * adjusted class schedules or grading * verbal testing * excused lateness, absence, or missed classwork * pre-approved nurse's office visits and accompaniment to visits * occupational or physical therapy |  |  | | --- | | **Modification Strategies** | | |  |  | | --- | --- | | • Extended Time | . | | • Frequent Breaks | . | | • Highlighted Text | . | | • Interactive Notebook | . | | • Modified Test | . | | • Oral Directions | . | | • Peer Tutoring | . | | • Preferential Seating | . | | • Re-Direct | . | | • Repeated Drill / Practice | . | | • Shortened Assignments | . | | • Teacher Notes | . | | • Tutorials | . | | • Use of Additional Reference Material | . | | • Use of Audio Resources | . | |  |  | | --- | | **High Preparation Differentiation** | | |  |  | | --- | --- | | • Alternative Assessments | . | | • Choice Boards | . | | • Games and Tournaments | . | | • Group Investigations | . | | • Guided Reading | . | | • Independent Research / Project | . | | • Interest Groups | . | | • Learning Contracts | . | | • Leveled Rubrics | . | | • Literature Circles | . | | • Menu Assignments | . | | • Multiple Intelligence Options | . | | • Multiple Texts | . | | • Personal Agendas | . | | • Project Based Learning (PBL) | . | | • Stations / Centers | . | | • Think-Tac-Toe | . | | • Tiered Activities / Assignments | . | | • Varying Graphic Organizers | . | |  |  | | --- | | **Low Preparation Differentiation** | | |  |  | | --- | --- | | • Choice of Book / Activity | . | | • Cubing Activities | . | | • Exploration by Interest (using interest inventories) | . | | • Flexible Grouping | . | | • Goal Setting With Student | . | | • Homework Options | . | | • Jigsaw | . | | • Mini Workshops to Extend Skills | . | | • Mini Workshops to Re-teach | . | | • Open-ended Activities | . | | • Think-Pair-Share by Interest | . | | • Think-Pair-Share by Learning Style | . | | • Think-Pair-Share by Learning Style | . | | • Think-Pair-Share by Readiness | . | | • Use of Collaboration | . | | • Use of Reading Buddies | . | | • Varied Journal Prompts | . | | • Varied Product Choice | . | | • Varied Supplemental Materials | . | | • Work Alone / Together | . | | |

|  |
| --- |
| **Additional Materials** |

|  |
| --- |
| LPS Adopted Textbooks and Programs  Savvas – Triola, Elementary Statistics, 13th Edition  Khan Academy  College Board |

|  |
| --- |
| **Unit 2** |
| **Overview:**  In unit 2, the following topic will be covered: Probability  Time Period: **Second Marking Period (Semester 1 Students) / Fourth Marking Period (Semester 2 Students)** Length: **10 Weeks** |

