**Subject**

**Math Prep**

**Curriculum Guide**

**­­\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**LINDEN PUBLIC SCHOOLS**

**LINDEN, NEW JERSEY**

**DR. MARNIE HAZELTON**

**SUPERINTENDENT**

**DENISE CLEARY**

**ASSISTANT SUPERINTENDENT**

**RICHARD MOLINARO**

**DIRECTOR OF MATHEMATICS, VOCATIONAL, & TECHNICAL SUBJECTS**

**The Linden Board of Education adopted the Curriculum Guide on:**

|  |  |  |
| --- | --- | --- |
| **July 28, 2022** |  | **Education Report #22** |
|  |  |  |
|  | | |
| **Rationale** | | |

**EDUCATION EQUITY:** The Linden Public School District guarantees each student equal educational opportunity regardless of age, race, color, creed, religion, gender, language, affectional or sexual orientation, ancestry, national origin, marital or economic status. For Information, contact District Educational Equity Officer Kevin Thurston at **(**908) 486-2800 x 8307**.**

**NONDISCRIMATION:** The Linden Public School District does not discriminate against handicapped persons in admission or access to or treatment or employment in its programs, activities, and vocational opportunities. For information contact District Public 504 Officer Annabell Louis at (908) 486-2800 x 8025.

**Linden Public Schools Vision**

The Linden Public School District is committed to developing respect for diversity, excellence in education, and a commitment to service, in order to promote global citizenship and ensure personal success for all students

**Linden Public Schools Mission**

The mission of the Linden Public School District is to promote distinction through the infinite resource that is Linden’s diversity, combined with our profound commitment to instructional excellence, so that each and every student achieves their maximum potential in an engaging, inspiring, and challenging learning environment.

**Math Department Vision**

To equip students with the understanding and application of mathematical skills and processes to foster a drive for advanced mathematics and higher-level thinking.

**Math Department Mission Statement**

To develop a community of learners who construct and communicate meaning from the mathematical world around them. Students will experience mathematics that encourage them to think critically, discover and apply concepts to solve problems strategically. Students will be encouraged to solve equations with accuracy, efficiency, and flexibility. Furthermore, students will have a multitude of opportunities to apply mathematical tools and practice standards to solve real-world and multi-step problems.

**Math Department Goals**

* Provide opportunities for student to develop computation skills, conceptual understanding, and problem-solving skills
* Require students to explain, justify or prove their thinking through mathematical reasoning, modeling, and speaking

Course Description

This course is targeted for students who have completed their 15 credits of required mathematics and need some additional instruction to prepare them for their college entrance exam. This course incorporates the New Jersey Student Learning Standards for Mathematical Practices as well as the following New Jersey Student Learning Standards for Mathematical Content: Expressions and Equations, The Number System, Functions, Algebra, Geometry, Number and Quantity, Statistics and Probability, and the New Jersey Student Learning Standards for High School Modeling. The standards align with the Mathematics Postsecondary Readiness Competencies deemed necessary for entry-level college courses.

Course Instructional Materials

* LPS Adopted Textbooks and Programs
  + Savvas – Lial, Algebra for College Students, 9th Edition
* Khan Academy
* College Board

Standards and NJDOE Mandates Guiding Instruction

* 1. New Jersey Student Learning Standards

<https://www.state.nj.us/education/cccs/>

Diversity, Equity, and Inclusion

* Use students’ interests in conceptualized tasks
* Expose students to a diverse group of mathematicians
* Design assessments and assignments with a variety of response types
* Use systematic grading and participation methods
* Encourage students to embrace a growth mindset

**Career Ready Practices**

CRP2.   Apply appropriate academic and technical skills.

CRP4.   Communicate clearly and effectively and with reason.

CRP6.   Demonstrate creativity and innovation.

CRP8.   Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11.   Use technology to enhance productivity.

CRP12.   Work productively in teams while using cultural global competence.

Pacing Guide

Linden Public Schools

Pacing Guide

Math Prep

2022-2023

**First Semester**

Marking Period 1: September 6, 2022 to November 15, 2022

Chapter R – Review of the Real Number System

Chapter 1 – Linear Equations, Inequalities, and Applications

Chapter 2 – Linear Equations, Graphs and Functions

Chapter 3 – Systems of Linear Equations

Chapter 4 – Exponents, Polynomials, and Polynomial Functions

Chapter 5 - Factoring

Marking Period 2: November 16, 2022 to January 31, 2023

Chapter 6 – Rational Expressions and Functions

Chapter 7 – Roots, Radicals, and Root Functions

Chapter 8 – Quadratic Equations and Inequalities

Chapter 9 – Additional Graphs of Functions and Relations

Chapter 10 – Inverse, Exponential, and Logarithmic Functions

Chapter 11 – Polynomial and Rational Functions (if time permits)

Chapter 12 – Conic Sections and Nonlinear Systems (if time permits)

Chapter 13 – Further Topics in Algebra (if time permits)

\***Assessment days are built into each chapter**.

