**Subject**

**Pre-Calculus**

**Curriculum Guide**

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**LINDEN PUBLIC SCHOOLS**

**LINDEN, NEW JERSEY**

**DR. MARNIE HAZELTON**

**SUPERINTENDENT**

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**DIRECTOR OF MATHEMATICS, VOCATIONAL, & TECHNICAL SUBJECTS**

**The Linden Board of Education adopted the Curriculum Guide on:**

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| **July 28, 2022** |  | **Education Report #22** |
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| **Rationale** | | |

**EDUCATION EQUITY:** The Linden Public School District guarantees each student equal educational opportunity regardless of age, race, color, creed, religion, gender, language, affectional or sexual orientation, ancestry, national origin, marital or economic status. For Information, contact District Educational Equity Officer Kevin Thurston at **(**908) 486-2800 x 8307**.**

**NONDISCRIMATION:** The Linden Public School District does not discriminate against handicapped persons in admission or access to or treatment or employment in its programs, activities, and vocational opportunities. For information contact District Public 504 Officer Annabell Louis at (908) 486-2800 x 8025.

**Linden Public Schools Vision**

The Linden Public School District is committed to developing respect for diversity, excellence in education, and a commitment to service, in order to promote global citizenship and ensure personal success for all students

**Linden Public Schools Mission**

The mission of the Linden Public School District is to promote distinction through the infinite resource that is Linden’s diversity, combined with our profound commitment to instructional excellence, so that each and every student achieves their maximum potential in an engaging, inspiring, and challenging learning environment.

**Math Department Vision**

To equip students with the understanding and application of mathematical skills and processes to foster a drive for advanced mathematics and higher-level thinking.

**Math Department Mission Statement**

To develop a community of learners who construct and communicate meaning from the mathematical world around them. Students will experience mathematics that encourage them to think critically, discover and apply concepts to solve problems strategically. Students will be encouraged to solve equations with accuracy, efficiency, and flexibility. Furthermore, students will have a multitude of opportunities to apply mathematical tools and practice standards to solve real-world and multi-step problems.

**Math Department Goals**

* Provide opportunities for student to develop computation skills, conceptual understanding, and problem-solving skills
* Require students to explain, justify or prove their thinking through mathematical reasoning, modeling, and speaking

Course Description

This course is a one-year elective designed for college preparatory students interested in taking calculus. Topics covered include a review of real numbers, algebra, basic trigonometry and trigonometric analysis, logarithms, coordinate geometry, functional relationships and their graphs and sequences and series. A summer math project is required for all students entering this course.

Course Instructional Materials

* LPS Adopted Textbooks and Programs
  + Cengage: Precalculus with Limits: A Graphing Approach
  + WebAssign (Computer Based program supplementing Cengage)
* Khan Academy
* Edmentum Exact Path

Standards and NJDOE Mandates Guiding Instruction

* 1. New Jersey Student Learning Standards

<https://www.state.nj.us/education/cccs/>

Diversity, Equity, and Inclusion

* Use students’ interests in conceptualized tasks
* Expose students to a diverse group of mathematicians
* Design assessments and assignments with a variety of response types
* Use systematic grading and participation methods
* Encourage students to embrace a growth mindset

**2022 – 2023 Pre-Calculus Pacing Guide**

Please adhere to the following guide for Pre-Calculus. The number of days per chapter is an estimate for your planning. When assigning exercises for homework, concentrate on the level appropriate for your students, tiering whenever possible according to the ability levels within the class.

* Include a sample of Skill Practice but concentrate on the Problem-Solving selections for homework.
* Include the “Explore the Concept” activities in your lessons
* Include the “Graphing Calculator-Technology Tips” in your lessons
* Incorporate “Essential Question” provided for each section as lesson objective to provide better focus for student learning

**Grading Policy**

**Tests: 25% (2-3/ Marking Period)**

**Quizzes: 40% (8-12/ Marking Period)**

**Classwork/Homework: 35% (3-5/ Week)**

**Quarter 1**

**September 6, 2022 –November 15, 2022**

**Quarter 2**

**November 16, 2022 –January 31, 2023**

**Quarter 3**

**February 1, 2023 – April 5, 2023**

**Quarter 4**

**April 17, 2023– June 22, 2023**

**Throughout the year be sure to acknowledge the BIG IDEAS of each chapter and the Key Concepts of the sections with your students.**

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| Chapter 1 | Estimated Time: 10 days | |
| Big Ideas   1. Finding the slopes of lines and writing equations of lines 2. Identify, categorize, describe and transform the 6 most commonly used functions in Algebra 3. Investigate the composition of more than one function, the inverse and the use of functions in real world data | | |
|  | **Required:**   * Explore the Concept * Technology Tips * What’s wrong activities * Chapter Summary Big Ideas p 80-81 | Select 3 Monday Sessions  Standardized Test Practice  Families of Functions |
| Chapter 2 | Estimated Time: 15 days | |
| Big Ideas   1. Analyze and graph polynomial and rational functions 2. Create graphs of quadratic and other polynomial functions 3. Use polynomial division to find real and complex roots 4. Find asymptotes, intercepts and holes of rational functions | | |
|  | **Required:**   * Explore the Concept * Technology Tips * What’s wrong activities * Chapter Summary Big Ideas p 168-169 | Select 2 Monday Sessions  Standardized Test Practice |

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| Chapter 3 | Estimated Time: 14 days | |
| Big Ideas   1. Writing, graphing and recognizing exponential and logarithmic functions 2. Applications to real world problems including compound interest, decay and human memory 3. Use properties logarithms and exponents to manipulate expressions and solve equations | | |
|  | **Required:**   * Explore the Concept * Technology Tips * What’s wrong activities * Chapter Summary Big Ideas p 242-243 | Select 2 Monday Sessions  Placement Exam questions (exponents and logs) |
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| Chapter 4 | Estimated Time: 19 days | |
| Big Ideas   1. Evaluate and graph trigonometric functions, their inverses and reciprocals 2. Use of unit circle to explore angles in both degrees and radians 3. Use right triangle trigonometric ratios to solve real world applications | | |
|  | **Required:**   * Explore the Concept * Technology Tips * What’s wrong activities * Chapter Summary Big Ideas p 338-339 | Select 2 Monday Sessions  Standardized Test Practice  Parent Function Review p.346-347 |

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| Chapter 5 | Estimated Time: 13 days | |
| Big Ideas   1. Simplification and verification of trigonometric identities 2. Solve trigonometric equations in quadratic form 3. Evaluate other angles using half-angle, double-angle and sum and difference formulas | | |
|  | **Required:**   * Explore the Concept * Technology Tips * What’s wrong activities * Chapter Summary Big Ideas p 394-395 | Select 2 Monday Sessions  Standardized Test Practice |
| Chapter 6 | Estimated Time: 11 days | |
| Big Ideas   1. Use Law of Sines and Law of Cosines to find side lengths, angles and areas of oblique triangles 2. Apply trigonometry to vectors and vector notation to solve real world problems 3. Learn how to write and perform operations on complex numbers in trigonometric form | | |
|  | **Required:**   * Explore the Concept * Technology Tips * What’s wrong activities * Chapter Summary Big Ideas p 456-457 | Standardized Test Practice |
| **Midterm Common Assessment Chapters 1-6** | | |

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| Chapter 7 | Estimated Time: 18 days | |
| Big Ideas   1. Solve systems of 2 equations by substitution, graphing and eliminations 2. Recognize when a system has no solution, infinitely many solutions or exactly one (or more) solution(s) 3. Apply the same techniques to multivariable systems 4. Represent systems of equations with matrices, and to perform elementary row operations 5. Add, subtract, multiply and find the inverse of matrices 6. Use matrix inverses to solve systems? | | |
|  | **Required:**   * Explore the Concept * Technology Tips * What’s wrong activities * Chapter Summary Big Ideas p 558-559 | Standardized Test Practice |

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| Chapter 8 | Estimated Time: 15 days | | |
| Big Ideas   1. Model and find sums of arithmetic and geometric sequences 2. Expand binomials by using the Binomials Theorem and Pascal’s Triangle 3. Determine the probability of events | | | |
|  | **Required:**   * Explore the Concept * Technology Tips * What’s wrong activities * Chapter Summary Big Ideas p 626-627 | | Standardized Test Practice |
| Chapter 9 | Estimated Time: 17 days | | |
| Big Ideas   1. Solve problems involving conic sections 2. Classify a conic section by its equation in general form 3. Rotate a conic section in order to simplify its equation 4. Use polar coordinates to represent and solve conic sections | | | |
|  | | **Required:**   * Explore the Concept * Technology Tips * What’s wrong activities * Chapter Summary Big Ideas p 698-699 | Standardized Test Practice |
| Chapter 10 | | Estimated Time: 12 days | |
| Big Ideas   1. Describe and analyze points, vectors, lines and planes in three-dimensional space 2. Use parametric and symmetric equations to compare lines and planes in space | | | |
|  | | **Required:**   * Explore the Concept * Technology Tips * What’s wrong activities * Chapter Summary Big Ideas p 743 | Standardized Test Practice |
| Chapter 11 | | Estimated Time: 12 days |
| Big Ideas   1. Find tangent lines of a function and the area of a region 2. Calculate the limit of a graph at a given value of x 3. Find the slope if a graph at a given point and calculate the derivative of a function 4. Find the limit of functions at infinity and the limits of sequences 5. Find the area of a region bounded by a function | | | |
|  | | **Required:**   * Explore the Concept * Technology Tips * What’s wrong activities * Chapter Summary Big Ideas p 796-797 |
| **Common Final Exam** | | | |

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| **Career Ready Practices** |
| CRP2.   Apply appropriate academic and technical skills.  CRP4.   Communicate clearly and effectively and with reason.  CRP6.   Demonstrate creativity and innovation.  CRP8.   Utilize critical thinking to make sense of problems and persevere in solving them.  CRP11.   Use technology to enhance productivity.  CRP12.   Work productively in teams while using cultural global competence. |

**Unit 1: Review of Algebra Skills**

Content Area: **Mathematics**  
Course(s): **Pre-Calculus**  
Time Period: **First Marking Period**  
Length: **4 Week**  
Status: **Published**

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| **Unit Overview** |
| Algebraic concepts are built from basic set properties which can be applied to various sets of numbers. The basic laws of integer exponents can be extended to rational exponents.  Rational exponents can be used to rewrite radical expressions.  Polynomials can be rewritten in factored form.  Quadratic equations and inequalities can always be solved, even it if the quadratic is not factorable.  More complex equations require techniques that remove the variable from its construct (e.g., radical, fraction or absolute value symbol). Polynomial and rational inequalities are solved by finding critical values and testing intervals. |

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| **STAGE 1- DESIRED RESULTS** |
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| **Educational Standards** |
| The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:  1. Make sense of problems and persevre in solving them.  2. Reason abstractly and quantitatively.  3. Construct viable arguments and critique the reasoning of others.  4. Model with mathematics.  5. Use appropriate tools strategically.  6. Attend to precision.  7. Look for and make use of structure.  8. Look for and express regularity in repeated reasoning. |

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| **New Jersey Student Learning Standards- Mathematics** |
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| **Standard Set: Number and Quantity** |
| |  |  | | --- | --- | |  | During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers. | |  | With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. | |  | Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that (5 to the 1/3 power)³ should be 5 to the (1/3)³ power = 5¹ = 5 and that 5 to the 1/3 power should be the cube root of 5. | |  | Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents. | |  | In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them. | |

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| **Domain: The Real Number System** |
| |  |  | | --- | --- | | MA.N-RN.B.3 | Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. | | MA.N-RN.A | Extend the properties of exponents to rational exponents. | | MA.N-RN.A.1 | Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. | | MA.N-RN.A.2 | Rewrite expressions involving radicals and rational exponents using the properties of exponents. | | MA.N-RN.B | Use properties of rational and irrational numbers. | |

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| **Domain: Quantities** |
| |  |  | | --- | --- | | MA.N-Q.A | Reason quantitatively and use units to solve problems. | | MA.N-Q.A.1 | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | | MA.N-Q.A.2 | Define appropriate quantities for the purpose of descriptive modeling. | | MA.N-Q.A.3 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | |

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| **Domain: The Complex Number System** |
| |  |  | | --- | --- | | MA.N-CN.B.6 | Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. | | MA.N-CN.A | Perform arithmetic operations with complex numbers. | | MA.N-CN.A.1 | Know there is a complex number 𝑖 such that 𝑖² = -1, and every complex number has the form 𝑎 + 𝑏𝑖 with 𝑎 and 𝑏 real. | | MA.N-CN.A.2 | Use the relation 𝑖² = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. | | MA.N-CN.A.3 | Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. | | MA.N-CN.B | Represent complex numbers and their operations on the complex plane. | | MA.N-CN.B.4 | Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. | | MA.N-CN.B.5 | Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. | | MA.N-CN.C | Use complex numbers in polynomial identities and equations. | | MA.N-CN.C.7 | Solve quadratic equations with real coefficients that have complex solutions. | | MA.N-CN.C.8 | Extend polynomial identities to the complex numbers. | | MA.N-CN.C.9 | Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. | |

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| **Domain: Vector and Matrix Quantities** |
| |  |  | | --- | --- | | MA.N-VM.B.5a | Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as 𝑐(𝒗ₓ, 𝒗 subscript 𝑦) = (𝑐𝒗ₓ, 𝑐𝒗 subscript 𝑦). | | MA.N-VM.B.4c | Understand vector subtraction 𝒗 − 𝑤 as 𝒗 + (−𝑤), where –𝑤 is the additive inverse of 𝑤, with the same magnitude as 𝑤 and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. | | MA.N-VM.B.5b | Compute the magnitude of a scalar multiple 𝑐𝒗 using ||𝑐𝒗|| = |𝑐|𝒗. Compute the direction of 𝑐𝒗 knowing that when |𝑐|𝒗 ≠ 0, the direction of 𝑐𝒗 is either along 𝒗 (for 𝑐 > 0) or against 𝒗 (for 𝑐 < 0). | | MA.N-VM.A | Represent and model with vector quantities. | | MA.N-VM.C | Perform operations on matrices and use matrices in applications. | | MA.N-VM.C.6 | Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. | | MA.N-VM.C.7 | Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. | | MA.N-VM.C.8 | Add, subtract, and multiply matrices of appropriate dimensions. | | MA.N-VM.C.9 | Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. | | MA.N-VM.C.10 | Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. | | MA.N-VM.C.11 | Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. | | MA.N-VM.C.12 | Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area. | | MA.N-VM.A.1 | Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., 𝒗, |𝒗|, ||𝒗||, 𝒗). | | MA.N-VM.A.2 | Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. | | MA.N-VM.A.3 | Solve problems involving velocity and other quantities that can be represented by vectors. | | MA.N-VM.B | Perform operations on vectors. | | MA.N-VM.B.4 | Add and subtract vectors. | | MA.N-VM.B.5 | Multiply a vector by a scalar. | | MA.N-VM.B.4a | Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. | | MA.N-VM.B.4b | Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. | |

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| **Standard Set: Statistics and Probability** |
| |  |  | | --- | --- | |  | Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account. | |  | Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. | |  | Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. | |  | Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. | |  | Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time. | |  | Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient. | |

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| **Domain: Interpreting Categorical and Quantitative Data** |
| |  |  | | --- | --- | | MA.S-ID.B.6c | Fit a linear function for a scatter plot that suggests a linear association. | | MA.S-ID.C | Interpret linear models | | MA.S-ID.C.7 | Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | | MA.S-ID.C.8 | Compute (using technology) and interpret the correlation coefficient of a linear fit. | | MA.S-ID.C.9 | Distinguish between correlation and causation. | | MA.S-ID.A | Summarize, represent, and interpret data on a single count or measurement variable | | MA.S-ID.A.1 | Represent data with plots on the real number line (dot plots, histograms, and box plots). | | MA.S-ID.A.2 | Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | | MA.S-ID.A.3 | Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | | MA.S-ID.A.4 | Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. | | MA.S-ID.B | Summarize, represent, and interpret data on two categorical and quantitative variables | | MA.S-ID.B.5 | Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. | | MA.S-ID.B.6 | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. | | MA.S-ID.B.6a | Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. | | MA.S-ID.B.6b | Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology. | |

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| **Domain: Making Inferences and Justifying Conclusions** |
| |  |  | | --- | --- | | MA.S-IC.A | Understand and evaluate random processes underlying statistical experiments | | MA.S-IC.A.1 | Understand statistics as a process for making inferences about population parameters based on a random sample from that population. | | MA.S-IC.A.2 | Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. | | MA.S-IC.B | Make inferences and justify conclusions from sample surveys, experiments, and observational studies | | MA.S-IC.B.3 | Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. | | MA.S-IC.B.4 | Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. | | MA.S-IC.B.5 | Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. | | MA.S-IC.B.6 | Evaluate reports based on data. | |