|  |
| --- |
| **STAGE 1 – Desired Results** |

|  |  |
| --- | --- |
| **Educational Standards**  The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:  1. Make sense of problems and persevere in solving them.  2. Reason abstractly and quantitatively.  3. Construct viable arguments and critique the reasoning of others.  4. Model with mathematics.  5. Use appropriate tools strategically.  6. Attend to precision.  7. Look for and make use of structure.  8. Look for and express regularity in repeated reasoning.   |  | | --- | | **Mathematics – High School**  **Statistics and Probability** |   Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.  Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.  Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.  Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.  Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.  Connections to Functions and Modeling. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.  **Conditional Probability and the Rules of Probability S-CP**  **A. Understand independence and conditional probability and use them to interpret data**  1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).  2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.  3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.  4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*  5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*  **B. Use the rules of probability to compute probabilities of compound events in a uniform probability model**  6. Find the conditional probability of *A* given *B* as the fraction of *B*’s outcomes that also belong to *A,* and interpret the answer in terms of the model.  7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.  8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.  9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.  **Using Probability to Make Decisions S-MD**  **A. Calculate expected values and use them to solve problems**  1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.  2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.  3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value  4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*  **B. Use probability to evaluate outcomes of decisions**  5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. *For example, find the expected winnings*  *from a state lottery ticket or a game at a fast food restaurant.* b. Evaluate and compare strategies on the basis of expected values. *For example, compare a*  *high-deductible versus a low-deductible automobile insurance policy using various, but*  *reasonable, chances of having a minor or a major accident.*  6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).  7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).  **Essential Questions…**   * What do you mean by saying that one person is *randomly selected* from the population? * What is the difference between *independent events* and *mutually exclusive events*? * When solving probability problems, how do I decide whether to use the *or* formulas or the *and* formulas?   **Enduring Understanding…**   * The number of ways in which a series of successive things can occur is founf by multiplying the number of ways un which each thing can occur. * A permutation from a group of items occurs when no item is used more than once and the order of arrangements makes a difference. * A combination from a group of items occurs when no item is used more than once and th order of items makes no difference. * Theoretical probabolity applies to exeriments in which the set of all equally likely outcomes called the sample space. * If it is impossible for two events to occur simultaneously, the events are mutually exclusive.   **Students will know...**   * Complement of an event * Independent events * Dependent events * The multiplication rule * The accounting rule * The factorial rule * Permutation role * Combination rule * Probability distribution * Standard normal distribution * Central limit theorem * The Population mean * Sample size   **Students will be able to...**   * Identify probabilities as values between zero and one, and interpret those values as expressions of likelihood of the events. * Developed the ability to calculate probabilities of events. * Define the complement of an event and calculate the probability of the compliment. * Develop the ability to calculate the probability that in a single trial, some event A occurs for some event B occurs or they both occur. * Distinguish between independent events and dependent events. * Apply the multiplication rule by computing the probability of some event, given that some event has already occurred. * Develop the ability to apply the multiplication counting rule, factorial rule, permutations rule and combinations rule. * Define random variable and probability distribution. * Describe a binomial probability distribution and find probability values for a binomial distribution. * Describe the characteristics of a standard normal distribution. * Develop the ability to describe a normal distribution. * Find the probability of some range of values in a normal distribution. * Describe what the central limit theorem states. * Construct a confidence interval estimate of a population proportion and interpret such confidence interval estimates. * Determine the sample size necessary to estimate a population mean. |

|  |
| --- |
| **STAGE 2 – Evidence of Learning** |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Formative Assessment Suggestions**   |  | | --- | | • 3- Minute Pause | | • A-B-C Summaries | | • Analogy Prompt | | • Choral Response | | • Debriefing | | • Exit Card / Ticket | | • Hand Signals | | • Idea Spinner | | • Index Card Summaries | | • Inside-Outside Circle Discussion (Fishbowl) | | • Journal Entry | | • Misconception Check | | • Observation | | • One Minute Essay | | • One Word Summary | | • Portfolio Check | | • Questions & Answers | | • Quiz | | • Self-Assessment | | • Student Conference | | • Think-Pair-Share | |  |   **Authentic Assessment Suggestions**  Through the following authentic assessments, students will develop traits regarding thinking and reasoning, settings, mathematical tools and attitudes and dispositions:   * Performance Assessments * Short Investigations * Open Ended Response Questions * Self-Assessments   **Assessments**  Chapter Tests |