**Second Semester**

Marking Period 3: February 1, 2023 to April 5, 2023

Chapter R – Review of the Real Number System

Chapter 1 – Linear Equations, Inequalities, and Applications

Chapter 2 – Linear Equations, Graphs and Functions

Chapter 3 – Systems of Linear Equations

Chapter 4 – Exponents, Polynomials, and Polynomial Functions

Chapter 5 - Factoring

Marking Period 4: April 17, 2023 to June 22, 2023

Chapter 6 – Rational Expressions and Functions

Chapter 7 – Roots, Radicals, and Root Functions

Chapter 8 – Quadratic Equations and Inequalities

Chapter 9 – Additional Graphs of Functions and Relations

Chapter 10 – Inverse, Exponential, and Logarithmic Functions

Chapter 11 – Polynomial and Rational Functions (if time permits)

Chapter 12 – Conic Sections and Nonlinear Systems (if time permits)

Chapter 13 – Further Topics in Algebra (if time permits)

\***Assessment days are built into each chapter**.

Vertical Integration – Program Mapping

N/A – Elective Course

Accommodations, Modifications, and Teacher Strategies

(specific recommendations are made in each unit)

|  |  |  |  |
| --- | --- | --- | --- |
| **Instructional Strategies**   * Teacher Presentation * Student Presentation * Class Discussion * Reading for Meaning * Inquiry Design Model * Interactive Lecture * Interactive Notetaking * Compare and Contrast * Research Based * Problem Based * Project Based   **504 Plans**  Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:   * walk, breathe, eat, or sleep * communicate, see, hear, or speak * read, concentrate, think, or learn * stand, bend, lift, or work   Examples of accommodations in 504 plans include:   * preferential seating * extended time on tests and assignments * reduced homework or classwork * verbal, visual, or technology aids * modified textbooks or audio-video materials * behavior management support * adjusted class schedules or grading * verbal testing * excused lateness, absence, or missed classwork * pre-approved nurse's office visits and accompaniment to visits occupational or physical therapy | **Gifted and Talent Accommodations and Modifications**   * Increase the level of complexity * Decrease scaffolding * Variety of finished products * Allow for greater independence * Learning stations, interest groups * Varied texts and supplementary materials * Use of technology * Flexibility in assignments * Varied questioning strategies * Encourage research * Strategy and flexible groups based on formative assessment or student choice * Acceleration within a unit of study * Exposure to more advanced or complex concepts, abstractions, and materials * Encourage students to move through content areas at their own pace * After mastery of a unit, provide students with more advanced learning activities, not more of the same activity * Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas | **Special Education and At-Risk Accommodations and Modifications**   * Remove unnecessary material, words, etc., that can distract from the content * Use of off-grade level materials * Provide appropriate scaffolding * Limit the number of steps required for completion * Time allowed * Level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Varied homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Ability to work at their own pace * Present ideas using auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment * Differentiated checklists and rubrics, if available and appropriate | **English Language Learners Accommodations and Modifications**   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials, including visuals * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Allow students to work at their own pace * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Role play * Provide graphic organizers, highlighted materials * Strategy and flexible groups based on formative assessment |

|  |
| --- |
| **Unit 1** |
| **Overview:**   In unit 1, the following topics will be covered: The Real Number System, Linear Equations, Linear Inequalities, Linear Graphs, Linear Functions, Systems of Linear Equations, Exponents, Polynomials, Polynomial Functions and Factoring.  Time Period: **First Marking Period (Semester 1 Students) / Third Marking Period (Semester 2 Students)** Length:  **10 Weeks** |