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| **Domain: Conditional Probability and the Rules of Probability** |
| |  |  | | --- | --- | | MA.S-CP.B | Use the rules of probability to compute probabilities of compound events in a uniform probability model | | MA.S-CP.B.6 | Find the conditional probability of 𝐴 given 𝐵 as the fraction of 𝐵’s outcomes that also belong to 𝐴, and interpret the answer in terms of the model. | | MA.S-CP.B.7 | Apply the Addition Rule, 𝑃(𝐴 𝑜𝑟 𝐵) = 𝑃(𝐴) + 𝑃(𝐵) – 𝑃(𝐴 𝑎𝑛𝑑 𝐵), and interpret the answer in terms of the model. | | MA.S-CP.B.8 | Apply the general Multiplication Rule in a uniform probability model, 𝑃(𝐴 𝑎𝑛𝑑 𝐵) = [𝑃(𝐴)] × [𝑃(𝐵|𝐴)] = [𝑃(𝐵)] × [𝑃(𝐴|𝐵)], and interpret the answer in terms of the model. | | MA.S-CP.B.9 | Use permutations and combinations to compute probabilities of compound events and solve problems. | | MA.S-CP.A | Understand independence and conditional probability and use them to interpret data | | MA.S-CP.A.1 | Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). | | MA.S-CP.A.2 | Understand that two events 𝐴 and 𝐵 are independent if the probability of 𝐴 and 𝐵 occurring together is the product of their probabilities, and use this characterization to determine if they are independent. | | MA.S-CP.A.3 | Understand the conditional probability of 𝐴 given 𝐵 as 𝑃(𝐴 𝑎𝑛𝑑 𝐵)/𝑃(𝐵), and interpret independence of 𝐴 and 𝐵 as saying that the conditional probability of 𝐴 given 𝐵 is the same as the probability of 𝐴, and the conditional probability of 𝐵 given 𝐴 is the same as the probability of 𝐵. | | MA.S-CP.A.4 | Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. | | MA.S-CP.A.5 | Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. | |

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| **Domain: Using Probability to Make Decisions** |
| |  |  | | --- | --- | | MA.S-MD.B.5 | Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. | | MA.S-MD.B.5a | Find the expected payoff for a game of chance. | | MA.S-MD.B.5b | Evaluate and compare strategies on the basis of expected values. | | MA.S-MD.A | Calculate expected values and use them to solve problems | | MA.S-MD.B.6 | Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). | | MA.S-MD.A.1 | Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. | | MA.S-MD.B.7 | Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). | | MA.S-MD.A.2 | Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. | | MA.S-MD.A.3 | Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. | | MA.S-MD.A.4 | Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. | | MA.S-MD.B | Use probability to evaluate outcomes of decisions | |

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| **Standard Set: Geometry** |
| |  |  | | --- | --- | |  | Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations. | |  | Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. | |  | The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. | |  | An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. | |  | Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) | |  | During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. | |  | The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. | |  | In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. | |  | Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. | |  | The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion. | |

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| **Domain: Congruence** |
| |  |  | | --- | --- | | MA.G-CO.A.1 | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. | | MA.G-CO.A.2 | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). | | MA.G-CO.A.3 | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. | | MA.G-CO.A.4 | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. | | MA.G-CO.A.5 | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. | | MA.G-CO.D.12 | Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). | | MA.G-CO.B | Understand congruence in terms of rigid motions | | MA.G-CO.D.13 | Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. | | MA.G-CO.B.6 | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. | | MA.G-CO.B.7 | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. | | MA.G-CO.B.8 | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. | | MA.G-CO.C | Prove geometric theorems | | MA.G-CO.C.9 | Prove theorems about lines and angles. | | MA.G-CO.C.10 | Prove theorems about triangles. | | MA.G-CO.C.11 | Prove theorems about parallelograms. | | MA.G-CO.D | Make geometric constructions | | MA.G-CO.A | Experiment with transformations in the plane | |

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| **Domain: Similarity, Right Triangles, and Trigonometry** |
| |  |  | | --- | --- | | MA.G-SRT.D.9 | Derive the formula 𝐴 = (1/2)𝑎𝑏 𝑠𝑖𝑛(𝐶) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. | | MA.G-SRT.D.10 | Prove the Laws of Sines and Cosines and use them to solve problems. | | MA.G-SRT.D.11 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). | | MA.G-SRT.A | Understand similarity in terms of similarity transformations | | MA.G-SRT.A.1 | Verify experimentally the properties of dilations given by a center and a scale factor: | | MA.G-SRT.A.1a | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. | | MA.G-SRT.A.1b | The dilation of a line segment is longer or shorter in the ratio given by the scale factor. | | MA.G-SRT.A.2 | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. | | MA.G-SRT.A.3 | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. | | MA.G-SRT.B | Prove theorems involving similarity | | MA.G-SRT.B.4 | Prove theorems about triangles. | | MA.G-SRT.B.5 | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | | MA.G-SRT.C | Define trigonometric ratios and solve problems involving right triangles | | MA.G-SRT.C.6 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. | | MA.G-SRT.C.7 | Explain and use the relationship between the sine and cosine of complementary angles. | | MA.G-SRT.C.8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. | | MA.G-SRT.D | Apply trigonometry to general triangles | |

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| **Domain: Circles** |
| |  |  | | --- | --- | | MA.G-C.A | Understand and apply theorems about circles | | MA.G-C.A.1 | Prove that all circles are similar. | | MA.G-C.A.2 | Identify and describe relationships among inscribed angles, radii, and chords. | | MA.G-C.A.3 | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. | | MA.G-C.A.4 | Construct a tangent line from a point outside a given circle to the circle. | | MA.G-C.B | Find arc lengths and areas of sectors of circles | | MA.G-C.B.5 | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. | |

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| **Domain: Expressing Geometric Properties with Equations** |
| |  |  | | --- | --- | | MA.G-GPE.B.4 | Use coordinates to prove simple geometric theorems algebraically. | | MA.G-GPE.B.5 | Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). | | MA.G-GPE.B.6 | Find the point on a directed line segment between two given points that partitions the segment in a given ratio. | | MA.G-GPE.B.7 | Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. | | MA.G-GPE.A | Translate between the geometric description and the equation for a conic section | | MA.G-GPE.A.1 | Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. | | MA.G-GPE.A.2 | Derive the equation of a parabola given a focus and directrix. | | MA.G-GPE.A.3 | Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. | | MA.G-GPE.B | Use coordinates to prove simple geometric theorems algebraically | |

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| **Domain: Geometric Measurement and Dimension** |
| |  |  | | --- | --- | | MA.G-GMD.B.4 | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. | | MA.G-GMD.A | Explain volume formulas and use them to solve problems | | MA.G-GMD.A.1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. | | MA.G-GMD.A.2 | Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures. | | MA.G-GMD.A.3 | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. | | MA.G-GMD.B | Visualize relationships between two-dimensional and three-dimensional objects | |

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| **Domain: Modeling with Geometry** |
| |  |  | | --- | --- | | MA.G-MG.A | Apply geometric concepts in modeling situations | | MA.G-MG.A.1 | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). | | MA.G-MG.A.2 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). | | MA.G-MG.A.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). | |

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| **Standard Set: Modeling** |
| |  |  | | --- | --- | |  | Designing the layout of the stalls in a school fair so as to raise as much money as possible. | |  | Analyzing stopping distance for a car. | |  | Modeling savings account balance, bacterial colony growth, or investment growth. | |  | Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport. | |  | Analyzing risk in situations such as extreme sports, pandemics, and terrorism. | |  | Relating population statistics to individual predictions. | |  | Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). | |  | Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. | |  | A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity. | |  | In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations. | |  | Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed. | |  | Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player. | |

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| **Standard Set: Functions** |
| |  |  | | --- | --- | |  | Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. | |  | In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car’s speed in miles per hour, v; the rule T(v) = 100/v expresses this relationship algebraically and defines a function whose name is T. | |  | The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. | |  | A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, “I’ll give you a state, you give me the capital city;” by an algebraic expression like f(x) = a + bx; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function’s properties. | |  | Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships. | |  | A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions. | |  | Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology. | |

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| **Domain: Interpreting Functions** |
| |  |  | | --- | --- | | MA.F-IF.C.8b | Use the properties of exponents to interpret expressions for exponential functions. | | MA.F-IF.A | Understand the concept of a function and use function notation | | MA.F-IF.A.1 | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If 𝑓 is a function and 𝑥 is an element of its domain, then 𝑓(𝑥) denotes the output of 𝑓 corresponding to the input 𝑥. The graph of 𝑓 is the graph of the equation𝑦 = 𝑓(𝑥). | | MA.F-IF.A.2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | | MA.F-IF.C.9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). | | MA.F-IF.A.3 | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. | | MA.F-IF.B | Interpret functions that arise in applications in terms of the context | | MA.F-IF.B.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. | | MA.F-IF.B.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. | | MA.F-IF.B.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | | MA.F-IF.C | Analyze functions using different representations | | MA.F-IF.C.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | | MA.F-IF.C.7a | Graph linear and quadratic functions and show intercepts, maxima, and minima. | | MA.F-IF.C.7b | Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. | | MA.F-IF.C.7c | Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7d | Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7e | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | | MA.F-IF.C.8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. | | MA.F-IF.C.8a | Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | |

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| **Domain: Building Functions** |
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| **Domain: Linear, Quadratic, and Exponential Models** |
| |  |  | | --- | --- | | MA.F-LE.A | Construct and compare linear and exponential models and solve problems | | MA.F-LE.A.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions. | | MA.F-LE.A.1a | Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. | | MA.F-LE.A.1b | Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. | | MA.F-LE.A.1c | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | | MA.F-LE.A.2 | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). | | MA.F-LE.A.3 | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | | MA.F-LE.A.4 | Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to 𝑎𝑏 to the 𝑐𝘵 power = 𝑑 where 𝑎, 𝑐, and 𝑑 are numbers and the base 𝑏 is 2, 10, or 𝑒; evaluate the logarithm using technology. | | MA.F-LE.B | Interpret expressions for functions in terms of the situation they model | | MA.F-LE.B.5 | Interpret the parameters in a linear or exponential function in terms of a context. | |

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| **Domain: Trigonometric Functions** |
| |  |  | | --- | --- | | MA.F-TF.A.3 | Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π – 𝑥, π + 𝑥, and 2π – 𝑥 in terms of their values for 𝑥, where 𝑥 is any real number. | | MA.F-TF.A.4 | Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. | | MA.F-TF.B | Model periodic phenomena with trigonometric functions | | MA.F-TF.B.5 | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. | | MA.F-TF.B.6 | Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. | | MA.F-TF.B.7 | Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. | | MA.F-TF.C | Prove and apply trigonometric identities | | MA.F-TF.C.8 | Prove the Pythagorean identity 𝑠𝑖𝑛²(θ) + 𝑐𝑜𝑠²(θ) = 1 and use it to find 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) given 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) and the quadrant of the angle. | | MA.F-TF.C.9 | Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. | | MA.F-TF.A | Extend the domain of trigonometric functions using the unit circle | | MA.F-TF.A.1 | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. | | MA.F-TF.A.2 | Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. | |

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| **Standard Set: Algebra** |
| |  |  | | --- | --- | |  | An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances. | |  | Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor. | |  | Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure. | |  | A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave. | |  | An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. | |  | The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system. | |  | An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. | |  | Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x² – 2 = 0 are real numbers, not rational numbers; and the solutions of x² + 2 = 0 are complex numbers, not real numbers. | |  | Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling. | |  | The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, 𝘈 = ((𝘣₁+𝘣₂)/2)𝘩, can be solved for 𝘩 using the same deductive process. | |  | Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them. | |

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| **Domain: Seeing Structures in Expressions** |
| |  |  | | --- | --- | | MA.A-SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. For example, see 𝑥⁴ – 𝑦⁴ as (𝑥²)² – (𝑦²)², thus recognizing it as a difference of squares that can be factored as (𝑥² – 𝑦²)(𝑥² + 𝑦²). | | MA.A-SSE.B | Write expressions in equivalent forms to solve problems | | MA.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. | | MA.A-SSE.B.3a | Factor a quadratic expression to reveal the zeros of the function it defines. | | MA.A-SSE.B.3b | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. | | MA.A-SSE.A | Interpret the structure of expressions | | MA.A-SSE.B.3c | Use the properties of exponents to transform expressions for exponential functions. | | MA.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context. | | MA.A-SSE.A.1a | Interpret parts of an expression, such as terms, factors, and coefficients. | | MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. | | MA.A-SSE.A.1b | Interpret complicated expressions by viewing one or more of their parts as a single entity. | |

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| **Domain: Arithmetic with Polynomials and Rational Functions** |
| |  |  | | --- | --- | | MA.A-APR.C.4 | Prove polynomial identities and use them to describe numerical relationships. | | MA.A-APR.A | Perform arithmetic operations on polynomials | | MA.A-APR.A.1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | | MA.A-APR.B | Understand the relationship between zeros and factors of polynomials | | MA.A-APR.B.2 | Know and apply the Remainder Theorem: For a polynomial 𝑝(𝑥) and a number 𝑎, the remainder on division by 𝑥 – 𝑎 is 𝑝(𝑎), so 𝑝(𝑎) = 0 if and only if (𝑥 – 𝑎) is a factor of 𝑝(𝑥). | | MA.A-APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. | | MA.A-APR.C.5 | Know and apply the Binomial Theorem for the expansion of (𝑥 + 𝑦)ⁿ in powers of 𝑥 and 𝑦 for a positive integer 𝑛, where 𝑥 and 𝑦 are any numbers, with coefficients determined for example by Pascal’s Triangle. | | MA.A-APR.D | Rewrite rational expressions | | MA.A-APR.D.6 | Rewrite simple rational expressions in different forms; write 𝑎(𝑥)/𝑏(𝑥) in the form 𝑞(𝑥) + 𝑟(𝑥)/𝑏(𝑥), where 𝑎(𝑥), 𝑏(𝑥), 𝑞(𝑥), and 𝑟(𝑥) are polynomials with the degree of 𝑟(𝑥) less than the degree of 𝑏(𝑥), using inspection, long division, or, for the more complicated examples, a computer algebra system. | | MA.A-APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. | | MA.A-APR.C | Use polynomial identities to solve problems | |

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| **Domain: Creating Equations** |
| |  |  | | --- | --- | | MA.A-CED.A | Create equations that describe numbers or relationships | | MA.A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems. | | MA.A-CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | | MA.A-CED.A.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. | | MA.A-CED.A.4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. | |

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| **Domain: Reasoning with Equations and Inequalities** |
| |  |  | | --- | --- | | MA.A-REI.B | Solve equations and inequalities in one variable | | MA.A-REI.B.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. | | MA.A-REI.B.4 | Solve quadratic equations in one variable. | | MA.A-REI.C | Solve systems of equations | | MA.A-REI.C.5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. | | MA.A-REI.C.6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. | | MA.A-REI.C.7 | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. | | MA.A-REI.C.8 | Represent a system of linear equations as a single matrix equation in a vector variable. | | MA.A-REI.C.9 | Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). | | MA.A-REI.D | Represent and solve equations and inequalities graphically | | MA.A-REI.B.4a | Use the method of completing the square to transform any quadratic equation in 𝑥 into an equation of the form (𝑥 – 𝑝)² = 𝑞 that has the same solutions. Derive the quadratic formula from this form. | | MA.A-REI.D.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). | | MA.A-REI.B.4b | Solve quadratic equations by inspection (e.g., for 𝑥² = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as 𝑎 ± 𝑏𝑖 for real numbers 𝑎 and 𝑏. | | MA.A-REI.A | Understand solving equations as a process of reasoning and explain the reasoning | | MA.A-REI.D.12 | Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | | MA.A-REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | | MA.A-REI.D.11 | Explain why the 𝑥-coordinates of the points where the graphs of the equations 𝑦 = 𝑓(𝑥) and 𝑦 = 𝑔(𝑥) intersect are the solutions of the equation𝑓(𝑥) = 𝑔(𝑥); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where 𝑓(𝑥) and/or 𝑔(𝑥) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | | MA.A-REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. | |

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| **Essential Questions** |
| How are different sets of numbers (natural numbers, real numbers, integers, etc.) related?  What is the meaning of a rational exponent? a negative exponent?  How can radical forms be converted exponential forms and vice-versa?  What transformations produce equivalent equations and inequalities?  How can solutions to linear inequalities be graphically represented on a number line?  What is the meaning of absolute value?  How can linear equations and inequalities be applied to the solution of word problems? |

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| **Enduring Understanding** |
| Algebraic concepts are built from a basic set properties which can be applied to various sets of numbers. The basic laws of integer exponents can be extended to rational exponents.  Rational exponents can be used to rewrite radical expressions.  Polynomials can be rewritten in factored form.  Quadratic equations and inequalities can always be solved, even it if the quadratic is not factorable.  More complex equations require techniques that remove the variable from its construct (e.g., radical, fraction or absolute value symbol). Polynomial and rational inequalities are solved by finding critical values and testing intervals. |

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| **Students will know...** |
| 1. The Real Number System 2. Exponents and Radicals 3. Polynomials: Special Products and Factoring 4. Fractional Expressions 5. Solving Equations and Inequalities |

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| **Students will be able to...** |
| 1. Distinguish between rational and irrational numbers. 2. Add, subtract, multiply and divide positive and negative numbers. 3. Use number properties to write equivalent expressions. 4. Evaluate algebraic expressions. 5. Simplify expressions with exponents and radicals. 6. Use exponential notation. 7. Find the nth root of a number. 8. Multiply, divide, add, or subtract radical expressions. 9. Find special products including sum and difference of same   terms, square and cube of a binomial. 10. Factor polynomials with a common factor. 11. Identify and factor special types of polynomials including trinomial square, difference of squares, and sum or difference of cubes. 12. Find the domain of an expression. 13. Reduce, multiply, divide, subtract, and combine rational expressions. 14. Simplify compound fractions. 15. Solve equations using factoring, square root principle, completing the square, and quadratic formula. 16. Solve equations involving radicals and absolute value. 17. Solve inequalities and give the answers in interval notation. 18. Identify and avoid common algebraic errors. |