|  |
| --- |
| **STAGE 3 – Learning Plan** |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Instructional Map**   |  | | --- | | **Modifications/Differentiation of Instruction** | | **Differentiation Strategies for Special Education Students**   * Remove unnecessary material, words, etc., that can distract from the content * Use of off-grade level materials * Provide appropriate scaffolding * Limit the number of steps required for completion * Time allowed * Level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Varied homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Ability to work at their own pace * Present ideas using auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment * Differentiated checklists and rubrics, if available and appropriate   **Differentiation Strategies for Gifted and Talented Students**   * Increase the level of complexity * Decrease scaffolding * Variety of finished products * Allow for greater independence * Learning stations, interest groups * Varied texts and supplementary materials * Use of technology * Flexibility in assignments * Varied questioning strategies * Encourage research * Strategy and flexible groups based on formative assessment or student choice * Acceleration within a unit of study * Exposure to more advanced or complex concepts, abstractions, and materials * Encourage students to move through content areas at their own pace * After mastery of a unit, provide students with more advanced learning activities, not more of the same activity * Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas   **Differentiated Strategies for ELL Students**   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials, including visuals * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Allow students to work at their own pace * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Role play * Provide graphic organizers, highlighted materials * Strategy and flexible groups based on formative assessment   **Differentiation Strategies for At Risk Students**   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessments   **504 Plans**  Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:   * walk, breathe, eat, or sleep * communicate, see, hear, or speak * read, concentrate, think, or learn * stand, bend, lift, or work   Examples of accommodations in 504 plans include:   * preferential seating * extended time on tests and assignments * reduced homework or classwork * verbal, visual, or technology aids * modified textbooks or audio-video materials * behavior management support * adjusted class schedules or grading * verbal testing * excused lateness, absence, or missed classwork * pre-approved nurse's office visits and accompaniment to visits * occupational or physical therapy |  |  | | --- | | **Modification Strategies** | | |  |  | | --- | --- | | • Extended Time | . | | • Frequent Breaks | . | | • Highlighted Text | . | | • Interactive Notebook | . | | • Modified Test | . | | • Oral Directions | . | | • Peer Tutoring | . | | • Preferential Seating | . | | • Re-Direct | . | | • Repeated Drill / Practice | . | | • Shortened Assignments | . | | • Teacher Notes | . | | • Tutorials | . | | • Use of Additional Reference Material | . | | • Use of Audio Resources | . | |  |  | | --- | | **High Preparation Differentiation** | | |  |  | | --- | --- | | • Alternative Assessments | . | | • Choice Boards | . | | • Games and Tournaments | . | | • Group Investigations | . | | • Guided Reading | . | | • Independent Research / Project | . | | • Interest Groups | . | | • Learning Contracts | . | | • Leveled Rubrics | . | | • Literature Circles | . | | • Menu Assignments | . | | • Multiple Intelligence Options | . | | • Multiple Texts | . | | • Personal Agendas | . | | • Project Based Learning (PBL) | . | | • Stations / Centers | . | | • Think-Tac-Toe | . | | • Tiered Activities / Assignments | . | | • Varying Graphic Organizers | . | |  |  | | --- | | **Low Preparation Differentiation** | | |  |  | | --- | --- | | • Choice of Book / Activity | . | | • Cubing Activities | . | | • Exploration by Interest (using interest inventories) | . | | • Flexible Grouping | . | | • Goal Setting With Student | . | | • Homework Options | . | | • Jigsaw | . | | • Mini Workshops to Extend Skills | . | | • Mini Workshops to Re-teach | . | | • Open-ended Activities | . | | • Think-Pair-Share by Interest | . | | • Think-Pair-Share by Learning Style | . | | • Think-Pair-Share by Learning Style | . | | • Think-Pair-Share by Readiness | . | | • Use of Collaboration | . | | • Use of Reading Buddies | . | | • Varied Journal Prompts | . | | • Varied Product Choice | . | | • Varied Supplemental Materials | . | | • Work Alone / Together | . | | |

|  |
| --- |
| **Additional Materials** |

|  |
| --- |
| LPS Adopted Textbooks and Programs  Savvas – Lial, Algebra for College Students, 9th Edition  Khan Academy  College Board |

**Interdisciplinary Connections & Standards**

With interdisciplinary instruction, the subject areas are woven together and explored through an overarching theme or concept. We use math to help us solve everyday problems in the kitchen, in the garden, and for many of us at our jobs.

Brain research has shown that information in our brains is organized in schematic structures. These structures are made up of interconnected bits of information and serve as a framework for the knowledge we acquire. When a learner’s knowledge is connected, it is much more likely that they will apply the prior knowledge to a wide variety of new situations. They will acquire new information in a way that is more accessible and will be better able to relate it to previously acquired knowledge.

Students learn about patterns in math, science, social studies, and even literature. Because of this, they are much more likely to “see” these patterns when they encounter new situations. Since patterns are not only studied in math they are able to make the connection and gain the understanding that patterns can be found in many areas of their lives. Interdisciplinary instruction allows students to understand the interconnectedness of the disciplines and makes learning more meaningful and relevant as fascinating connections are made across the subject areas.

**Science**

HS-LS3-3 Apply concepts of statistics and probability to explain the variation and distribution of expressed traits in a population.

HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

**Language Arts**

RL.11-12.1. Cite strong and thorough textual evidence and make relevant connections to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain.

RI.11-12.1. Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.

RI.11-12.2. Determine two or more central ideas of a text, and analyze their development and how they interact to provide a complex analysis; provide an objective summary of the text.

**Social Studies**

6.1.12.EconEM.2 Analyze how technological developments transformed the economy, created international markets, and affected the environment in New Jersey and the nation.

6.1.12.EconGE.16 Use quantitative data and other sources to assess the impact of international, global business organizations, and oversees competition on the United States economy and workforce.