|  |
| --- |
| **STAGE 1 – Desired Results** |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Educational Standards**  The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:  1. Make sense of problems and persevere in solving them.  2. Reason abstractly and quantitatively.  3. Construct viable arguments and critique the reasoning of others.  4. Model with mathematics.  5. Use appropriate tools strategically.  6. Attend to precision.  7. Look for and make use of structure.  8. Look for and express regularity in repeated reasoning   |  | | --- | | **New Jersey Student Learning Standards- Mathematics** | |  |  |  | | --- | | **Mathematics – High School - Number and Quantity**  **Mathematics – High School - Algebra**  **Mathematics – High School - Functions** | | **Number and Number Systems**  During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.  With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.  Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that (51/3)3 should be 5(1/3)3 = 51 = 5 and that 51/3 should be the cube root of 5.  Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.  **Quantities**  In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.  **Expressions**  An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.  Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, *p* + 0.05*p* can be interpreted as the addition of a 5% tax to a price *p*. Rewriting *p* + 0.05*p* as 1.05*p* shows that adding a tax is the same as multiplying the price by a constant factor.  Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, *p* + 0.05*p* is the sum of the simpler expressions p and 0.05*p*. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.  A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.  **Equations and Inequalities**  An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.  The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.  An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.  Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of *x* + 1 = 0 is an integer, not a whole number; the solution of 2*x* + 1 = 0 is a rational number, not an integer; the solutions of *x*2 – 2 = 0 are real numbers, not rational numbers; and the solutions of *x*2 + 2 = 0 are complex numbers, not real numbers.  The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, *A* = ((*b*1+*b*2)/2)*h*, can be solved for *h* using the same deductive process. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.  **Functions**  Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.  In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car’s speed in miles per hour, *v*; the rule *T*(*v*) = 100/*v* expresses this relationship algebraically and defines a function whose name is *T*.  The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.  A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, “I’ll give you a state, you give me the capital city;” by an algebraic expression like *f*(*x*) = *a* + *bx*; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function’s properties.  Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.  A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.  **Connections to Expressions, Equations, Modeling and Coordinates**  Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology. | |  | |  |   **The Real Number System N-RN**  **A. Extend the properties of exponents to rational exponents.**  1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.*  2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.  **B. Use properties of rational and irrational numbers.**  3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.  **Quantities N-Q**  **A. Reason quantitatively and use units to solve problems.**  1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.  2. Define appropriate quantities for the purpose of descriptive modeling. 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.  **Seeing Structure in Expressions A-SSE**  **A. Interpret the structure of expressions**  1. Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.  *For example, interpret P*(1+*r*)n *as the product of P and a factor not depending on P* 2. Use the structure of an expression to identify ways to rewrite it. *For example, see x*4 – *y*4 *as*  (*x*2)2 – (*y*2)2, *thus recognizing it as a difference of squares that can be factored as* (*x*2 – *y*2)(*x*2 + *y*2).  **B. Write expressions in equivalent forms to solve problems**  3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.  a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* 1.15t *can be rewritten as* (1.151/12)12*t* ≈1.01212t *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*  4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*  **Creating Equations, A-CED**  **A. Create equations that describe numbers or relationships**  1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*  2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*  4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s law V = IR to highlight resistance R.*  **Reasoning with Equations and Inequalities A-REI**  **A. Understand solving equations as a process of reasoning and explain the reasoning**  1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.  2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.  **B. Solve equations and inequalities in one variable**  3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.  4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form (*x* – *p*)2 = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for *x*2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as *a* ± *bi* for real numbers *a* and *b*.  **C. Solve systems of equations**  5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.  6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.  7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y*=–*3x* and the circle *x*2+*y*2 =*3.