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| **STAGE 2- EVIDENCE OF LEARNING** |
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| **Formative Assessment Suggestions** |
| Edementum Exact Path (BOY, MOY, EOY)  Class Discussions  Do Nows  Notebook Checks  Homework  Presentations  Webassign   |  |  | | --- | --- | | • 3- Minute Pause | **.** | | • A-B-C Summaries | **.** | | • Analogy Prompt | **.** | | • Choral Response | **.** | | • Debriefing | **.** | | • Exit Card / Ticket | **.** | | • Hand Signals | **.** | | • Idea Spinner | **.** | | • Index Card Summaries | **.** | | • Inside-Outside Circle Discussion (Fishbowl) | **.** | | • Journal Entry | **.** | | • Misconception Check | **.** | | • Observation | **.** | | • One Minute Essay | **.** | | • One Word Summary | **.** | | • Portfolio Check | **.** | | • Questions & Answers | **.** | | • Quiz | **.** | | • Self-Assessment | **.** | | • Student Conference | **.** | | • Think-Pair-Share | **.** | | • Web or Concept Map | **.** | |

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| **Authentic Assessments Suggestions** |
| Edementum Exact Path (BOY, MOY, EOY)  Class Discussions  Do Nows  Notebook Checks  Homework  Presentations  Webassign |

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| **Benchmark Assessments** |
| Edementum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3- LEARNING PLAN** |
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| **Instructional Map** |
| Topic #1 – The Real Number System  Topic #2 – Exponents and Radicals  Topic #3 – Polynomials: Special Products and Factoring  Topic #4 – Fractional Expressions  Topic #5 – Solving Equations and Inequalities |

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| **Modifications/Differentiation of Instruction** |
| Differentiation Strategies for Special Education Students   * Remove unnecessary material, words, etc., that can distract from the content * Use of off-grade level materials * Provide appropriate scaffolding * Limit the number of steps required for completion * Time allowed * Level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Varied homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Ability to work at their own pace * Present ideas using auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment * Differentiated checklists and rubrics, if available and appropriate       Differentiation Strategies for Gifted and Talented Students   * Increase the level of complexity * Decrease scaffolding * Variety of finished products * Allow for greater independence * Learning stations, interest groups * Varied texts and supplementary materials * Use of technology * Flexibility in assignments * Varied questioning strategies * Encourage research * Strategy and flexible groups based on formative assessment or student choice * Acceleration within a unit of study * Exposure to more advanced or complex concepts, abstractions, and materials * Encourage students to move through content areas at their own pace * After mastery of a unit, provide students with more advanced learning activities, not more of the same activity * Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas     Differentiated Strategies for ELL Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials, including visuals * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Allow students to work at their own pace * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Role play * Provide graphic organizers, highlighted materials * Strategy and flexible groups based on formative assessment     Differentiation Strategies for At Risk Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment       **504 Plans**  Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:   * walk, breathe, eat, or sleep * communicate, see, hear, or speak * read, concentrate, think, or learn * stand, bend, lift, or work     Examples of accommodations in 504 plans include:   * preferential seating * extended time on tests and assignments * reduced homework or classwork * verbal, visual, or technology aids * modified textbooks or audio-video materials * behavior management support * adjusted class schedules or grading * verbal testing * excused lateness, absence, or missed classwork * pre-approved nurse's office visits and accompaniment to visits * occupational or physical therapy |

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| **Modification Strategies** |
| |  |  | | --- | --- | | • Extended Time | **.** | | • Frequent Breaks | **.** | | • Highlighted Text | **.** | | • Interactive Notebook | **.** | | • Modified Test | **.** | | • Oral Directions | **.** | | • Peer Tutoring | **.** | | • Preferential Seating | **.** | | • Re-Direct | **.** | | • Repeated Drill / Practice | **.** | | • Shortened Assignments | **.** | | • Teacher Notes | **.** | | • Tutorials | **.** | | • Use of Additional Reference Material | **.** | | • Use of Audio Resources | **.** | |

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| **High Preparation Differentiation** |
| |  |  | | --- | --- | | • Alternative Assessments | **.** | | • Choice Boards | **.** | | • Games and Tournaments | **.** | | • Group Investigations | **.** | | • Guided Reading | **.** | | • Independent Research / Project | **.** | | • Interest Groups | **.** | | • Learning Contracts | **.** | | • Leveled Rubrics | **.** | | • Literature Circles | **.** | | • Menu Assignments | **.** | | • Multiple Intelligence Options | **.** | | • Multiple Texts | **.** | | • Personal Agendas | **.** | | • Project Based Learning (PBL) | **.** | | • Stations / Centers | **.** | | • Think-Tac-Toe | **.** | | • Tiered Activities / Assignments | **.** | | • Varying Graphic Organizers | **.** | |

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| **Low Preparation Differentiation** |
| |  |  | | --- | --- | | • Choice of Book / Activity | **.** | | • Cubing Activities | **.** | | • Exploration by Interest (using interest inventories) | **.** | | • Flexible Grouping | **.** | | • Goal Setting With Student | **.** | | • Homework Options | **.** | | • Jigsaw | **.** | | • Mini Workshops to Extend Skills | **.** | | • Mini Workshops to Re-teach | **.** | | • Open-ended Activities | **.** | | • Think-Pair-Share by Interest | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Readiness | **.** | | • Use of Collaboration | **.** | | • Use of Reading Buddies | **.** | | • Varied Journal Prompts | **.** | | • Varied Product Choice | **.** | | • Varied Supplemental Materials | **.** | | • Work Alone / Together | **.** | |

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| **Horizontal Integration- Interdisciplinary Connections** |
| Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other. |

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| **Vertical Integration- Discipline Mapping** |
| The standards in this unit were introduced in Algebra I, Geometry and Algebra II. Precalculus coursework focuses on preparing the students be proficient in Calculus standards. |

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| **Additional Materials** |
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LPS Adopted Textbooks and Programs

* Cengage: Precalculus with Limits: A Graphing Approach
* WebAssign (Computer Based program supplementing Cengage)

Khan Academy

Edmentum Exact Path

**Unit 2: Functions and Graphs**

Content Area: **Mathematics**  
Course(s): **Pre-Calculus**  
Time Period: **First Marking Period**  
Length: **4 Weeks**  
Status: **Published**

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| **Unit Overview** |
| Functions are an important topic in the study of mathematics.  Functions may be defined and described in various ways one of which is a two-dimensional graph generated from a table of input/output values. Two functions can be combined to produce functions that are more complex.  Such combinations impact the graph, domain, and range of the functions. Functions can be inverted to create new functions.   Linear functions and those involving variation can be used to model real world problems. |

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| **STAGE 1- DESIRED RESULTS** |
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| **Educational Standards** |
| The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:  1. Make sense of problems and persevre in solving them.  2. Reason abstractly and quantitatively.  3. Construct viable arguments and critique the reasoning of others.  4. Model with mathematics.  5. Use appropriate tools strategically.  6. Attend to precision.  7. Look for and make use of structure.  8. Look for and express regularity in repeated reasoning. |

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| **New Jersey Student Learning Standards- Mathematics** |
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| **Standard Set: Number and Quantity** |
| |  |  | | --- | --- | |  | During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers. | |  | With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. | |  | Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that (5 to the 1/3 power)³ should be 5 to the (1/3)³ power = 5¹ = 5 and that 5 to the 1/3 power should be the cube root of 5. | |  | Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents. | |  | In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them. | |

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| **Domain: The Real Number System** |
| |  |  | | --- | --- | | MA.N-RN.B.3 | Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. | | MA.N-RN.A | Extend the properties of exponents to rational exponents. | | MA.N-RN.A.1 | Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. | | MA.N-RN.A.2 | Rewrite expressions involving radicals and rational exponents using the properties of exponents. | | MA.N-RN.B | Use properties of rational and irrational numbers. | |

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| **Domain: Quantities** |
| |  |  | | --- | --- | | MA.N-Q.A | Reason quantitatively and use units to solve problems. | | MA.N-Q.A.1 | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | | MA.N-Q.A.2 | Define appropriate quantities for the purpose of descriptive modeling. | | MA.N-Q.A.3 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | |

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| **Domain: The Complex Number System** |
| |  |  | | --- | --- | | MA.N-CN.B.6 | Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. | | MA.N-CN.A | Perform arithmetic operations with complex numbers. | | MA.N-CN.A.1 | Know there is a complex number 𝑖 such that 𝑖² = -1, and every complex number has the form 𝑎 + 𝑏𝑖 with 𝑎 and 𝑏 real. | | MA.N-CN.A.2 | Use the relation 𝑖² = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. | | MA.N-CN.A.3 | Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. | | MA.N-CN.B | Represent complex numbers and their operations on the complex plane. | | MA.N-CN.B.4 | Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. | | MA.N-CN.B.5 | Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. | | MA.N-CN.C | Use complex numbers in polynomial identities and equations. | | MA.N-CN.C.7 | Solve quadratic equations with real coefficients that have complex solutions. | | MA.N-CN.C.8 | Extend polynomial identities to the complex numbers. | | MA.N-CN.C.9 | Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. | |

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| **Domain: Vector and Matrix Quantities** |
| |  |  | | --- | --- | | MA.N-VM.B.5a | Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as 𝑐(𝒗ₓ, 𝒗 subscript 𝑦) = (𝑐𝒗ₓ, 𝑐𝒗 subscript 𝑦). | | MA.N-VM.B.4c | Understand vector subtraction 𝒗 − 𝑤 as 𝒗 + (−𝑤), where –𝑤 is the additive inverse of 𝑤, with the same magnitude as 𝑤 and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. | | MA.N-VM.B.5b | Compute the magnitude of a scalar multiple 𝑐𝒗 using ||𝑐𝒗|| = |𝑐|𝒗. Compute the direction of 𝑐𝒗 knowing that when |𝑐|𝒗 ≠ 0, the direction of 𝑐𝒗 is either along 𝒗 (for 𝑐 > 0) or against 𝒗 (for 𝑐 < 0). | | MA.N-VM.A | Represent and model with vector quantities. | | MA.N-VM.C | Perform operations on matrices and use matrices in applications. | | MA.N-VM.C.6 | Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. | | MA.N-VM.C.7 | Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. | | MA.N-VM.C.8 | Add, subtract, and multiply matrices of appropriate dimensions. | | MA.N-VM.C.9 | Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. | | MA.N-VM.C.10 | Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. | | MA.N-VM.C.11 | Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. | | MA.N-VM.C.12 | Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area. | | MA.N-VM.A.1 | Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., 𝒗, |𝒗|, ||𝒗||, 𝒗). | | MA.N-VM.A.2 | Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. | | MA.N-VM.A.3 | Solve problems involving velocity and other quantities that can be represented by vectors. | | MA.N-VM.B | Perform operations on vectors. | | MA.N-VM.B.4 | Add and subtract vectors. | | MA.N-VM.B.5 | Multiply a vector by a scalar. | | MA.N-VM.B.4a | Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. | | MA.N-VM.B.4b | Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. | |

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| **Standard Set: Statistics and Probability** |
| |  |  | | --- | --- | |  | Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account. | |  | Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. | |  | Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. | |  | Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. | |  | Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time. | |  | Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient. | |

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| **Domain: Interpreting Categorical and Quantitative Data** |
| |  |  | | --- | --- | | MA.S-ID.B.6c | Fit a linear function for a scatter plot that suggests a linear association. | | MA.S-ID.C | Interpret linear models | | MA.S-ID.C.7 | Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | | MA.S-ID.C.8 | Compute (using technology) and interpret the correlation coefficient of a linear fit. | | MA.S-ID.C.9 | Distinguish between correlation and causation. | | MA.S-ID.A | Summarize, represent, and interpret data on a single count or measurement variable | | MA.S-ID.A.1 | Represent data with plots on the real number line (dot plots, histograms, and box plots). | | MA.S-ID.A.2 | Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | | MA.S-ID.A.3 | Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | | MA.S-ID.A.4 | Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. | | MA.S-ID.B | Summarize, represent, and interpret data on two categorical and quantitative variables | | MA.S-ID.B.5 | Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. | | MA.S-ID.B.6 | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. | | MA.S-ID.B.6a | Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. | | MA.S-ID.B.6b | Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology. | |

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| **Domain: Making Inferences and Justifying Conclusions** |
| |  |  | | --- | --- | | MA.S-IC.A | Understand and evaluate random processes underlying statistical experiments | | MA.S-IC.A.1 | Understand statistics as a process for making inferences about population parameters based on a random sample from that population. | | MA.S-IC.A.2 | Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. | | MA.S-IC.B | Make inferences and justify conclusions from sample surveys, experiments, and observational studies | | MA.S-IC.B.3 | Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. | | MA.S-IC.B.4 | Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. | | MA.S-IC.B.5 | Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. | | MA.S-IC.B.6 | Evaluate reports based on data. | |

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| **Domain: Conditional Probability and the Rules of Probability** |
| |  |  | | --- | --- | | MA.S-CP.B | Use the rules of probability to compute probabilities of compound events in a uniform probability model | | MA.S-CP.B.6 | Find the conditional probability of 𝐴 given 𝐵 as the fraction of 𝐵’s outcomes that also belong to 𝐴, and interpret the answer in terms of the model. | | MA.S-CP.B.7 | Apply the Addition Rule, 𝑃(𝐴 𝑜𝑟 𝐵) = 𝑃(𝐴) + 𝑃(𝐵) – 𝑃(𝐴 𝑎𝑛𝑑 𝐵), and interpret the answer in terms of the model. | | MA.S-CP.B.8 | Apply the general Multiplication Rule in a uniform probability model, 𝑃(𝐴 𝑎𝑛𝑑 𝐵) = [𝑃(𝐴)] × [𝑃(𝐵|𝐴)] = [𝑃(𝐵)] × [𝑃(𝐴|𝐵)], and interpret the answer in terms of the model. | | MA.S-CP.B.9 | Use permutations and combinations to compute probabilities of compound events and solve problems. | | MA.S-CP.A | Understand independence and conditional probability and use them to interpret data | | MA.S-CP.A.1 | Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). | | MA.S-CP.A.2 | Understand that two events 𝐴 and 𝐵 are independent if the probability of 𝐴 and 𝐵 occurring together is the product of their probabilities, and use this characterization to determine if they are independent. | | MA.S-CP.A.3 | Understand the conditional probability of 𝐴 given 𝐵 as 𝑃(𝐴 𝑎𝑛𝑑 𝐵)/𝑃(𝐵), and interpret independence of 𝐴 and 𝐵 as saying that the conditional probability of 𝐴 given 𝐵 is the same as the probability of 𝐴, and the conditional probability of 𝐵 given 𝐴 is the same as the probability of 𝐵. | | MA.S-CP.A.4 | Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. | | MA.S-CP.A.5 | Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. | |

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| **Domain: Using Probability to Make Decisions** |
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| **Standard Set: Geometry** |
| |  |  | | --- | --- | |  | Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations. | |  | Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. | |  | The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. | |  | An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. | |  | Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) | |  | During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. | |  | The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. | |  | In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. | |  | Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. | |  | The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion. | |

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| **Domain: Congruence** |
| |  |  | | --- | --- | | MA.G-CO.A.1 | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. | | MA.G-CO.A.2 | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). | | MA.G-CO.A.3 | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. | | MA.G-CO.A.4 | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. | | MA.G-CO.A.5 | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. | | MA.G-CO.D.12 | Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). | | MA.G-CO.B | Understand congruence in terms of rigid motions | | MA.G-CO.D.13 | Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. | | MA.G-CO.B.6 | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. | | MA.G-CO.B.7 | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. | | MA.G-CO.B.8 | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. | | MA.G-CO.C | Prove geometric theorems | | MA.G-CO.C.9 | Prove theorems about lines and angles. | | MA.G-CO.C.10 | Prove theorems about triangles. | | MA.G-CO.C.11 | Prove theorems about parallelograms. | | MA.G-CO.D | Make geometric constructions | | MA.G-CO.A | Experiment with transformations in the plane | |

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| **Domain: Similarity, Right Triangles, and Trigonometry** |
| |  |  | | --- | --- | | MA.G-SRT.D.9 | Derive the formula 𝐴 = (1/2)𝑎𝑏 𝑠𝑖𝑛(𝐶) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. | | MA.G-SRT.D.10 | Prove the Laws of Sines and Cosines and use them to solve problems. | | MA.G-SRT.D.11 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). | | MA.G-SRT.A | Understand similarity in terms of similarity transformations | | MA.G-SRT.A.1 | Verify experimentally the properties of dilations given by a center and a scale factor: | | MA.G-SRT.A.1a | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. | | MA.G-SRT.A.1b | The dilation of a line segment is longer or shorter in the ratio given by the scale factor. | | MA.G-SRT.A.2 | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. | | MA.G-SRT.A.3 | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. | | MA.G-SRT.B | Prove theorems involving similarity | | MA.G-SRT.B.4 | Prove theorems about triangles. | | MA.G-SRT.B.5 | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | | MA.G-SRT.C | Define trigonometric ratios and solve problems involving right triangles | | MA.G-SRT.C.6 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. | | MA.G-SRT.C.7 | Explain and use the relationship between the sine and cosine of complementary angles. | | MA.G-SRT.C.8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. | | MA.G-SRT.D | Apply trigonometry to general triangles | |

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| **Domain: Circles** |
| |  |  | | --- | --- | | MA.G-C.A | Understand and apply theorems about circles | | MA.G-C.A.1 | Prove that all circles are similar. | | MA.G-C.A.2 | Identify and describe relationships among inscribed angles, radii, and chords. | | MA.G-C.A.3 | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. | | MA.G-C.A.4 | Construct a tangent line from a point outside a given circle to the circle. | | MA.G-C.B | Find arc lengths and areas of sectors of circles | | MA.G-C.B.5 | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. | |

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| **Domain: Expressing Geometric Properties with Equations** |
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| **Domain: Geometric Measurement and Dimension** |
| |  |  | | --- | --- | | MA.G-GMD.B.4 | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. | | MA.G-GMD.A | Explain volume formulas and use them to solve problems | | MA.G-GMD.A.1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. | | MA.G-GMD.A.2 | Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures. | | MA.G-GMD.A.3 | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. | | MA.G-GMD.B | Visualize relationships between two-dimensional and three-dimensional objects | |

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| **Domain: Modeling with Geometry** |
| |  |  | | --- | --- | | MA.G-MG.A | Apply geometric concepts in modeling situations | | MA.G-MG.A.1 | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). | | MA.G-MG.A.2 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). | | MA.G-MG.A.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). | |

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| **Standard Set: Modeling** |
| |  |  | | --- | --- | |  | Designing the layout of the stalls in a school fair so as to raise as much money as possible. | |  | Analyzing stopping distance for a car. | |  | Modeling savings account balance, bacterial colony growth, or investment growth. | |  | Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport. | |  | Analyzing risk in situations such as extreme sports, pandemics, and terrorism. | |  | Relating population statistics to individual predictions. | |  | Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). | |  | Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. | |  | A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity. | |  | In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations. | |  | Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed. | |  | Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player. | |

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| **Standard Set: Functions** |
| |  |  | | --- | --- | |  | Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. | |  | In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car’s speed in miles per hour, v; the rule T(v) = 100/v expresses this relationship algebraically and defines a function whose name is T. | |  | The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. | |  | A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, “I’ll give you a state, you give me the capital city;” by an algebraic expression like f(x) = a + bx; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function’s properties. | |  | Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships. | |  | A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions. | |  | Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology. | |

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| **Domain: Interpreting Functions** |
| |  |  | | --- | --- | | MA.F-IF.C.8b | Use the properties of exponents to interpret expressions for exponential functions. | | MA.F-IF.A | Understand the concept of a function and use function notation | | MA.F-IF.A.1 | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If 𝑓 is a function and 𝑥 is an element of its domain, then 𝑓(𝑥) denotes the output of 𝑓 corresponding to the input 𝑥. The graph of 𝑓 is the graph of the equation𝑦 = 𝑓(𝑥). | | MA.F-IF.A.2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | | MA.F-IF.C.9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). | | MA.F-IF.A.3 | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. | | MA.F-IF.B | Interpret functions that arise in applications in terms of the context | | MA.F-IF.B.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. | | MA.F-IF.B.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. | | MA.F-IF.B.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | | MA.F-IF.C | Analyze functions using different representations | | MA.F-IF.C.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | | MA.F-IF.C.7a | Graph linear and quadratic functions and show intercepts, maxima, and minima. | | MA.F-IF.C.7b | Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. | | MA.F-IF.C.7c | Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7d | Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7e | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | | MA.F-IF.C.8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. | | MA.F-IF.C.8a | Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | |

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| **Domain: Building Functions** |
| |  |  | | --- | --- | | MA.F-BF.B.4b | Verify by composition that one function is the inverse of another. | | MA.F-BF.B.4c | Read values of an inverse function from a graph or a table, given that the function has an inverse. | | MA.F-BF.B.4d | Produce an invertible function from a non-invertible function by restricting the domain. | | MA.F-BF.B.5 | Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents. | | MA.F-BF.A.1 | Write a function that describes a relationship between two quantities. | | MA.F-BF.A.1a | Determine an explicit expression, a recursive process, or steps for calculation from a context. | | MA.F-BF.A.1b | Combine standard function types using arithmetic operations. | | MA.F-BF.A | Build a function that models a relationship between two quantities | | MA.F-BF.A.1c | Compose functions. | | MA.F-BF.A.2 | Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. | | MA.F-BF.B | Build new functions from existing functions | | MA.F-BF.B.3 | Identify the effect on the graph of replacing 𝑓(𝑥) by 𝑓(𝑥) + 𝑘, 𝑘𝑓(𝑥), 𝑓(𝑘𝑥), and 𝑓(𝑥 + 𝑘) for specific values of 𝑘 (both positive and negative); find the value of 𝑘 given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. | | MA.F-BF.B.4 | Find inverse functions. | | MA.F-BF.B.4a | Solve an equation of the form 𝑓(𝑥) = 𝑐 for a simple function 𝑓 that has an inverse and write an expression for the inverse. | |

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| **Domain: Linear, Quadratic, and Exponential Models** |
| |  |  | | --- | --- | | MA.F-LE.A | Construct and compare linear and exponential models and solve problems | | MA.F-LE.A.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions. | | MA.F-LE.A.1a | Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. | | MA.F-LE.A.1b | Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. | | MA.F-LE.A.1c | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | | MA.F-LE.A.2 | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). | | MA.F-LE.A.3 | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | | MA.F-LE.A.4 | Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to 𝑎𝑏 to the 𝑐𝘵 power = 𝑑 where 𝑎, 𝑐, and 𝑑 are numbers and the base 𝑏 is 2, 10, or 𝑒; evaluate the logarithm using technology. | | MA.F-LE.B | Interpret expressions for functions in terms of the situation they model | | MA.F-LE.B.5 | Interpret the parameters in a linear or exponential function in terms of a context. | |

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| **Domain: Trigonometric Functions** |
| |  |  | | --- | --- | | MA.F-TF.A.3 | Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π – 𝑥, π + 𝑥, and 2π – 𝑥 in terms of their values for 𝑥, where 𝑥 is any real number. | | MA.F-TF.A.4 | Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. | | MA.F-TF.B | Model periodic phenomena with trigonometric functions | | MA.F-TF.B.5 | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. | | MA.F-TF.B.6 | Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. | | MA.F-TF.B.7 | Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. | | MA.F-TF.C | Prove and apply trigonometric identities | | MA.F-TF.C.8 | Prove the Pythagorean identity 𝑠𝑖𝑛²(θ) + 𝑐𝑜𝑠²(θ) = 1 and use it to find 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) given 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) and the quadrant of the angle. | | MA.F-TF.C.9 | Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. | | MA.F-TF.A | Extend the domain of trigonometric functions using the unit circle | | MA.F-TF.A.1 | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. | | MA.F-TF.A.2 | Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. | |

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| **Standard Set: Algebra** |
| |  |  | | --- | --- | |  | An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances. | |  | Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor. | |  | Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure. | |  | A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave. | |  | An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. | |  | The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system. | |  | An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. | |  | Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x² – 2 = 0 are real numbers, not rational numbers; and the solutions of x² + 2 = 0 are complex numbers, not real numbers. | |  | Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling. | |  | The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, 𝘈 = ((𝘣₁+𝘣₂)/2)𝘩, can be solved for 𝘩 using the same deductive process. | |  | Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them. | |

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| **Domain: Seeing Structures in Expressions** |
| |  |  | | --- | --- | | MA.A-SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. For example, see 𝑥⁴ – 𝑦⁴ as (𝑥²)² – (𝑦²)², thus recognizing it as a difference of squares that can be factored as (𝑥² – 𝑦²)(𝑥² + 𝑦²). | | MA.A-SSE.B | Write expressions in equivalent forms to solve problems | | MA.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. | | MA.A-SSE.B.3a | Factor a quadratic expression to reveal the zeros of the function it defines. | | MA.A-SSE.B.3b | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. | | MA.A-SSE.A | Interpret the structure of expressions | | MA.A-SSE.B.3c | Use the properties of exponents to transform expressions for exponential functions. | | MA.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context. | | MA.A-SSE.A.1a | Interpret parts of an expression, such as terms, factors, and coefficients. | | MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. | | MA.A-SSE.A.1b | Interpret complicated expressions by viewing one or more of their parts as a single entity. | |

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| **Domain: Arithmetic with Polynomials and Rational Functions** |
| |  |  | | --- | --- | | MA.A-APR.C.4 | Prove polynomial identities and use them to describe numerical relationships. | | MA.A-APR.A | Perform arithmetic operations on polynomials | | MA.A-APR.A.1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | | MA.A-APR.B | Understand the relationship between zeros and factors of polynomials | | MA.A-APR.B.2 | Know and apply the Remainder Theorem: For a polynomial 𝑝(𝑥) and a number 𝑎, the remainder on division by 𝑥 – 𝑎 is 𝑝(𝑎), so 𝑝(𝑎) = 0 if and only if (𝑥 – 𝑎) is a factor of 𝑝(𝑥). | | MA.A-APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. | | MA.A-APR.C.5 | Know and apply the Binomial Theorem for the expansion of (𝑥 + 𝑦)ⁿ in powers of 𝑥 and 𝑦 for a positive integer 𝑛, where 𝑥 and 𝑦 are any numbers, with coefficients determined for example by Pascal’s Triangle. | | MA.A-APR.D | Rewrite rational expressions | | MA.A-APR.D.6 | Rewrite simple rational expressions in different forms; write 𝑎(𝑥)/𝑏(𝑥) in the form 𝑞(𝑥) + 𝑟(𝑥)/𝑏(𝑥), where 𝑎(𝑥), 𝑏(𝑥), 𝑞(𝑥), and 𝑟(𝑥) are polynomials with the degree of 𝑟(𝑥) less than the degree of 𝑏(𝑥), using inspection, long division, or, for the more complicated examples, a computer algebra system. | | MA.A-APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. | | MA.A-APR.C | Use polynomial identities to solve problems | |

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| **Domain: Creating Equations** |
| |  |  | | --- | --- | | MA.A-CED.A | Create equations that describe numbers or relationships | | MA.A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems. | | MA.A-CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | | MA.A-CED.A.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. | | MA.A-CED.A.4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. | |

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| **Domain: Reasoning with Equations and Inequalities** |
| |  |  | | --- | --- | | MA.A-REI.B | Solve equations and inequalities in one variable | | MA.A-REI.B.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. | | MA.A-REI.B.4 | Solve quadratic equations in one variable. | | MA.A-REI.C | Solve systems of equations | | MA.A-REI.C.5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. | | MA.A-REI.C.6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. | | MA.A-REI.C.7 | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. | | MA.A-REI.C.8 | Represent a system of linear equations as a single matrix equation in a vector variable. | | MA.A-REI.C.9 | Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). | | MA.A-REI.D | Represent and solve equations and inequalities graphically | | MA.A-REI.B.4a | Use the method of completing the square to transform any quadratic equation in 𝑥 into an equation of the form (𝑥 – 𝑝)² = 𝑞 that has the same solutions. Derive the quadratic formula from this form. | | MA.A-REI.D.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). | | MA.A-REI.B.4b | Solve quadratic equations by inspection (e.g., for 𝑥² = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as 𝑎 ± 𝑏𝑖 for real numbers 𝑎 and 𝑏. | | MA.A-REI.A | Understand solving equations as a process of reasoning and explain the reasoning | | MA.A-REI.D.12 | Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | | MA.A-REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | | MA.A-REI.D.11 | Explain why the 𝑥-coordinates of the points where the graphs of the equations 𝑦 = 𝑓(𝑥) and 𝑦 = 𝑔(𝑥) intersect are the solutions of the equation𝑓(𝑥) = 𝑔(𝑥); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where 𝑓(𝑥) and/or 𝑔(𝑥) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | | MA.A-REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. | |

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| **Essential Questions** |
| Given two points in a plane, how is the distance between them determined?  How is the midpoint between the points determined?  What is the equation of a line given partial information?  What is a function?  What is its domain and range?  What is the impact on the domain and range when functions are combined arithmetically or via composition?  How are the graphs of inverse functions related?  How can functions be transformed via translation, reflection, stretches or shrinks?  How can a function’s symmetry be determined and used in understanding the behavior of the graph of a function?  What does it mean for a function to be even or odd? How is a rational function graphed?  How are graphing calculators used to visualize and verify results? |

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| **Enduring Understanding** |
| Functions are an important topic in the study of mathematics.  Functions may be defined and described in various ways one of which is a two-dimensional graph generated from a table of input/output values. Two functions can be combined to produce functions that are more complex.  Such combinations impact the graph, domain, and range of the functions. Functions can be inverted to create new functions.   Linear functions and those involving variation can be used to model real world problems. |

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| **Students will know...** |
| 1. Graphs of Equations on Cartesian Planes 2. Functions and Graphs of Functions 3. Combinations of Functions 4. Inverse Functions 5. Variation and Mathematical Models |

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| **Students will be able to...** |
| 1. Find the midpoint and distance between two points on the Cartesian Planes. 2. Sketch the graph and find the x, and y-intercepts of an equation. 3. Test and equation for symmetry, and if present use symmetry to assist in graphing the equation. 4. Find the equation of a circle. 5. Find the slope and write an equation of a line. 6. Use equations to determine if lines are parallel or perpendicular. 7. Determine the domain, range and zeros of a function, and sketch the graph. 8. Explore standard transformations (stretches, shifts, and reflections) on standard graphs. 9. Write the equation and identify the domain of the composite of two functions. 10. Test for one-to-one functions. 11. Find the equation and sketch the graph of the inverse of a function. 12. Find mathematical models using direct, inverse and joint variations. |

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| **STAGE 2- EVIDENCE OF LEARNING** |
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| **Formative Assessment Suggestions** |
| |  |  | | --- | --- | | • 3- Minute Pause | **.** | | • A-B-C Summaries | **.** | | • Analogy Prompt | **.** | | • Choral Response | **.** | | • Debriefing | **.** | | • Exit Card / Ticket | **.** | | • Hand Signals | **.** | | • Idea Spinner | **.** | | • Index Card Summaries | **.** | | • Inside-Outside Circle Discussion (Fishbowl) | **.** | | • Journal Entry | **.** | | • Misconception Check | **.** | | • Observation | **.** | | • One Minute Essay | **.** | | • One Word Summary | **.** | | • Portfolio Check | **.** | | • Questions & Answers | **.** | | • Quiz | **.** | | • Self-Assessment | **.** | | • Student Conference | **.** | | • Think-Pair-Share | **.** | | • Web or Concept Map | **.** | |

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| **Authentic Assessments Suggestions** |
| Edementum Exact Path (BOY, MOY, EOY)  Class Discussions  Do Nows  Notebook Checks  Homework  Presentations  Webassign |

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| **Benchmark Assessments** |
| Edementum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3- LEARNING PLAN** |
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| **Instructional Map** |
| Topic #1 – Graphs of Equations on Cartesian Planes  Topic #2 – Functions and Graphs of Functions  Topic #3 – Combinations of Functions  Topic #4 – Inverse Functions  Topic #5 – Variation and Mathematical Models |

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| **Modifications/Differentiation of Instruction** |
| Differentiation Strategies for Special Education Students   * Remove unnecessary material, words, etc., that can distract from the content * Use of off-grade level materials * Provide appropriate scaffolding * Limit the number of steps required for completion * Time allowed * Level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Varied homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Ability to work at their own pace * Present ideas using auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment * Differentiated checklists and rubrics, if available and appropriate     Differentiation Strategies for Gifted and Talented Students   * Increase the level of complexity * Decrease scaffolding * Variety of finished products * Allow for greater independence * Learning stations, interest groups * Varied texts and supplementary materials * Use of technology * Flexibility in assignments * Varied questioning strategies * Encourage research * Strategy and flexible groups based on formative assessment or student choice * Acceleration within a unit of study * Exposure to more advanced or complex concepts, abstractions, and materials * Encourage students to move through content areas at their own pace * After mastery of a unit, provide students with more advanced learning activities, not more of the same activity * Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas     Differentiated Strategies for ELL Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials, including visuals * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Allow students to work at their own pace * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Role play * Provide graphic organizers, highlighted materials * Strategy and flexible groups based on formative assessment       Differentiation Strategies for At Risk Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment       **504 Plans**  Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:   * walk, breathe, eat, or sleep * communicate, see, hear, or speak * read, concentrate, think, or learn * stand, bend, lift, or work       Examples of accommodations in 504 plans include:   * preferential seating * extended time on tests and assignments * reduced homework or classwork * verbal, visual, or technology aids * modified textbooks or audio-video materials * behavior management support * adjusted class schedules or grading * verbal testing * excused lateness, absence, or missed classwork * pre-approved nurse's office visits and accompaniment to visits * occupational or physical therapy |

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| **Modification Strategies** |
| |  |  | | --- | --- | | • Extended Time | **.** | | • Frequent Breaks | **.** | | • Highlighted Text | **.** | | • Interactive Notebook | **.** | | • Modified Test | **.** | | • Oral Directions | **.** | | • Peer Tutoring | **.** | | • Preferential Seating | **.** | | • Re-Direct | **.** | | • Repeated Drill / Practice | **.** | | • Shortened Assignments | **.** | | • Teacher Notes | **.** | | • Tutorials | **.** | | • Use of Additional Reference Material | **.** | | • Use of Audio Resources | **.** | |

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| **High Preparation Differentiation** |
| |  |  | | --- | --- | | • Alternative Assessments | **.** | | • Choice Boards | **.** | | • Games and Tournaments | **.** | | • Group Investigations | **.** | | • Guided Reading | **.** | | • Independent Research / Project | **.** | | • Interest Groups | **.** | | • Learning Contracts | **.** | | • Leveled Rubrics | **.** | | • Literature Circles | **.** | | • Menu Assignments | **.** | | • Multiple Intelligence Options | **.** | | • Multiple Texts | **.** | | • Personal Agendas | **.** | | • Project Based Learning (PBL) | **.** | | • Stations / Centers | **.** | | • Think-Tac-Toe | **.** | | • Tiered Activities / Assignments | **.** | | • Varying Graphic Organizers | **.** | |