*  8. (+) Represent a system of linear equations as a single matrix equation in a vector variable. 9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations  (using technology for matrices of dimension 3 × 3 or greater).  **D. Represent and solve equations and inequalities graphically**  10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  11. Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★  12. Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.  **Interpreting Functions F-IF**  **A. Understand the concept of a function and use function notation**  1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).  2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.  3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n* ≥ *1.*  **B. Interpret functions that arise in applications in terms of the context**  4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*  5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*  6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.  **C. Analyze functions using different representations**  7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*  9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*  **Building Functions F-BF**  **A. Build a function that models a relationship between two quantities**  1. Write a function that describes a relationship between two quantities a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*  2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.  **B. Build new functions from existing functions**  3. Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*  4. Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x* ≠*1.* b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.  5. (+) Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.  **Essential Questions…**   * Why do we use variables? * Why are “the order of operations” and other properties of mathematics important? * When would we want to use the absolute value of numbers? * Why is advantageous to use and solve equations algebraically for real- world problems? * How does the slope affect direct variation? * What does the slope-intercept form of an equation tell us? * Why should we know different forms of linear equations? * How do we write the equations of parallel and perpendicular lines? * How do we use real-world data to write the equation of a line? * How might one determine the most efficient method for solving a system of equations?   **Enduring Understanding…**   * Solving linear equations and inequalities in an essential skill required to solve other types of equations. * Mathematical rules can be used to describe quantitative relationships.   **Students will know...**   * Reason quantitatively and use units to solve probles. * Interpret the structure of expressions * Create equations that describe numbers or relationships * Understand solving equations as a process of reasoning and explain the reasoning * Solve equations and inequalities in one variable * Solve systems of equations * Represent and solve equations and inequalities graphically * Understand the concept of a function and use function notation * Interpret functions that arise in applications in terms of a context * Analyze functions using different representations * Write expressions in equivalent forms to solve problems * Perform arithmetic operations on polynomials   **Students will be able to...**   * identify different parts of an expression, including terms, factors and constants. * explain the meaning of parts of an expression in context. * solve linear equations with coefficients represented by letters in one variable.• use the properties of equality to justify steps in solving linear equations. * solve linear inequalities in one variable. * rearrange linear formulas and literal equations, isolating a specific variable. * identify and describe relationships between quantities in word problems. * create linear equations in one variable. * create linear inequalities in one variable. * use equations and inequalities to solve real world problems. * explain each step in the solution process. * select appropriate scales for constructing a graph. * interpret the origin in graphs. * graph equations on coordinate axes, including labels and scales. * identify and describe the solutions in the graph of an equation. * create linear equations in two variables, including those from a context. * select appropriate scales for constructing a graph. * interpret the origin in graphs. * graph equations on coordinate axes, including labels and scales. * identify and describe the solutions in the graph of an equation. * explain the relationship between the x-coordinate of a point of intersection and the solution to the equation f(x) = g(x) for linear equations y = f(x) and y = g(x). * find approximate solutions to the system by making a table of values, graphing, and finding successive approximations. * identify and define variables representing essential features for the model. * model real world situations by creating a system of linear equations. * solve systems of linear equations using the elimination or substitution method.• solve systems of linear equations by graphing. * interpret the solution(s) in context. * model real world situations by creating a system of linear inequalities given a context. * interpret the solution(s) in context. * use the definition of a function to determine whether a relationship is a function.• use function notation once a relation is determined to be a function. * evaluate functions for given inputs in the domain. * explain statements involving function notation in the context of the problem. * identify and describe situations in which one quantity changes at a constant rate. * use the properties of exponents to simplify or expand exponential expressions, recognizing these are equivalent forms. * add and subtract polynomials. * multiply polynomials. |