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| **Low Preparation Differentiation** |
| |  |  | | --- | --- | | • Choice of Book / Activity | **.** | | • Cubing Activities | **.** | | • Exploration by Interest (using interest inventories) | **.** | | • Flexible Grouping | **.** | | • Goal Setting With Student | **.** | | • Homework Options | **.** | | • Jigsaw | **.** | | • Mini Workshops to Extend Skills | **.** | | • Mini Workshops to Re-teach | **.** | | • Open-ended Activities | **.** | | • Think-Pair-Share by Interest | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Readiness | **.** | | • Use of Collaboration | **.** | | • Use of Reading Buddies | **.** | | • Varied Journal Prompts | **.** | | • Varied Product Choice | **.** | | • Varied Supplemental Materials | **.** | | • Work Alone / Together | **.** | |

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| **Horizontal Integration- Interdisciplinary Connections** |
| Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other. |

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| **Vertical Integration- Discipline Mapping** |
| The standards in this unit were introduced in Algebra I, Geometry and Algebra II. Precalculus coursework focuses on preparing the students be proficient in Calculus standards. |

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| **Additional Materials** |
| LPS Adopted Textbooks and Programs   * Cengage: Precalculus with Limits: A Graphing Approach * WebAssign (Computer Based program supplementing Cengage)   Khan Academy  Edmentum Exact Path |

**Unit 3: Polynomial and Rational Functions**

Content Area: **Mathematics**  
Course(s): **Pre-Calculus**  
Time Period: **Second Marking period**  
Length: **4 Week**  
Status: **Published**

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| **Unit Overview** |
| Quadratic functions produce parabolic curves. Higher level polynomial functions produce smooth curves which cross the x-axis at the function’s zeros or roots.   Complex numbers extend the real number system to represent the roots of polynomial functions that do not cross the x-axis when the function is graphed. Rational functions include domain issues and their graphs display asymptotic behavior. |

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| **STAGE 1- DESIRED RESULTS** |
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| **Educational Standards** |
| The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:  1. Make sense of problems and persevre in solving them.  2. Reason abstractly and quantitatively.  3. Construct viable arguments and critique the reasoning of others.  4. Model with mathematics.  5. Use appropriate tools strategically.  6. Attend to precision.  7. Look for and make use of structure.  8. Look for and express regularity in repeated reasoning. |

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| **New Jersey Student Learning Standards- Mathematics** |
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| **Standard Set: Number and Quantity** |
| |  |  | | --- | --- | |  | During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers. | |  | With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. | |  | Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that (5 to the 1/3 power)³ should be 5 to the (1/3)³ power = 5¹ = 5 and that 5 to the 1/3 power should be the cube root of 5. | |  | Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents. | |  | In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them. | |

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| **Domain: The Real Number System** |
| |  |  | | --- | --- | | MA.N-RN.B.3 | Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. | | MA.N-RN.A | Extend the properties of exponents to rational exponents. | | MA.N-RN.A.1 | Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. | | MA.N-RN.A.2 | Rewrite expressions involving radicals and rational exponents using the properties of exponents. | | MA.N-RN.B | Use properties of rational and irrational numbers. | |

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| **Domain: Quantities** |
| |  |  | | --- | --- | | MA.N-Q.A | Reason quantitatively and use units to solve problems. | | MA.N-Q.A.1 | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | | MA.N-Q.A.2 | Define appropriate quantities for the purpose of descriptive modeling. | | MA.N-Q.A.3 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | |

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| **Domain: The Complex Number System** |
| |  |  | | --- | --- | | MA.N-CN.B.6 | Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. | | MA.N-CN.A | Perform arithmetic operations with complex numbers. | | MA.N-CN.A.1 | Know there is a complex number 𝑖 such that 𝑖² = -1, and every complex number has the form 𝑎 + 𝑏𝑖 with 𝑎 and 𝑏 real. | | MA.N-CN.A.2 | Use the relation 𝑖² = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. | | MA.N-CN.A.3 | Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. | | MA.N-CN.B | Represent complex numbers and their operations on the complex plane. | | MA.N-CN.B.4 | Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. | | MA.N-CN.B.5 | Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. | | MA.N-CN.C | Use complex numbers in polynomial identities and equations. | | MA.N-CN.C.7 | Solve quadratic equations with real coefficients that have complex solutions. | | MA.N-CN.C.8 | Extend polynomial identities to the complex numbers. | | MA.N-CN.C.9 | Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. | |

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| **Domain: Vector and Matrix Quantities** |
| |  |  | | --- | --- | | MA.N-VM.B.5a | Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as 𝑐(𝒗ₓ, 𝒗 subscript 𝑦) = (𝑐𝒗ₓ, 𝑐𝒗 subscript 𝑦). | | MA.N-VM.B.4c | Understand vector subtraction 𝒗 − 𝑤 as 𝒗 + (−𝑤), where –𝑤 is the additive inverse of 𝑤, with the same magnitude as 𝑤 and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. | | MA.N-VM.B.5b | Compute the magnitude of a scalar multiple 𝑐𝒗 using ||𝑐𝒗|| = |𝑐|𝒗. Compute the direction of 𝑐𝒗 knowing that when |𝑐|𝒗 ≠ 0, the direction of 𝑐𝒗 is either along 𝒗 (for 𝑐 > 0) or against 𝒗 (for 𝑐 < 0). | | MA.N-VM.A | Represent and model with vector quantities. | | MA.N-VM.C | Perform operations on matrices and use matrices in applications. | | MA.N-VM.C.6 | Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. | | MA.N-VM.C.7 | Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. | | MA.N-VM.C.8 | Add, subtract, and multiply matrices of appropriate dimensions. | | MA.N-VM.C.9 | Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. | | MA.N-VM.C.10 | Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. | | MA.N-VM.C.11 | Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. | | MA.N-VM.C.12 | Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area. | | MA.N-VM.A.1 | Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., 𝒗, |𝒗|, ||𝒗||, 𝒗). | | MA.N-VM.A.2 | Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. | | MA.N-VM.A.3 | Solve problems involving velocity and other quantities that can be represented by vectors. | | MA.N-VM.B | Perform operations on vectors. | | MA.N-VM.B.4 | Add and subtract vectors. | | MA.N-VM.B.5 | Multiply a vector by a scalar. | | MA.N-VM.B.4a | Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. | | MA.N-VM.B.4b | Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. | |

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| **Standard Set: Statistics and Probability** |
| |  |  | | --- | --- | |  | Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account. | |  | Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. | |  | Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. | |  | Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. | |  | Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time. | |  | Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient. | |

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| **Domain: Interpreting Categorical and Quantitative Data** |
| |  |  | | --- | --- | | MA.S-ID.B.6c | Fit a linear function for a scatter plot that suggests a linear association. | | MA.S-ID.C | Interpret linear models | | MA.S-ID.C.7 | Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | | MA.S-ID.C.8 | Compute (using technology) and interpret the correlation coefficient of a linear fit. | | MA.S-ID.C.9 | Distinguish between correlation and causation. | | MA.S-ID.A | Summarize, represent, and interpret data on a single count or measurement variable | | MA.S-ID.A.1 | Represent data with plots on the real number line (dot plots, histograms, and box plots). | | MA.S-ID.A.2 | Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | | MA.S-ID.A.3 | Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | | MA.S-ID.A.4 | Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. | | MA.S-ID.B | Summarize, represent, and interpret data on two categorical and quantitative variables | | MA.S-ID.B.5 | Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. | | MA.S-ID.B.6 | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. | | MA.S-ID.B.6a | Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. | | MA.S-ID.B.6b | Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology. | |

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| **Domain: Making Inferences and Justifying Conclusions** |
| |  |  | | --- | --- | | MA.S-IC.A | Understand and evaluate random processes underlying statistical experiments | | MA.S-IC.A.1 | Understand statistics as a process for making inferences about population parameters based on a random sample from that population. | | MA.S-IC.A.2 | Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. | | MA.S-IC.B | Make inferences and justify conclusions from sample surveys, experiments, and observational studies | | MA.S-IC.B.3 | Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. | | MA.S-IC.B.4 | Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. | | MA.S-IC.B.5 | Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. | | MA.S-IC.B.6 | Evaluate reports based on data. | |

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| **Domain: Conditional Probability and the Rules of Probability** |
| |  |  | | --- | --- | | MA.S-CP.B | Use the rules of probability to compute probabilities of compound events in a uniform probability model | | MA.S-CP.B.6 | Find the conditional probability of 𝐴 given 𝐵 as the fraction of 𝐵’s outcomes that also belong to 𝐴, and interpret the answer in terms of the model. | | MA.S-CP.B.7 | Apply the Addition Rule, 𝑃(𝐴 𝑜𝑟 𝐵) = 𝑃(𝐴) + 𝑃(𝐵) – 𝑃(𝐴 𝑎𝑛𝑑 𝐵), and interpret the answer in terms of the model. | | MA.S-CP.B.8 | Apply the general Multiplication Rule in a uniform probability model, 𝑃(𝐴 𝑎𝑛𝑑 𝐵) = [𝑃(𝐴)] × [𝑃(𝐵|𝐴)] = [𝑃(𝐵)] × [𝑃(𝐴|𝐵)], and interpret the answer in terms of the model. | | MA.S-CP.B.9 | Use permutations and combinations to compute probabilities of compound events and solve problems. | | MA.S-CP.A | Understand independence and conditional probability and use them to interpret data | | MA.S-CP.A.1 | Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). | | MA.S-CP.A.2 | Understand that two events 𝐴 and 𝐵 are independent if the probability of 𝐴 and 𝐵 occurring together is the product of their probabilities, and use this characterization to determine if they are independent. | | MA.S-CP.A.3 | Understand the conditional probability of 𝐴 given 𝐵 as 𝑃(𝐴 𝑎𝑛𝑑 𝐵)/𝑃(𝐵), and interpret independence of 𝐴 and 𝐵 as saying that the conditional probability of 𝐴 given 𝐵 is the same as the probability of 𝐴, and the conditional probability of 𝐵 given 𝐴 is the same as the probability of 𝐵. | | MA.S-CP.A.4 | Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. | | MA.S-CP.A.5 | Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. | |

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| **Domain: Using Probability to Make Decisions** |
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| **Standard Set: Geometry** |
| |  |  | | --- | --- | |  | Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations. | |  | Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. | |  | The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. | |  | An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. | |  | Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) | |  | During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. | |  | The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. | |  | In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. | |  | Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. | |  | The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion. | |

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| **Domain: Congruence** |
| |  |  | | --- | --- | | MA.G-CO.A.1 | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. | | MA.G-CO.A.2 | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). | | MA.G-CO.A.3 | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. | | MA.G-CO.A.4 | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. | | MA.G-CO.A.5 | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. | | MA.G-CO.D.12 | Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). | | MA.G-CO.B | Understand congruence in terms of rigid motions | | MA.G-CO.D.13 | Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. | | MA.G-CO.B.6 | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. | | MA.G-CO.B.7 | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. | | MA.G-CO.B.8 | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. | | MA.G-CO.C | Prove geometric theorems | | MA.G-CO.C.9 | Prove theorems about lines and angles. | | MA.G-CO.C.10 | Prove theorems about triangles. | | MA.G-CO.C.11 | Prove theorems about parallelograms. | | MA.G-CO.D | Make geometric constructions | | MA.G-CO.A | Experiment with transformations in the plane | |

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| **Domain: Similarity, Right Triangles, and Trigonometry** |
| |  |  | | --- | --- | | MA.G-SRT.D.9 | Derive the formula 𝐴 = (1/2)𝑎𝑏 𝑠𝑖𝑛(𝐶) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. | | MA.G-SRT.D.10 | Prove the Laws of Sines and Cosines and use them to solve problems. | | MA.G-SRT.D.11 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). | | MA.G-SRT.A | Understand similarity in terms of similarity transformations | | MA.G-SRT.A.1 | Verify experimentally the properties of dilations given by a center and a scale factor: | | MA.G-SRT.A.1a | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. | | MA.G-SRT.A.1b | The dilation of a line segment is longer or shorter in the ratio given by the scale factor. | | MA.G-SRT.A.2 | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. | | MA.G-SRT.A.3 | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. | | MA.G-SRT.B | Prove theorems involving similarity | | MA.G-SRT.B.4 | Prove theorems about triangles. | | MA.G-SRT.B.5 | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | | MA.G-SRT.C | Define trigonometric ratios and solve problems involving right triangles | | MA.G-SRT.C.6 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. | | MA.G-SRT.C.7 | Explain and use the relationship between the sine and cosine of complementary angles. | | MA.G-SRT.C.8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. | | MA.G-SRT.D | Apply trigonometry to general triangles | |

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| **Domain: Circles** |
| |  |  | | --- | --- | | MA.G-C.A | Understand and apply theorems about circles | | MA.G-C.A.1 | Prove that all circles are similar. | | MA.G-C.A.2 | Identify and describe relationships among inscribed angles, radii, and chords. | | MA.G-C.A.3 | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. | | MA.G-C.A.4 | Construct a tangent line from a point outside a given circle to the circle. | | MA.G-C.B | Find arc lengths and areas of sectors of circles | | MA.G-C.B.5 | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. | |

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| **Domain: Expressing Geometric Properties with Equations** |
| |  |  | | --- | --- | | MA.G-GPE.B.4 | Use coordinates to prove simple geometric theorems algebraically. | | MA.G-GPE.B.5 | Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). | | MA.G-GPE.B.6 | Find the point on a directed line segment between two given points that partitions the segment in a given ratio. | | MA.G-GPE.B.7 | Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. | | MA.G-GPE.A | Translate between the geometric description and the equation for a conic section | | MA.G-GPE.A.1 | Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. | | MA.G-GPE.A.2 | Derive the equation of a parabola given a focus and directrix. | | MA.G-GPE.A.3 | Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. | | MA.G-GPE.B | Use coordinates to prove simple geometric theorems algebraically | |

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| **Domain: Geometric Measurement and Dimension** |
| |  |  | | --- | --- | | MA.G-GMD.B.4 | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. | | MA.G-GMD.A | Explain volume formulas and use them to solve problems | | MA.G-GMD.A.1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. | | MA.G-GMD.A.2 | Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures. | | MA.G-GMD.A.3 | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. | | MA.G-GMD.B | Visualize relationships between two-dimensional and three-dimensional objects | |

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| **Domain: Modeling with Geometry** |
| |  |  | | --- | --- | | MA.G-MG.A | Apply geometric concepts in modeling situations | | MA.G-MG.A.1 | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). | | MA.G-MG.A.2 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). | | MA.G-MG.A.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). | |

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| **Standard Set: Modeling** |
| |  |  | | --- | --- | |  | Designing the layout of the stalls in a school fair so as to raise as much money as possible. | |  | Analyzing stopping distance for a car. | |  | Modeling savings account balance, bacterial colony growth, or investment growth. | |  | Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport. | |  | Analyzing risk in situations such as extreme sports, pandemics, and terrorism. | |  | Relating population statistics to individual predictions. | |  | Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). | |  | Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. | |  | A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity. | |  | In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations. | |  | Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed. | |  | Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player. | |

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| **Standard Set: Functions** |
| |  |  | | --- | --- | |  | Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. | |  | In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car’s speed in miles per hour, v; the rule T(v) = 100/v expresses this relationship algebraically and defines a function whose name is T. | |  | The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. | |  | A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, “I’ll give you a state, you give me the capital city;” by an algebraic expression like f(x) = a + bx; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function’s properties. | |  | Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships. | |  | A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions. | |  | Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology. | |

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| **Domain: Interpreting Functions** |
| |  |  | | --- | --- | | MA.F-IF.C.8b | Use the properties of exponents to interpret expressions for exponential functions. | | MA.F-IF.A | Understand the concept of a function and use function notation | | MA.F-IF.A.1 | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If 𝑓 is a function and 𝑥 is an element of its domain, then 𝑓(𝑥) denotes the output of 𝑓 corresponding to the input 𝑥. The graph of 𝑓 is the graph of the equation𝑦 = 𝑓(𝑥). | | MA.F-IF.A.2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | | MA.F-IF.C.9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). | | MA.F-IF.A.3 | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. | | MA.F-IF.B | Interpret functions that arise in applications in terms of the context | | MA.F-IF.B.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. | | MA.F-IF.B.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. | | MA.F-IF.B.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | | MA.F-IF.C | Analyze functions using different representations | | MA.F-IF.C.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | | MA.F-IF.C.7a | Graph linear and quadratic functions and show intercepts, maxima, and minima. | | MA.F-IF.C.7b | Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. | | MA.F-IF.C.7c | Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7d | Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7e | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | | MA.F-IF.C.8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. | | MA.F-IF.C.8a | Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | |

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| **Domain: Building Functions** |
| |  |  | | --- | --- | | MA.F-BF.B.4b | Verify by composition that one function is the inverse of another. | | MA.F-BF.B.4c | Read values of an inverse function from a graph or a table, given that the function has an inverse. | | MA.F-BF.B.4d | Produce an invertible function from a non-invertible function by restricting the domain. | | MA.F-BF.B.5 | Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents. | | MA.F-BF.A.1 | Write a function that describes a relationship between two quantities. | | MA.F-BF.A.1a | Determine an explicit expression, a recursive process, or steps for calculation from a context. | | MA.F-BF.A.1b | Combine standard function types using arithmetic operations. | | MA.F-BF.A | Build a function that models a relationship between two quantities | | MA.F-BF.A.1c | Compose functions. | | MA.F-BF.A.2 | Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. | | MA.F-BF.B | Build new functions from existing functions | | MA.F-BF.B.3 | Identify the effect on the graph of replacing 𝑓(𝑥) by 𝑓(𝑥) + 𝑘, 𝑘𝑓(𝑥), 𝑓(𝑘𝑥), and 𝑓(𝑥 + 𝑘) for specific values of 𝑘 (both positive and negative); find the value of 𝑘 given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. | | MA.F-BF.B.4 | Find inverse functions. | | MA.F-BF.B.4a | Solve an equation of the form 𝑓(𝑥) = 𝑐 for a simple function 𝑓 that has an inverse and write an expression for the inverse. | |

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| **Domain: Linear, Quadratic, and Exponential Models** |
| |  |  | | --- | --- | | MA.F-LE.A | Construct and compare linear and exponential models and solve problems | | MA.F-LE.A.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions. | | MA.F-LE.A.1a | Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. | | MA.F-LE.A.1b | Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. | | MA.F-LE.A.1c | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | | MA.F-LE.A.2 | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). | | MA.F-LE.A.3 | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | | MA.F-LE.A.4 | Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to 𝑎𝑏 to the 𝑐𝘵 power = 𝑑 where 𝑎, 𝑐, and 𝑑 are numbers and the base 𝑏 is 2, 10, or 𝑒; evaluate the logarithm using technology. | | MA.F-LE.B | Interpret expressions for functions in terms of the situation they model | | MA.F-LE.B.5 | Interpret the parameters in a linear or exponential function in terms of a context. | |

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| **Domain: Trigonometric Functions** |
| |  |  | | --- | --- | | MA.F-TF.A.3 | Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π – 𝑥, π + 𝑥, and 2π – 𝑥 in terms of their values for 𝑥, where 𝑥 is any real number. | | MA.F-TF.A.4 | Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. | | MA.F-TF.B | Model periodic phenomena with trigonometric functions | | MA.F-TF.B.5 | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. | | MA.F-TF.B.6 | Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. | | MA.F-TF.B.7 | Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. | | MA.F-TF.C | Prove and apply trigonometric identities | | MA.F-TF.C.8 | Prove the Pythagorean identity 𝑠𝑖𝑛²(θ) + 𝑐𝑜𝑠²(θ) = 1 and use it to find 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) given 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) and the quadrant of the angle. | | MA.F-TF.C.9 | Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. | | MA.F-TF.A | Extend the domain of trigonometric functions using the unit circle | | MA.F-TF.A.1 | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. | | MA.F-TF.A.2 | Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. | |

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| **Standard Set: Algebra** |
| |  |  | | --- | --- | |  | An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances. | |  | Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor. | |  | Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure. | |  | A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave. | |  | An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. | |  | The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system. | |  | An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. | |  | Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x² – 2 = 0 are real numbers, not rational numbers; and the solutions of x² + 2 = 0 are complex numbers, not real numbers. | |  | Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling. | |  | The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, 𝘈 = ((𝘣₁+𝘣₂)/2)𝘩, can be solved for 𝘩 using the same deductive process. | |  | Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them. | |

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| **Domain: Seeing Structures in Expressions** |
| |  |  | | --- | --- | | MA.A-SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. For example, see 𝑥⁴ – 𝑦⁴ as (𝑥²)² – (𝑦²)², thus recognizing it as a difference of squares that can be factored as (𝑥² – 𝑦²)(𝑥² + 𝑦²). | | MA.A-SSE.B | Write expressions in equivalent forms to solve problems | | MA.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. | | MA.A-SSE.B.3a | Factor a quadratic expression to reveal the zeros of the function it defines. | | MA.A-SSE.B.3b | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. | | MA.A-SSE.A | Interpret the structure of expressions | | MA.A-SSE.B.3c | Use the properties of exponents to transform expressions for exponential functions. | | MA.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context. | | MA.A-SSE.A.1a | Interpret parts of an expression, such as terms, factors, and coefficients. | | MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. | | MA.A-SSE.A.1b | Interpret complicated expressions by viewing one or more of their parts as a single entity. | |

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| **Domain: Arithmetic with Polynomials and Rational Functions** |
| |  |  | | --- | --- | | MA.A-APR.C.4 | Prove polynomial identities and use them to describe numerical relationships. | | MA.A-APR.A | Perform arithmetic operations on polynomials | | MA.A-APR.A.1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | | MA.A-APR.B | Understand the relationship between zeros and factors of polynomials | | MA.A-APR.B.2 | Know and apply the Remainder Theorem: For a polynomial 𝑝(𝑥) and a number 𝑎, the remainder on division by 𝑥 – 𝑎 is 𝑝(𝑎), so 𝑝(𝑎) = 0 if and only if (𝑥 – 𝑎) is a factor of 𝑝(𝑥). | | MA.A-APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. | | MA.A-APR.C.5 | Know and apply the Binomial Theorem for the expansion of (𝑥 + 𝑦)ⁿ in powers of 𝑥 and 𝑦 for a positive integer 𝑛, where 𝑥 and 𝑦 are any numbers, with coefficients determined for example by Pascal’s Triangle. | | MA.A-APR.D | Rewrite rational expressions | | MA.A-APR.D.6 | Rewrite simple rational expressions in different forms; write 𝑎(𝑥)/𝑏(𝑥) in the form 𝑞(𝑥) + 𝑟(𝑥)/𝑏(𝑥), where 𝑎(𝑥), 𝑏(𝑥), 𝑞(𝑥), and 𝑟(𝑥) are polynomials with the degree of 𝑟(𝑥) less than the degree of 𝑏(𝑥), using inspection, long division, or, for the more complicated examples, a computer algebra system. | | MA.A-APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. | | MA.A-APR.C | Use polynomial identities to solve problems | |

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| **Domain: Creating Equations** |
| |  |  | | --- | --- | | MA.A-CED.A | Create equations that describe numbers or relationships | | MA.A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems. | | MA.A-CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | | MA.A-CED.A.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. | | MA.A-CED.A.4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. | |

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| **Domain: Reasoning with Equations and Inequalities** |
| |  |  | | --- | --- | | MA.A-REI.B | Solve equations and inequalities in one variable | | MA.A-REI.B.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. | | MA.A-REI.B.4 | Solve quadratic equations in one variable. | | MA.A-REI.C | Solve systems of equations | | MA.A-REI.C.5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. | | MA.A-REI.C.6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. | | MA.A-REI.C.7 | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. | | MA.A-REI.C.8 | Represent a system of linear equations as a single matrix equation in a vector variable. | | MA.A-REI.C.9 | Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). | | MA.A-REI.D | Represent and solve equations and inequalities graphically | | MA.A-REI.B.4a | Use the method of completing the square to transform any quadratic equation in 𝑥 into an equation of the form (𝑥 – 𝑝)² = 𝑞 that has the same solutions. Derive the quadratic formula from this form. | | MA.A-REI.D.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). | | MA.A-REI.B.4b | Solve quadratic equations by inspection (e.g., for 𝑥² = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as 𝑎 ± 𝑏𝑖 for real numbers 𝑎 and 𝑏. | | MA.A-REI.A | Understand solving equations as a process of reasoning and explain the reasoning | | MA.A-REI.D.12 | Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | | MA.A-REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | | MA.A-REI.D.11 | Explain why the 𝑥-coordinates of the points where the graphs of the equations 𝑦 = 𝑓(𝑥) and 𝑦 = 𝑔(𝑥) intersect are the solutions of the equation𝑓(𝑥) = 𝑔(𝑥); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where 𝑓(𝑥) and/or 𝑔(𝑥) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | | MA.A-REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. | |

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| **Essential Questions** |
| What is the equation of a line given partial information?  What are some key characteristics of the graph of a quadratic function and how are they related to the coefficients of the equation?  What techniques and methods can be used to find the roots of higher degree polynomial functions?  What is the Fundamental Theorem of Algebra?  How do you determine the end-behavior of polynomial graphs?  How do you detect the degree of a polynomial?  How is a rational function graphed?  How are the domains of rational functions determined?  How are graphing calculators used to visualize and verify results? |

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| **Enduring Understanding** |
| Quadratic functions produce parabolic curves. Higher level polynomial functions produce smooth curves which cross the x-axis at the function’s zeros or roots.   Complex numbers extend the real number system to represent the roots of polynomial functions that do not cross the x-axis when the function is graphed. Rational functions include domain issues and their graphs display asymptotic behavior. |

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| **Students will know...** |
| 1. Quadratic Functions 2. Polynomial Functions 3. Complex Numbers 4. Rational Functions |

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| **Students will be able to...** |
| 1. Sketch parabolas. 2. Write equations of quadratic functions in standard form. 3. Graph polynomial functions, including their end behavior. 4. Use the Intermediate Value Theorem to help locate real roots or zeros. 5. Divide polynomials. 6. Use the Rational Root Theorem to identify potential rational roots and use synthetic division to test those roots. 7. Use Descartes Theorem to identify the number of positive and negative roots [Optional]. 8. Write polynomial equations given the roots. 9. Use the Fundamental Theorem of Algebra to find zeros of a polynomial function. 10. Perform arithmetic operations with complex numbers and write the results in standard form. 11. Find the domain, vertical and horizontal asymptotes, and sketch the graph of a rational function. |

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| **STAGE 2- EVIDENCE OF LEARNING** |
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| **Formative Assessment Suggestions** |
| |  |  | | --- | --- | | • 3- Minute Pause | **.** | | • A-B-C Summaries | **.** | | • Analogy Prompt | **.** | | • Choral Response | **.** | | • Debriefing | **.** | | • Exit Card / Ticket | **.** | | • Hand Signals | **.** | | • Idea Spinner | **.** | | • Index Card Summaries | **.** | | • Inside-Outside Circle Discussion (Fishbowl) | **.** | | • Journal Entry | **.** | | • Misconception Check | **.** | | • Observation | **.** | | • One Minute Essay | **.** | | • One Word Summary | **.** | | • Portfolio Check | **.** | | • Questions & Answers | **.** | | • Quiz | **.** | | • Self-Assessment | **.** | | • Student Conference | **.** | | • Think-Pair-Share | **.** | | • Web or Concept Map | **.** | |

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| **Authentic Assessments Suggestions** |
| Edementum Exact Path (BOY, MOY, EOY)  Class Discussions  Do Nows  Notebook Checks  Homework  Presentations  Webassign |

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| **Benchmark Assessments** |
| Edementum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3- LEARNING PLAN** |
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| **Instructional Map** |
| Topic #1 – Quadratic Functions  Topic #2 – Polynomial Functions  Topic #3 – Complex Numbers  Topic #4 – Rational Functions |

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| **Modifications/Differentiation of Instruction** |
| Differentiation Strategies for Special Education Students   * Remove unnecessary material, words, etc., that can distract from the content * Use of off-grade level materials * Provide appropriate scaffolding * Limit the number of steps required for completion * Time allowed * Level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Varied homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Ability to work at their own pace * Present ideas using auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment * Differentiated checklists and rubrics, if available and appropriate       Differentiation Strategies for Gifted and Talented Students   * Increase the level of complexity * Decrease scaffolding * Variety of finished products * Allow for greater independence * Learning stations, interest groups * Varied texts and supplementary materials * Use of technology * Flexibility in assignments * Varied questioning strategies * Encourage research * Strategy and flexible groups based on formative assessment or student choice * Acceleration within a unit of study * Exposure to more advanced or complex concepts, abstractions, and materials * Encourage students to move through content areas at their own pace * After mastery of a unit, provide students with more advanced learning activities, not more of the same activity * Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas       Differentiated Strategies for ELL Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials, including visuals * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Allow students to work at their own pace * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Role play * Provide graphic organizers, highlighted materials * Strategy and flexible groups based on formative assessment       Differentiation Strategies for At Risk Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment       **504 Plans**  Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:   * walk, breathe, eat, or sleep * communicate, see, hear, or speak * read, concentrate, think, or learn * stand, bend, lift, or work       Examples of accommodations in 504 plans include:   * preferential seating * extended time on tests and assignments * reduced homework or classwork * verbal, visual, or technology aids * modified textbooks or audio-video materials * behavior management support * adjusted class schedules or grading * verbal testing * excused lateness, absence, or missed classwork * pre-approved nurse's office visits and accompaniment to visits * occupational or physical therapy |

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| **Modification Strategies** |
| |  |  | | --- | --- | | • Extended Time | **.** | | • Frequent Breaks | **.** | | • Highlighted Text | **.** | | • Interactive Notebook | **.** | | • Modified Test | **.** | | • Oral Directions | **.** | | • Peer Tutoring | **.** | | • Preferential Seating | **.** | | • Re-Direct | **.** | | • Repeated Drill / Practice | **.** | | • Shortened Assignments | **.** | | • Teacher Notes | **.** | | • Tutorials | **.** | | • Use of Additional Reference Material | **.** | | • Use of Audio Resources | **.** | |

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| **High Preparation Differentiation** |
| |  |  | | --- | --- | | • Alternative Assessments | **.** | | • Choice Boards | **.** | | • Games and Tournaments | **.** | | • Group Investigations | **.** | | • Guided Reading | **.** | | • Independent Research / Project | **.** | | • Interest Groups | **.** | | • Learning Contracts | **.** | | • Leveled Rubrics | **.** | | • Literature Circles | **.** | | • Menu Assignments | **.** | | • Multiple Intelligence Options | **.** | | • Multiple Texts | **.** | | • Personal Agendas | **.** | | • Project Based Learning (PBL) | **.** | | • Stations / Centers | **.** | | • Think-Tac-Toe | **.** | | • Tiered Activities / Assignments | **.** | | • Varying Graphic Organizers | **.** | |

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| **Low Preparation Differentiation** |
| |  |  | | --- | --- | | • Choice of Book / Activity | **.** | | • Cubing Activities | **.** | | • Exploration by Interest (using interest inventories) | **.** | | • Flexible Grouping | **.** | | • Goal Setting With Student | **.** | | • Homework Options | **.** | | • Jigsaw | **.** | | • Mini Workshops to Extend Skills | **.** | | • Mini Workshops to Re-teach | **.** | | • Open-ended Activities | **.** | | • Think-Pair-Share by Interest | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Readiness | **.** | | • Use of Collaboration | **.** | | • Use of Reading Buddies | **.** | | • Varied Journal Prompts | **.** | | • Varied Product Choice | **.** | | • Varied Supplemental Materials | **.** | | • Work Alone / Together | **.** | |

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| **Horizontal Integration- Interdisciplinary Connections** |
| Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other. |

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| **Vertical Integration- Discipline Mapping** |
| The standards in this unit were introduced in Algebra I, Geometry and Algebra II. Precalculus coursework focuses on preparing the students be proficient in Calculus standards. |

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| **Additional Materials** |
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LPS Adopted Textbooks and Programs

* Cengage: Precalculus with Limits: A Graphing Approach
* WebAssign (Computer Based program supplementing Cengage)

Khan Academy

Edmentum Exact Path

**Unit 4: Exponential and Logarithmic functions.**

Content Area: **Mathematics**  
Course(s): **Pre-Calculus**  
Time Period: **Second Marking period**  
Length: **4 Weeks**  
Status: **Published**

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| **Unit Overview** |
| Exponential and logarithmic functions are inverses of each other.  Properties of exponents and logarithms are closely related and can be used to simplify expressions and solve equations.  Many applications can be modeled using exponential or logarithmic functions. |

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| **STAGE 1- DESIRED RESULTS** |
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| **Educational Standards** |
| The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:  1. Make sense of problems and persevre in solving them.  2. Reason abstractly and quantitatively.  3. Construct viable arguments and critique the reasoning of others.  4. Model with mathematics.  5. Use appropriate tools strategically.  6. Attend to precision.  7. Look for and make use of structure.  8. Look for and express regularity in repeated reasoning. |

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| **New Jersey Student Learning Standards- Mathematics** |
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| **Standard Set: Number and Quantity** |
| |  |  | | --- | --- | |  | During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers. | |  | With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. | |  | Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that (5 to the 1/3 power)³ should be 5 to the (1/3)³ power = 5¹ = 5 and that 5 to the 1/3 power should be the cube root of 5. | |  | Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents. | |  | In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them. | |

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| **Domain: The Real Number System** |
| |  |  | | --- | --- | | MA.N-RN.B.3 | Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. | | MA.N-RN.A | Extend the properties of exponents to rational exponents. | | MA.N-RN.A.1 | Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. | | MA.N-RN.A.2 | Rewrite expressions involving radicals and rational exponents using the properties of exponents. | | MA.N-RN.B | Use properties of rational and irrational numbers. | |

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| **Domain: Quantities** |
| |  |  | | --- | --- | | MA.N-Q.A | Reason quantitatively and use units to solve problems. | | MA.N-Q.A.1 | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | | MA.N-Q.A.2 | Define appropriate quantities for the purpose of descriptive modeling. | | MA.N-Q.A.3 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | |

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| **Domain: The Complex Number System** |
| |  |  | | --- | --- | | MA.N-CN.B.6 | Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. | | MA.N-CN.A | Perform arithmetic operations with complex numbers. | | MA.N-CN.A.1 | Know there is a complex number 𝑖 such that 𝑖² = -1, and every complex number has the form 𝑎 + 𝑏𝑖 with 𝑎 and 𝑏 real. | | MA.N-CN.A.2 | Use the relation 𝑖² = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. | | MA.N-CN.A.3 | Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. | | MA.N-CN.B | Represent complex numbers and their operations on the complex plane. | | MA.N-CN.B.4 | Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. | | MA.N-CN.B.5 | Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. | | MA.N-CN.C | Use complex numbers in polynomial identities and equations. | | MA.N-CN.C.7 | Solve quadratic equations with real coefficients that have complex solutions. | | MA.N-CN.C.8 | Extend polynomial identities to the complex numbers. | | MA.N-CN.C.9 | Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. | |