|  |
| --- |
| **STAGE 2 – Evidence of Learning** |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Formative Assessment Suggestions**   |  | | --- | | • 3- Minute Pause | | • A-B-C Summaries | | • Analogy Prompt | | • Choral Response | | • Debriefing | | • Exit Card / Ticket | | • Hand Signals | | • Idea Spinner | | • Index Card Summaries | | • Inside-Outside Circle Discussion (Fishbowl) | | • Journal Entry | | • Misconception Check | | • Observation | | • One Minute Essay | | • One Word Summary | | • Portfolio Check | | • Questions & Answers | | • Quiz | | • Self-Assessment | | • Student Conference | | • Think-Pair-Share | | • Web or Concept Map |   **Authentic Assessment Suggestions**  Through the following authentic assessments, students will develop traits regarding thinking and reasoning, settings, mathematical tools and attitudes and dispositions:   * Performance Assessments * Short Investigations * Open Ended Response Questions * Self-Assessments   **Assessments**   * Chapter Tests |

|  |
| --- |
| **STAGE 3 – Learning Plan** |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Instructional Map**   |  | | --- | | **Modifications/Differentiation of Instruction** | | **Differentiation Strategies for Special Education Students**   * Remove unnecessary material, words, etc., that can distract from the content * Use of off-grade level materials * Provide appropriate scaffolding * Limit the number of steps required for completion * Time allowed * Level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Varied homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Ability to work at their own pace * Present ideas using auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment * Differentiated checklists and rubrics, if available and appropriate   **Differentiation Strategies for Gifted and Talented Students**   * Increase the level of complexity * Decrease scaffolding * Variety of finished products * Allow for greater independence * Learning stations, interest groups * Varied texts and supplementary materials * Use of technology * Flexibility in assignments * Varied questioning strategies * Encourage research * Strategy and flexible groups based on formative assessment or student choice * Acceleration within a unit of study * Exposure to more advanced or complex concepts, abstractions, and materials * Encourage students to move through content areas at their own pace * After mastery of a unit, provide students with more advanced learning activities, not more of the same activity * Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas   **Differentiated Strategies for ELL Students**   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials, including visuals * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Allow students to work at their own pace * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Role play * Provide graphic organizers, highlighted materials * Strategy and flexible groups based on formative assessment   **Differentiation Strategies for At Risk Students**   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessments   **504 Plans**  Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:   * walk, breathe, eat, or sleep * communicate, see, hear, or speak * read, concentrate, think, or learn * stand, bend, lift, or work   Examples of accommodations in 504 plans include:   * preferential seating * extended time on tests and assignments * reduced homework or classwork * verbal, visual, or technology aids * modified textbooks or audio-video materials * behavior management support * adjusted class schedules or grading * verbal testing * excused lateness, absence, or missed classwork * pre-approved nurse's office visits and accompaniment to visits * occupational or physical therapy |  |  | | --- | | **Modification Strategies** | | |  |  | | --- | --- | | • Extended Time | . | | • Frequent Breaks | . | | • Highlighted Text | . | | • Interactive Notebook | . | | • Modified Test | . | | • Oral Directions | . | | • Peer Tutoring | . | | • Preferential Seating | . | | • Re-Direct | . | | • Repeated Drill / Practice | . | | • Shortened Assignments | . | | • Teacher Notes | . | | • Tutorials | . | | • Use of Additional Reference Material | . | | • Use of Audio Resources | . | |  |  | | --- | | **High Preparation Differentiation** | | |  |  | | --- | --- | | • Alternative Assessments | . | | • Choice Boards | . | | • Games and Tournaments | . | | • Group Investigations | . | | • Guided Reading | . | | • Independent Research / Project | . | | • Interest Groups | . | | • Learning Contracts | . | | • Leveled Rubrics | . | | • Literature Circles | . | | • Menu Assignments | . | | • Multiple Intelligence Options | . | | • Multiple Texts | . | | • Personal Agendas | . | | • Project Based Learning (PBL) | . | | • Stations / Centers | . | | • Think-Tac-Toe | . | | • Tiered Activities / Assignments | . | | • Varying Graphic Organizers | . | |  |  | | --- | | **Low Preparation Differentiation** | | |  |  | | --- | --- | | • Choice of Book / Activity | . | | • Cubing Activities | . | | • Exploration by Interest (using interest inventories) | . | | • Flexible Grouping | . | | • Goal Setting With Student | . | | • Homework Options | . | | • Jigsaw | . | | • Mini Workshops to Extend Skills | . | | • Mini Workshops to Re-teach | . | | • Open-ended Activities | . | | • Think-Pair-Share by Interest | . | | • Think-Pair-Share by Learning Style | . | | • Think-Pair-Share by Learning Style | . | | • Think-Pair-Share by Readiness | . | | • Use of Collaboration | . | | • Use of Reading Buddies | . | | • Varied Journal Prompts | . | | • Varied Product Choice | . | | • Varied Supplemental Materials | . | | • Work Alone / Together | . | | |

|  |
| --- |
| **Additional Materials** |

|  |
| --- |
| LPS Adopted Textbooks and Programs  Savvas – Lial, Algebra for College Students, 9th Edition  Khan Academy  College Board |
|  |

|  |
| --- |
| **Unit 2** |
| **Overview:**  In unit 2, the following topics will be covered: Rational Expressions, Rational Functions, Roots, Radicals, Quadratic Equations, Quadratic Inequalities, Additional Graphs of Functions, Exponential and Logarithmic Functions  Time Period: **Second Marking Period (Semester 1 Students) / Fourth Marking Period (Semester 2 Students)** Length: **10 Weeks** |