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| **Domain: Vector and Matrix Quantities** |
| |  |  | | --- | --- | | MA.N-VM.B.5a | Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as 𝑐(𝒗ₓ, 𝒗 subscript 𝑦) = (𝑐𝒗ₓ, 𝑐𝒗 subscript 𝑦). | | MA.N-VM.B.4c | Understand vector subtraction 𝒗 − 𝑤 as 𝒗 + (−𝑤), where –𝑤 is the additive inverse of 𝑤, with the same magnitude as 𝑤 and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. | | MA.N-VM.B.5b | Compute the magnitude of a scalar multiple 𝑐𝒗 using ||𝑐𝒗|| = |𝑐|𝒗. Compute the direction of 𝑐𝒗 knowing that when |𝑐|𝒗 ≠ 0, the direction of 𝑐𝒗 is either along 𝒗 (for 𝑐 > 0) or against 𝒗 (for 𝑐 < 0). | | MA.N-VM.A | Represent and model with vector quantities. | | MA.N-VM.C | Perform operations on matrices and use matrices in applications. | | MA.N-VM.C.6 | Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. | | MA.N-VM.C.7 | Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. | | MA.N-VM.C.8 | Add, subtract, and multiply matrices of appropriate dimensions. | | MA.N-VM.C.9 | Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. | | MA.N-VM.C.10 | Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. | | MA.N-VM.C.11 | Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. | | MA.N-VM.C.12 | Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area. | | MA.N-VM.A.1 | Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., 𝒗, |𝒗|, ||𝒗||, 𝒗). | | MA.N-VM.A.2 | Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. | | MA.N-VM.A.3 | Solve problems involving velocity and other quantities that can be represented by vectors. | | MA.N-VM.B | Perform operations on vectors. | | MA.N-VM.B.4 | Add and subtract vectors. | | MA.N-VM.B.5 | Multiply a vector by a scalar. | | MA.N-VM.B.4a | Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. | | MA.N-VM.B.4b | Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. | |

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| **Standard Set: Statistics and Probability** |
| |  |  | | --- | --- | |  | Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account. | |  | Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. | |  | Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. | |  | Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. | |  | Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time. | |  | Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient. | |

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| **Domain: Interpreting Categorical and Quantitative Data** |
| |  |  | | --- | --- | | MA.S-ID.B.6c | Fit a linear function for a scatter plot that suggests a linear association. | | MA.S-ID.C | Interpret linear models | | MA.S-ID.C.7 | Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | | MA.S-ID.C.8 | Compute (using technology) and interpret the correlation coefficient of a linear fit. | | MA.S-ID.C.9 | Distinguish between correlation and causation. | | MA.S-ID.A | Summarize, represent, and interpret data on a single count or measurement variable | | MA.S-ID.A.1 | Represent data with plots on the real number line (dot plots, histograms, and box plots). | | MA.S-ID.A.2 | Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | | MA.S-ID.A.3 | Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | | MA.S-ID.A.4 | Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. | | MA.S-ID.B | Summarize, represent, and interpret data on two categorical and quantitative variables | | MA.S-ID.B.5 | Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. | | MA.S-ID.B.6 | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. | | MA.S-ID.B.6a | Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. | | MA.S-ID.B.6b | Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology. | |

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| **Domain: Making Inferences and Justifying Conclusions** |
| |  |  | | --- | --- | | MA.S-IC.A | Understand and evaluate random processes underlying statistical experiments | | MA.S-IC.A.1 | Understand statistics as a process for making inferences about population parameters based on a random sample from that population. | | MA.S-IC.A.2 | Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. | | MA.S-IC.B | Make inferences and justify conclusions from sample surveys, experiments, and observational studies | | MA.S-IC.B.3 | Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. | | MA.S-IC.B.4 | Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. | | MA.S-IC.B.5 | Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. | | MA.S-IC.B.6 | Evaluate reports based on data. | |

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| **Domain: Conditional Probability and the Rules of Probability** |
| |  |  | | --- | --- | | MA.S-CP.B | Use the rules of probability to compute probabilities of compound events in a uniform probability model | | MA.S-CP.B.6 | Find the conditional probability of 𝐴 given 𝐵 as the fraction of 𝐵’s outcomes that also belong to 𝐴, and interpret the answer in terms of the model. | | MA.S-CP.B.7 | Apply the Addition Rule, 𝑃(𝐴 𝑜𝑟 𝐵) = 𝑃(𝐴) + 𝑃(𝐵) – 𝑃(𝐴 𝑎𝑛𝑑 𝐵), and interpret the answer in terms of the model. | | MA.S-CP.B.8 | Apply the general Multiplication Rule in a uniform probability model, 𝑃(𝐴 𝑎𝑛𝑑 𝐵) = [𝑃(𝐴)] × [𝑃(𝐵|𝐴)] = [𝑃(𝐵)] × [𝑃(𝐴|𝐵)], and interpret the answer in terms of the model. | | MA.S-CP.B.9 | Use permutations and combinations to compute probabilities of compound events and solve problems. | | MA.S-CP.A | Understand independence and conditional probability and use them to interpret data | | MA.S-CP.A.1 | Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). | | MA.S-CP.A.2 | Understand that two events 𝐴 and 𝐵 are independent if the probability of 𝐴 and 𝐵 occurring together is the product of their probabilities, and use this characterization to determine if they are independent. | | MA.S-CP.A.3 | Understand the conditional probability of 𝐴 given 𝐵 as 𝑃(𝐴 𝑎𝑛𝑑 𝐵)/𝑃(𝐵), and interpret independence of 𝐴 and 𝐵 as saying that the conditional probability of 𝐴 given 𝐵 is the same as the probability of 𝐴, and the conditional probability of 𝐵 given 𝐴 is the same as the probability of 𝐵. | | MA.S-CP.A.4 | Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. | | MA.S-CP.A.5 | Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. | |

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| **Domain: Using Probability to Make Decisions** |
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| **Standard Set: Geometry** |
| |  |  | | --- | --- | |  | Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations. | |  | Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. | |  | The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. | |  | An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. | |  | Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) | |  | During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. | |  | The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. | |  | In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. | |  | Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. | |  | The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion. | |

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| **Domain: Congruence** |
| |  |  | | --- | --- | | MA.G-CO.A.1 | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. | | MA.G-CO.A.2 | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). | | MA.G-CO.A.3 | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. | | MA.G-CO.A.4 | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. | | MA.G-CO.A.5 | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. | | MA.G-CO.D.12 | Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). | | MA.G-CO.B | Understand congruence in terms of rigid motions | | MA.G-CO.D.13 | Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. | | MA.G-CO.B.6 | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. | | MA.G-CO.B.7 | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. | | MA.G-CO.B.8 | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. | | MA.G-CO.C | Prove geometric theorems | | MA.G-CO.C.9 | Prove theorems about lines and angles. | | MA.G-CO.C.10 | Prove theorems about triangles. | | MA.G-CO.C.11 | Prove theorems about parallelograms. | | MA.G-CO.D | Make geometric constructions | | MA.G-CO.A | Experiment with transformations in the plane | |

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| **Domain: Similarity, Right Triangles, and Trigonometry** |
| |  |  | | --- | --- | | MA.G-SRT.D.9 | Derive the formula 𝐴 = (1/2)𝑎𝑏 𝑠𝑖𝑛(𝐶) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. | | MA.G-SRT.D.10 | Prove the Laws of Sines and Cosines and use them to solve problems. | | MA.G-SRT.D.11 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). | | MA.G-SRT.A | Understand similarity in terms of similarity transformations | | MA.G-SRT.A.1 | Verify experimentally the properties of dilations given by a center and a scale factor: | | MA.G-SRT.A.1a | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. | | MA.G-SRT.A.1b | The dilation of a line segment is longer or shorter in the ratio given by the scale factor. | | MA.G-SRT.A.2 | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. | | MA.G-SRT.A.3 | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. | | MA.G-SRT.B | Prove theorems involving similarity | | MA.G-SRT.B.4 | Prove theorems about triangles. | | MA.G-SRT.B.5 | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | | MA.G-SRT.C | Define trigonometric ratios and solve problems involving right triangles | | MA.G-SRT.C.6 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. | | MA.G-SRT.C.7 | Explain and use the relationship between the sine and cosine of complementary angles. | | MA.G-SRT.C.8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. | | MA.G-SRT.D | Apply trigonometry to general triangles | |

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| **Domain: Circles** |
| |  |  | | --- | --- | | MA.G-C.A | Understand and apply theorems about circles | | MA.G-C.A.1 | Prove that all circles are similar. | | MA.G-C.A.2 | Identify and describe relationships among inscribed angles, radii, and chords. | | MA.G-C.A.3 | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. | | MA.G-C.A.4 | Construct a tangent line from a point outside a given circle to the circle. | | MA.G-C.B | Find arc lengths and areas of sectors of circles | | MA.G-C.B.5 | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. | |

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| **Domain: Expressing Geometric Properties with Equations** |
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| **Domain: Geometric Measurement and Dimension** |
| |  |  | | --- | --- | | MA.G-GMD.B.4 | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. | | MA.G-GMD.A | Explain volume formulas and use them to solve problems | | MA.G-GMD.A.1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. | | MA.G-GMD.A.2 | Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures. | | MA.G-GMD.A.3 | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. | | MA.G-GMD.B | Visualize relationships between two-dimensional and three-dimensional objects | |

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| **Domain: Modeling with Geometry** |
| |  |  | | --- | --- | | MA.G-MG.A | Apply geometric concepts in modeling situations | | MA.G-MG.A.1 | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). | | MA.G-MG.A.2 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). | | MA.G-MG.A.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). | |

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| **Standard Set: Modeling** |
| |  |  | | --- | --- | |  | Designing the layout of the stalls in a school fair so as to raise as much money as possible. | |  | Analyzing stopping distance for a car. | |  | Modeling savings account balance, bacterial colony growth, or investment growth. | |  | Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport. | |  | Analyzing risk in situations such as extreme sports, pandemics, and terrorism. | |  | Relating population statistics to individual predictions. | |  | Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). | |  | Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. | |  | A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity. | |  | In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations. | |  | Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed. | |  | Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player. | |

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| **Standard Set: Functions** |
| |  |  | | --- | --- | |  | Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. | |  | In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car’s speed in miles per hour, v; the rule T(v) = 100/v expresses this relationship algebraically and defines a function whose name is T. | |  | The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. | |  | A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, “I’ll give you a state, you give me the capital city;” by an algebraic expression like f(x) = a + bx; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function’s properties. | |  | Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships. | |  | A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions. | |  | Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology. | |

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| **Domain: Interpreting Functions** |
| |  |  | | --- | --- | | MA.F-IF.C.8b | Use the properties of exponents to interpret expressions for exponential functions. | | MA.F-IF.A | Understand the concept of a function and use function notation | | MA.F-IF.A.1 | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If 𝑓 is a function and 𝑥 is an element of its domain, then 𝑓(𝑥) denotes the output of 𝑓 corresponding to the input 𝑥. The graph of 𝑓 is the graph of the equation𝑦 = 𝑓(𝑥). | | MA.F-IF.A.2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | | MA.F-IF.C.9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). | | MA.F-IF.A.3 | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. | | MA.F-IF.B | Interpret functions that arise in applications in terms of the context | | MA.F-IF.B.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. | | MA.F-IF.B.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. | | MA.F-IF.B.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | | MA.F-IF.C | Analyze functions using different representations | | MA.F-IF.C.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | | MA.F-IF.C.7a | Graph linear and quadratic functions and show intercepts, maxima, and minima. | | MA.F-IF.C.7b | Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. | | MA.F-IF.C.7c | Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7d | Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7e | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | | MA.F-IF.C.8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. | | MA.F-IF.C.8a | Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | |

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| **Domain: Building Functions** |
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| **Domain: Linear, Quadratic, and Exponential Models** |
| |  |  | | --- | --- | | MA.F-LE.A | Construct and compare linear and exponential models and solve problems | | MA.F-LE.A.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions. | | MA.F-LE.A.1a | Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. | | MA.F-LE.A.1b | Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. | | MA.F-LE.A.1c | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | | MA.F-LE.A.2 | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). | | MA.F-LE.A.3 | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | | MA.F-LE.A.4 | Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to 𝑎𝑏 to the 𝑐𝘵 power = 𝑑 where 𝑎, 𝑐, and 𝑑 are numbers and the base 𝑏 is 2, 10, or 𝑒; evaluate the logarithm using technology. | | MA.F-LE.B | Interpret expressions for functions in terms of the situation they model | | MA.F-LE.B.5 | Interpret the parameters in a linear or exponential function in terms of a context. | |

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| **Domain: Trigonometric Functions** |
| |  |  | | --- | --- | | MA.F-TF.A.3 | Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π – 𝑥, π + 𝑥, and 2π – 𝑥 in terms of their values for 𝑥, where 𝑥 is any real number. | | MA.F-TF.A.4 | Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. | | MA.F-TF.B | Model periodic phenomena with trigonometric functions | | MA.F-TF.B.5 | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. | | MA.F-TF.B.6 | Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. | | MA.F-TF.B.7 | Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. | | MA.F-TF.C | Prove and apply trigonometric identities | | MA.F-TF.C.8 | Prove the Pythagorean identity 𝑠𝑖𝑛²(θ) + 𝑐𝑜𝑠²(θ) = 1 and use it to find 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) given 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) and the quadrant of the angle. | | MA.F-TF.C.9 | Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. | | MA.F-TF.A | Extend the domain of trigonometric functions using the unit circle | | MA.F-TF.A.1 | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. | | MA.F-TF.A.2 | Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. | |

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| **Standard Set: Algebra** |
| |  |  | | --- | --- | |  | An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances. | |  | Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor. | |  | Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure. | |  | A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave. | |  | An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. | |  | The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system. | |  | An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. | |  | Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x² – 2 = 0 are real numbers, not rational numbers; and the solutions of x² + 2 = 0 are complex numbers, not real numbers. | |  | Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling. | |  | The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, 𝘈 = ((𝘣₁+𝘣₂)/2)𝘩, can be solved for 𝘩 using the same deductive process. | |  | Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them. | |

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| **Domain: Seeing Structures in Expressions** |
| |  |  | | --- | --- | | MA.A-SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. For example, see 𝑥⁴ – 𝑦⁴ as (𝑥²)² – (𝑦²)², thus recognizing it as a difference of squares that can be factored as (𝑥² – 𝑦²)(𝑥² + 𝑦²). | | MA.A-SSE.B | Write expressions in equivalent forms to solve problems | | MA.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. | | MA.A-SSE.B.3a | Factor a quadratic expression to reveal the zeros of the function it defines. | | MA.A-SSE.B.3b | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. | | MA.A-SSE.A | Interpret the structure of expressions | | MA.A-SSE.B.3c | Use the properties of exponents to transform expressions for exponential functions. | | MA.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context. | | MA.A-SSE.A.1a | Interpret parts of an expression, such as terms, factors, and coefficients. | | MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. | | MA.A-SSE.A.1b | Interpret complicated expressions by viewing one or more of their parts as a single entity. | |

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| **Domain: Arithmetic with Polynomials and Rational Functions** |
| |  |  | | --- | --- | | MA.A-APR.C.4 | Prove polynomial identities and use them to describe numerical relationships. | | MA.A-APR.A | Perform arithmetic operations on polynomials | | MA.A-APR.A.1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | | MA.A-APR.B | Understand the relationship between zeros and factors of polynomials | | MA.A-APR.B.2 | Know and apply the Remainder Theorem: For a polynomial 𝑝(𝑥) and a number 𝑎, the remainder on division by 𝑥 – 𝑎 is 𝑝(𝑎), so 𝑝(𝑎) = 0 if and only if (𝑥 – 𝑎) is a factor of 𝑝(𝑥). | | MA.A-APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. | | MA.A-APR.C.5 | Know and apply the Binomial Theorem for the expansion of (𝑥 + 𝑦)ⁿ in powers of 𝑥 and 𝑦 for a positive integer 𝑛, where 𝑥 and 𝑦 are any numbers, with coefficients determined for example by Pascal’s Triangle. | | MA.A-APR.D | Rewrite rational expressions | | MA.A-APR.D.6 | Rewrite simple rational expressions in different forms; write 𝑎(𝑥)/𝑏(𝑥) in the form 𝑞(𝑥) + 𝑟(𝑥)/𝑏(𝑥), where 𝑎(𝑥), 𝑏(𝑥), 𝑞(𝑥), and 𝑟(𝑥) are polynomials with the degree of 𝑟(𝑥) less than the degree of 𝑏(𝑥), using inspection, long division, or, for the more complicated examples, a computer algebra system. | | MA.A-APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. | | MA.A-APR.C | Use polynomial identities to solve problems | |

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| **Domain: Creating Equations** |
| |  |  | | --- | --- | | MA.A-CED.A | Create equations that describe numbers or relationships | | MA.A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems. | | MA.A-CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | | MA.A-CED.A.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. | | MA.A-CED.A.4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. | |