|  |
| --- |
| **STAGE 1 – Desired Results** |

|  |
| --- |
| **Educational Standards**  The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:  1. Make sense of problems and persevere in solving them.  2. Reason abstractly and quantitatively.  3. Construct viable arguments and critique the reasoning of others.  4. Model with mathematics.  5. Use appropriate tools strategically.  6. Attend to precision.  7. Look for and make use of structure.  8. Look for and express regularity in repeated reasoning.  **Mathematics – High School – Functions**  **Functions**  Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.  In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car’s speed in miles per hour, *v*; the rule *T*(*v*) = 100/*v* expresses this relationship algebraically and defines a function whose name is *T*.  The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.  A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, “I’ll give you a state, you give me the capital city;” by an algebraic expression like *f*(*x*) = *a* + *bx*; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function’s properties.  Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.  A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.  **Connections to Expressions, Equations, Modeling and Coordinates**  Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.  **Interpreting Functions F-IF**  **A. Understand the concept of a function and use function notation**  1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).  2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.  3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n* ≥ *1.*  **B. Interpret functions that arise in applications in terms of the context**  4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*  5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*  6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.  **C. Analyze functions using different representations**  7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*  9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*  **Building Functions F-BF**  **A. Build a function that models a relationship between two quantities**  1. Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*  2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.  **B. Build new functions from existing functions**  3. Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*  4. Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x* ≠*1.* b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.  5. (+) Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.  **Linear and Exponential Models F-LE**  **A. Construct and compare linear and exponential models and solve problems**  1. Distinguish between situations that can be modeled with linear functions and with exponential functions.  a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.  b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.  c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.  2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).  3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.  4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.  **B. Interpret expressions for functions in terms of the situation they model**  5. Interpret the parameters in a linear or exponential function in terms of a context.  **Essential Questions…**   * + How do equations show a relationship between two quantities in real-life?   + How can algebraic properties by used in problem solving?   + How can solution to equation(s) or inequality(s) be represented in multiple ways?   + What distinguishes equations from inequalities?   + How do quadratic functions compare to linear functions?   + How can we determine which way the parabola wil be facing before you graph it?   + How can we find the vertex when the equation is given? A graph?   + How does the quadratic equation transform on a coordinate plane?   + How can we recognize solutions on a parabola?   + What do solutions of quadratics represent?   + How do quadratics compare to linear functions?   + How can we apply quadratics to real life situations?   + In what different ways can data be represented on a real number line, and how can statistics appropriate to the shape of the data distribution serve to compare two or more data sets?   + How do relative frequencies, two-way frequency tables,and residual analysis help us summarize and better understand categorical data?   + What information can a slope and intercept of a linear model provide regarding the context of a situation?   **Enduring Understanding…**   * An expression can be manipulated to produce an equivalent yet different looking expression. Real numbers and their calculations can be represented in different manners. * Functions give us the power to organize, compare, and make sense of relationships around us. * Statistical analysis and data models allow us to accurately interpret the information given to us by functions, equations and systems.   **Students will know...**   * + Extend the properties of exponents to rational exponents   + Use properties of rational and irrational numbers   + Interpret functions that arise in applications in terms of a context   + Analyze functions using different representations   + Build a function that models a relationship between two quantities   + Build new functions from existing functions.   + Construct and compare linear, quadratic and exponential models and solve problems   + Interpret expressions for functions in terms of the situation they model   + Summarize, represent, and interpret data on a single count or measurement variable   + Summarize, represent, and interpret data on two categorical and quantitative variables   + Interpret linear models   + Build a function that models a relationship between two quantities   + Construct and compare linear, quadratic, and exponential models and solve problems   **Students will be able to...**   * Recognize numerical expressions as a difference of squares and rewrite the expression as the product of sums/differences. * recognize polynomial expressions in one variable as a difference of squares and rewrite the expression as the product of sums/differences. * use the method of completing the square to transform a quadratic equation in x into an equation of the form (x - p)2 = q. * derive the quadratic formula from (x - p)2 = q. * solve a quadratic equation in one variable by inspection. * solve quadratic equations in one variable by taking square roots. * solve a quadratic equation in one variable by completing the square. * solve a quadratic equation in one variable using the quadratic formula. * solve a quadratic equation in one variable by factoring. * strategically select, as appropriate to the initial form of the equation, a method for solving a quadratic equation in one variable. * create quadratic equations in one variable. * use quadratic equations to solve real world problems. * interpret maximum/minimum and intercepts of quadratic functions from graphs and tables in the context of the problem. * sketch graphs of quadratic functions given a verbal description of the relationship between the quantities. * identify intercepts and intervals where function is increasing/decreasing * determine the practical domain of a function. * factor a quadratic expression for the purpose of revealing the zeros of a function. * complete the square for the purpose of revealing the maximum or minimum of a function. * identify and describe key features of the graphs of quadratic functions. * given two quadratic functions, each represented in a different way, compare the properties of the functions. * distinguish linear models representing approximately linear data from linear equations representing “perfectly” linear relationships. * create a scatter plot and sketch a line of best fit. * fit a linear function to data using technology. * solve problems using prediction equations. * interpret the slope and the intercepts of the linear model in context. * determine the correlation coefficient for the linear model using technology. * determine the direction and strength of the linear association between two variables. * identify and describe situations in which one quantity changes at a constant rate. * identify and describe situations in which a quantity grows or decays by a constant percent. * show that linear functions grow by equal differences over equal intervals. * show that exponential functions grow by equal factors over equal intervals. * create arithmetic and geometric sequences from verbal descriptions. * create arithmetic sequences from linear functions. * create geometric sequences from exponential functions. * identify recursively defined sequences as functions. * create linear and exponential functions given   + a graph;   + a description of a relationship;   + a table of values. * use the properties of exponents to simplify or expand exponential expressions, recognizing these are equivalent forms. * given a verbal description of a relationship, sketch linear and exponential functions. * identify intercepts and intervals where the function is positive/negative. * interpret parameters in context. * determine the domain of a function. * compare key features of two linear functions represented in different ways. * compare key features of two exponential functions represented in different ways. * calculate the rate of change from a table of values or from a function presented symbolically. * estimate the rate of change from a graph. * graph linear, square root, cube root, and piecewise-defined functions. * graph more complicated cases of functions using technology. * identify and describe key features of the graphs of square root, cube root, and piecewise-defined functions . * perform transformations on graphs of linear and quadratic functions. * identify the effect on the graph of replacing f(x) by   + f(x) + k;   + k f(x);− f(kx);   + and f(x + k) for specific values of k (both positive and negative). * identify the effect on the graph of combinations of transformations. * given the graph, find the value of k. * illustrate an explanation of the effects on linear and quadratic graphs using technology. * represent two or more data sets with plots and use appropriate statistics to compare their center and spread. * interpret differences in shape, center, and spread in context. * explain possible effects of extreme data points (outliers) when summarizing data and interpreting shape, center and spread. * explain possible associations between categorical data in two-way tables. * identify and describe trends in the data * fit a function to data using technology. * solve problems using functions fitted to data (prediction equations). * interpret the intercepts of models in context. * plot residuals of linear and non-linear functions. * analyze residuals in order to informally evaluate the fit of linear and non-linear functions. |

|  |
| --- |
| **STAGE 2 – Evidence of Learning** |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Formative Assessment Suggestions**   |  | | --- | | • 3- Minute Pause | | • A-B-C Summaries | | • Analogy Prompt | | • Choral Response | | • Debriefing | | • Exit Card / Ticket | | • Hand Signals | | • Idea Spinner | | • Index Card Summaries | | • Inside-Outside Circle Discussion (Fishbowl) | | • Journal Entry | | • Misconception Check | | • Observation | | • One Minute Essay | | • One Word Summary | | • Portfolio Check | | • Questions & Answers | | • Quiz | | • Self-Assessment | | • Student Conference | | • Think-Pair-Share | |  |   **Authentic Assessment Suggestions**  Through the following authentic assessments, students will develop traits regarding thinking and reasoning, settings, mathematical tools and attitudes and dispositions:   * Performance Assessments * Short Investigations * Open Ended Response Questions * Self-Assessments   **Assessments**  Chapter Tests |

|  |
| --- |
| **STAGE 3 – Learning Plan** |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Instructional Map**   |  | | --- | | **Modifications/Differentiation of Instruction** | | **Differentiation Strategies for Special Education Students**   * Remove unnecessary material, words, etc., that can distract from the content * Use of off-grade level materials * Provide appropriate scaffolding * Limit the number of steps required for completion * Time allowed * Level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Varied homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Ability to work at their own pace * Present ideas using auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment * Differentiated checklists and rubrics, if available and appropriate   **Differentiation Strategies for Gifted and Talented Students**   * Increase the level of complexity * Decrease scaffolding * Variety of finished products * Allow for greater independence * Learning stations, interest groups * Varied texts and supplementary materials * Use of technology * Flexibility in assignments * Varied questioning strategies * Encourage research * Strategy and flexible groups based on formative assessment or student choice * Acceleration within a unit of study * Exposure to more advanced or complex concepts, abstractions, and materials * Encourage students to move through content areas at their own pace * After mastery of a unit, provide students with more advanced learning activities, not more of the same activity * Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas   **Differentiated Strategies for ELL Students**   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials, including visuals * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Allow students to work at their own pace * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Role play * Provide graphic organizers, highlighted materials * Strategy and flexible groups based on formative assessment   **Differentiation Strategies for At Risk Students**   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessments   **504 Plans**  Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:   * walk, breathe, eat, or sleep * communicate, see, hear, or speak * read, concentrate, think, or learn * stand, bend, lift, or work   Examples of accommodations in 504 plans include:   * preferential seating * extended time on tests and assignments * reduced homework or classwork * verbal, visual, or technology aids * modified textbooks or audio-video materials * behavior management support * adjusted class schedules or grading * verbal testing * excused lateness, absence, or missed classwork * pre-approved nurse's office visits and accompaniment to visits * occupational or physical therapy |  |  | | --- | | **Modification Strategies** | | |  |  | | --- | --- | | • Extended Time | . | | • Frequent Breaks | . | | • Highlighted Text | . | | • Interactive Notebook | . | | • Modified Test | . | | • Oral Directions | . | | • Peer Tutoring | . | | • Preferential Seating | . | | • Re-Direct | . | | • Repeated Drill / Practice | . | | • Shortened Assignments | . | | • Teacher Notes | . | | • Tutorials | . | | • Use of Additional Reference Material | . | | • Use of Audio Resources | . | |  |  | | --- | | **High Preparation Differentiation** | | |  |  | | --- | --- | | • Alternative Assessments | . | | • Choice Boards | . | | • Games and Tournaments | . | | • Group Investigations | . | | • Guided Reading | . | | • Independent Research / Project | . | | • Interest Groups | . | | • Learning Contracts | . | | • Leveled Rubrics | . | | • Literature Circles | . | | • Menu Assignments | . | | • Multiple Intelligence Options | . | | • Multiple Texts | . | | • Personal Agendas | . | | • Project Based Learning (PBL) | . | | • Stations / Centers | . | | • Think-Tac-Toe | . | | • Tiered Activities / Assignments | . | | • Varying Graphic Organizers | . | |  |  | | --- | | **Low Preparation Differentiation** | | |  |  | | --- | --- | | • Choice of Book / Activity | . | | • Cubing Activities | . | | • Exploration by Interest (using interest inventories) | . | | • Flexible Grouping | . | | • Goal Setting With Student | . | | • Homework Options | . | | • Jigsaw | . | | • Mini Workshops to Extend Skills | . | | • Mini Workshops to Re-teach | . | | • Open-ended Activities | . | | • Think-Pair-Share by Interest | . | | • Think-Pair-Share by Learning Style | . | | • Think-Pair-Share by Learning Style | . | | • Think-Pair-Share by Readiness | . | | • Use of Collaboration | . | | • Use of Reading Buddies | . | | • Varied Journal Prompts | . | | • Varied Product Choice | . | | • Varied Supplemental Materials | . | | • Work Alone / Together | . | | |

|  |
| --- |
| **Additional Materials** |

|  |
| --- |
| LPS Adopted Textbooks and Programs  Savvas – Lial, Algebra for College Students, 9th Edition  Khan Academy  College Board |

**Interdisciplinary Connections & Standards**

With interdisciplinary instruction, the subject areas are woven together and explored through an overarching theme or concept. We use math to help us solve everyday problems in the kitchen, in the garden, and for many of us at our jobs.

Brain research has shown that information in our brains is organized in schematic structures. These structures are made up of interconnected bits of information and serve as a framework for the knowledge we acquire. When a learner’s knowledge is connected, it is much more likely that they will apply the prior knowledge to a wide variety of new situations. They will acquire new information in a way that is more accessible and will be better able to relate it to previously acquired knowledge.

Students learn about patterns in math, science, social studies, and even literature. Because of this, they are much more likely to “see” these patterns when they encounter new situations. Since patterns are not only studied in math they are able to make the connection and gain the understanding that patterns can be found in many areas of their lives. Interdisciplinary instruction allows students to understand the interconnectedness of the disciplines and makes learning more meaningful and relevant as fascinating connections are made across the subject areas.

**Science**

HS-LS3-3 Apply concepts of statistics and probability to explain the variation and distribution of expressed traits in a population.

HS-ESS3-2 Evaluate competing design solutions for developing, managing, and utilizing energy and mineral resources based on cost-benefit ratios.

HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

**Language Arts**

RL.11-12.1. Cite strong and thorough textual evidence and make relevant connections to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain.

RI.11-12.1. Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.

RI.11-12.2. Determine two or more central ideas of a text, and analyze their development and how they interact to provide a complex analysis; provide an objective summary of the text.

**Social Studies**

6.1.12.EconEM.2 Analyze how technological developments transformed the economy, created international markets, and affected the environment in New Jersey and the nation.

6.1.12.EconGE.16 Use quantitative data and other sources to assess the impact of international, global business organizations, and oversees competition on the United States economy and workforce.