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| **Domain: Reasoning with Equations and Inequalities** |
| |  |  | | --- | --- | | MA.A-REI.B | Solve equations and inequalities in one variable | | MA.A-REI.B.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. | | MA.A-REI.B.4 | Solve quadratic equations in one variable. | | MA.A-REI.C | Solve systems of equations | | MA.A-REI.C.5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. | | MA.A-REI.C.6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. | | MA.A-REI.C.7 | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. | | MA.A-REI.C.8 | Represent a system of linear equations as a single matrix equation in a vector variable. | | MA.A-REI.C.9 | Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). | | MA.A-REI.D | Represent and solve equations and inequalities graphically | | MA.A-REI.B.4a | Use the method of completing the square to transform any quadratic equation in 𝑥 into an equation of the form (𝑥 – 𝑝)² = 𝑞 that has the same solutions. Derive the quadratic formula from this form. | | MA.A-REI.D.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). | | MA.A-REI.B.4b | Solve quadratic equations by inspection (e.g., for 𝑥² = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as 𝑎 ± 𝑏𝑖 for real numbers 𝑎 and 𝑏. | | MA.A-REI.A | Understand solving equations as a process of reasoning and explain the reasoning | | MA.A-REI.D.12 | Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | | MA.A-REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | | MA.A-REI.D.11 | Explain why the 𝑥-coordinates of the points where the graphs of the equations 𝑦 = 𝑓(𝑥) and 𝑦 = 𝑔(𝑥) intersect are the solutions of the equation𝑓(𝑥) = 𝑔(𝑥); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where 𝑓(𝑥) and/or 𝑔(𝑥) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | | MA.A-REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. | |

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| **Essential Questions** |
| What is the meaning of rational exponents?  What is the equation of an exponential function given partial information?  How can population growth, compound interest and radioactive decay be modeled using exponential functions?  What is the natural base? What is a logarithm?  How are exponential and logarithmic functions related?  What basic operations apply to logarithms? |

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| **Enduring Understanding** |
| Exponential and logarithmic functions are inverses of each other.  Properties of exponents and logarithms are closely related and can be used to simplify expressions and solve equations.  Many applications can be modeled using exponential or logarithmic functions. |

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| **Students will know...** |
| 1. Exponential Functions 2. Logarithmic Functions |

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| **Students will be able to...** |
| 1. Define and graph exponential functions. 2. Solve exponential equations. 3. Use exponential functions in applications. 4. Define and graph logarithmic functions. 5. Apply the properties of logarithms. 6. Solve logarithmic equations. 7. Use logarithmic functions in applications. |

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| **STAGE 2- EVIDENCE OF LEARNING** |
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| **Formative Assessment Suggestions** |
| |  |  | | --- | --- | | • 3- Minute Pause | **.** | | • A-B-C Summaries | **.** | | • Analogy Prompt | **.** | | • Choral Response | **.** | | • Debriefing | **.** | | • Exit Card / Ticket | **.** | | • Hand Signals | **.** | | • Idea Spinner | **.** | | • Index Card Summaries | **.** | | • Inside-Outside Circle Discussion (Fishbowl) | **.** | | • Journal Entry | **.** | | • Misconception Check | **.** | | • Observation | **.** | | • One Minute Essay | **.** | | • One Word Summary | **.** | | • Portfolio Check | **.** | | • Questions & Answers | **.** | | • Quiz | **.** | | • Self-Assessment | **.** | | • Student Conference | **.** | | • Think-Pair-Share | **.** | | • Web or Concept Map | **.** | |

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| **Authentic Assessments Suggestions** |
| Edementum Exact Path (BOY, MOY, EOY)  Class Discussions  Do Nows  Notebook Checks  Homework  Presentations  Webassign |

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| **Benchmark Assessments** |
| Edementum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3- LEARNING PLAN** |
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| **Instructional Map** |
| Topic #1 – Exponential Functions  Topic #2 – Logarithmic Functions |

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| **Modifications/Differentiation of Instruction** |
| Differentiation Strategies for Special Education Students   * Remove unnecessary material, words, etc., that can distract from the content * Use of off-grade level materials * Provide appropriate scaffolding * Limit the number of steps required for completion * Time allowed * Level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Varied homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Ability to work at their own pace * Present ideas using auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment * Differentiated checklists and rubrics, if available and appropriate       Differentiation Strategies for Gifted and Talented Students   * Increase the level of complexity * Decrease scaffolding * Variety of finished products * Allow for greater independence * Learning stations, interest groups * Varied texts and supplementary materials * Use of technology * Flexibility in assignments * Varied questioning strategies * Encourage research * Strategy and flexible groups based on formative assessment or student choice * Acceleration within a unit of study * Exposure to more advanced or complex concepts, abstractions, and materials * Encourage students to move through content areas at their own pace * After mastery of a unit, provide students with more advanced learning activities, not more of the same activity * Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas       Differentiated Strategies for ELL Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials, including visuals * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Allow students to work at their own pace * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Role play * Provide graphic organizers, highlighted materials * Strategy and flexible groups based on formative assessment       Differentiation Strategies for At Risk Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment       **504 Plans**  Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:   * walk, breathe, eat, or sleep * communicate, see, hear, or speak * read, concentrate, think, or learn * stand, bend, lift, or work       Examples of accommodations in 504 plans include:   * preferential seating * extended time on tests and assignments * reduced homework or classwork * verbal, visual, or technology aids * modified textbooks or audio-video materials * behavior management support * adjusted class schedules or grading * verbal testing * excused lateness, absence, or missed classwork * pre-approved nurse's office visits and accompaniment to visits * occupational or physical therapy |

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| **Modification Strategies** |
| |  |  | | --- | --- | | • Extended Time | **.** | | • Frequent Breaks | **.** | | • Highlighted Text | **.** | | • Interactive Notebook | **.** | | • Modified Test | **.** | | • Oral Directions | **.** | | • Peer Tutoring | **.** | | • Preferential Seating | **.** | | • Re-Direct | **.** | | • Repeated Drill / Practice | **.** | | • Shortened Assignments | **.** | | • Teacher Notes | **.** | | • Tutorials | **.** | | • Use of Additional Reference Material | **.** | | • Use of Audio Resources | **.** | |

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| **High Preparation Differentiation** |
| |  |  | | --- | --- | | • Alternative Assessments | **.** | | • Choice Boards | **.** | | • Games and Tournaments | **.** | | • Group Investigations | **.** | | • Guided Reading | **.** | | • Independent Research / Project | **.** | | • Interest Groups | **.** | | • Learning Contracts | **.** | | • Leveled Rubrics | **.** | | • Literature Circles | **.** | | • Menu Assignments | **.** | | • Multiple Intelligence Options | **.** | | • Multiple Texts | **.** | | • Personal Agendas | **.** | | • Project Based Learning (PBL) | **.** | | • Stations / Centers | **.** | | • Think-Tac-Toe | **.** | | • Tiered Activities / Assignments | **.** | | • Varying Graphic Organizers | **.** | |

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| **Low Preparation Differentiation** |
| |  |  | | --- | --- | | • Choice of Book / Activity | **.** | | • Cubing Activities | **.** | | • Exploration by Interest (using interest inventories) | **.** | | • Flexible Grouping | **.** | | • Goal Setting With Student | **.** | | • Homework Options | **.** | | • Jigsaw | **.** | | • Mini Workshops to Extend Skills | **.** | | • Mini Workshops to Re-teach | **.** | | • Open-ended Activities | **.** | | • Think-Pair-Share by Interest | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Readiness | **.** | | • Use of Collaboration | **.** | | • Use of Reading Buddies | **.** | | • Varied Journal Prompts | **.** | | • Varied Product Choice | **.** | | • Varied Supplemental Materials | **.** | | • Work Alone / Together | **.** | |

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| **Horizontal Integration- Interdisciplinary Connections** |
| Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other. |

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| **Vertical Integration- Discipline Mapping** |
| The standards in this unit were introduced in Algebra I, Geometry and Algebra II. Precalculus coursework focuses on preparing the students be proficient in Calculus standards. |

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| **Additional Materials** |
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LPS Adopted Textbooks and Programs

* Cengage: Precalculus with Limits: A Graphing Approach
* WebAssign (Computer Based program supplementing Cengage)

Khan Academy

Edmentum Exact Path

**Unit 5: Trigonometric Functions**

Content Area: **Mathematics**  
Course(s): **Pre-Calculus**  
Time Period: **Second Marking period**  
Length: **4 Week**  
Status: **Published**

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| **Unit Overview** |
| Angles can be measured in revolutions, degrees or radians.  The six trigonometric functions can be extended to angles of any measure using the circular definition and periodic nature of trigonometric functions.  Trigonometric functions can be transformed by translation, reflection or non-rigid stretches or shrinks.  Non-rigid transformations result in period or amplitude changes whereas horizontal translations are called phase shifts. Restricting the domains of trigonometric functions allows us to define inverse trigonometric functions. |

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| **STAGE 1- DESIRED RESULTS** |
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| **Educational Standards** |
| The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:  1. Make sense of problems and persevre in solving them.  2. Reason abstractly and quantitatively.  3. Construct viable arguments and critique the reasoning of others.  4. Model with mathematics.  5. Use appropriate tools strategically.  6. Attend to precision.  7. Look for and make use of structure.  8. Look for and express regularity in repeated reasoning. |

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| **New Jersey Student Learning Standards- Mathematics** |
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| **Standard Set: Number and Quantity** |
| |  |  | | --- | --- | |  | During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers. | |  | With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. | |  | Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that (5 to the 1/3 power)³ should be 5 to the (1/3)³ power = 5¹ = 5 and that 5 to the 1/3 power should be the cube root of 5. | |  | Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents. | |  | In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them. | |

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| **Domain: The Real Number System** |
| |  |  | | --- | --- | | MA.N-RN.B.3 | Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. | | MA.N-RN.A | Extend the properties of exponents to rational exponents. | | MA.N-RN.A.1 | Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. | | MA.N-RN.A.2 | Rewrite expressions involving radicals and rational exponents using the properties of exponents. | | MA.N-RN.B | Use properties of rational and irrational numbers. | |

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| **Domain: Quantities** |
| |  |  | | --- | --- | | MA.N-Q.A | Reason quantitatively and use units to solve problems. | | MA.N-Q.A.1 | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | | MA.N-Q.A.2 | Define appropriate quantities for the purpose of descriptive modeling. | | MA.N-Q.A.3 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | |

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| **Domain: The Complex Number System** |
| |  |  | | --- | --- | | MA.N-CN.B.6 | Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. | | MA.N-CN.A | Perform arithmetic operations with complex numbers. | | MA.N-CN.A.1 | Know there is a complex number 𝑖 such that 𝑖² = -1, and every complex number has the form 𝑎 + 𝑏𝑖 with 𝑎 and 𝑏 real. | | MA.N-CN.A.2 | Use the relation 𝑖² = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. | | MA.N-CN.A.3 | Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. | | MA.N-CN.B | Represent complex numbers and their operations on the complex plane. | | MA.N-CN.B.4 | Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. | | MA.N-CN.B.5 | Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. | | MA.N-CN.C | Use complex numbers in polynomial identities and equations. | | MA.N-CN.C.7 | Solve quadratic equations with real coefficients that have complex solutions. | | MA.N-CN.C.8 | Extend polynomial identities to the complex numbers. | | MA.N-CN.C.9 | Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. | |

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| **Domain: Vector and Matrix Quantities** |
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| **Standard Set: Statistics and Probability** |
| |  |  | | --- | --- | |  | Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account. | |  | Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. | |  | Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. | |  | Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. | |  | Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time. | |  | Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient. | |

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| **Domain: Interpreting Categorical and Quantitative Data** |
| |  |  | | --- | --- | | MA.S-ID.B.6c | Fit a linear function for a scatter plot that suggests a linear association. | | MA.S-ID.C | Interpret linear models | | MA.S-ID.C.7 | Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | | MA.S-ID.C.8 | Compute (using technology) and interpret the correlation coefficient of a linear fit. | | MA.S-ID.C.9 | Distinguish between correlation and causation. | | MA.S-ID.A | Summarize, represent, and interpret data on a single count or measurement variable | | MA.S-ID.A.1 | Represent data with plots on the real number line (dot plots, histograms, and box plots). | | MA.S-ID.A.2 | Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | | MA.S-ID.A.3 | Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | | MA.S-ID.A.4 | Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. | | MA.S-ID.B | Summarize, represent, and interpret data on two categorical and quantitative variables | | MA.S-ID.B.5 | Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. | | MA.S-ID.B.6 | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. | | MA.S-ID.B.6a | Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. | | MA.S-ID.B.6b | Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology. | |

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| **Domain: Making Inferences and Justifying Conclusions** |
| |  |  | | --- | --- | | MA.S-IC.A | Understand and evaluate random processes underlying statistical experiments | | MA.S-IC.A.1 | Understand statistics as a process for making inferences about population parameters based on a random sample from that population. | | MA.S-IC.A.2 | Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. | | MA.S-IC.B | Make inferences and justify conclusions from sample surveys, experiments, and observational studies | | MA.S-IC.B.3 | Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. | | MA.S-IC.B.4 | Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. | | MA.S-IC.B.5 | Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. | | MA.S-IC.B.6 | Evaluate reports based on data. | |

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| **Domain: Conditional Probability and the Rules of Probability** |
| |  |  | | --- | --- | | MA.S-CP.B | Use the rules of probability to compute probabilities of compound events in a uniform probability model | | MA.S-CP.B.6 | Find the conditional probability of 𝐴 given 𝐵 as the fraction of 𝐵’s outcomes that also belong to 𝐴, and interpret the answer in terms of the model. | | MA.S-CP.B.7 | Apply the Addition Rule, 𝑃(𝐴 𝑜𝑟 𝐵) = 𝑃(𝐴) + 𝑃(𝐵) – 𝑃(𝐴 𝑎𝑛𝑑 𝐵), and interpret the answer in terms of the model. | | MA.S-CP.B.8 | Apply the general Multiplication Rule in a uniform probability model, 𝑃(𝐴 𝑎𝑛𝑑 𝐵) = [𝑃(𝐴)] × [𝑃(𝐵|𝐴)] = [𝑃(𝐵)] × [𝑃(𝐴|𝐵)], and interpret the answer in terms of the model. | | MA.S-CP.B.9 | Use permutations and combinations to compute probabilities of compound events and solve problems. | | MA.S-CP.A | Understand independence and conditional probability and use them to interpret data | | MA.S-CP.A.1 | Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). | | MA.S-CP.A.2 | Understand that two events 𝐴 and 𝐵 are independent if the probability of 𝐴 and 𝐵 occurring together is the product of their probabilities, and use this characterization to determine if they are independent. | | MA.S-CP.A.3 | Understand the conditional probability of 𝐴 given 𝐵 as 𝑃(𝐴 𝑎𝑛𝑑 𝐵)/𝑃(𝐵), and interpret independence of 𝐴 and 𝐵 as saying that the conditional probability of 𝐴 given 𝐵 is the same as the probability of 𝐴, and the conditional probability of 𝐵 given 𝐴 is the same as the probability of 𝐵. | | MA.S-CP.A.4 | Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. | | MA.S-CP.A.5 | Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. | |

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| **Domain: Using Probability to Make Decisions** |
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| **Standard Set: Geometry** |
| |  |  | | --- | --- | |  | Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations. | |  | Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. | |  | The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. | |  | An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. | |  | Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) | |  | During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. | |  | The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. | |  | In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. | |  | Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. | |  | The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion. | |

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| **Domain: Congruence** |
| |  |  | | --- | --- | | MA.G-CO.A.1 | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. | | MA.G-CO.A.2 | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). | | MA.G-CO.A.3 | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. | | MA.G-CO.A.4 | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. | | MA.G-CO.A.5 | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. | | MA.G-CO.D.12 | Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). | | MA.G-CO.B | Understand congruence in terms of rigid motions | | MA.G-CO.D.13 | Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. | | MA.G-CO.B.6 | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. | | MA.G-CO.B.7 | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. | | MA.G-CO.B.8 | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. | | MA.G-CO.C | Prove geometric theorems | | MA.G-CO.C.9 | Prove theorems about lines and angles. | | MA.G-CO.C.10 | Prove theorems about triangles. | | MA.G-CO.C.11 | Prove theorems about parallelograms. | | MA.G-CO.D | Make geometric constructions | | MA.G-CO.A | Experiment with transformations in the plane | |

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| **Domain: Similarity, Right Triangles, and Trigonometry** |
| |  |  | | --- | --- | | MA.G-SRT.D.9 | Derive the formula 𝐴 = (1/2)𝑎𝑏 𝑠𝑖𝑛(𝐶) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. | | MA.G-SRT.D.10 | Prove the Laws of Sines and Cosines and use them to solve problems. | | MA.G-SRT.D.11 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). | | MA.G-SRT.A | Understand similarity in terms of similarity transformations | | MA.G-SRT.A.1 | Verify experimentally the properties of dilations given by a center and a scale factor: | | MA.G-SRT.A.1a | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. | | MA.G-SRT.A.1b | The dilation of a line segment is longer or shorter in the ratio given by the scale factor. | | MA.G-SRT.A.2 | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. | | MA.G-SRT.A.3 | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. | | MA.G-SRT.B | Prove theorems involving similarity | | MA.G-SRT.B.4 | Prove theorems about triangles. | | MA.G-SRT.B.5 | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | | MA.G-SRT.C | Define trigonometric ratios and solve problems involving right triangles | | MA.G-SRT.C.6 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. | | MA.G-SRT.C.7 | Explain and use the relationship between the sine and cosine of complementary angles. | | MA.G-SRT.C.8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. | | MA.G-SRT.D | Apply trigonometry to general triangles | |

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| **Domain: Circles** |
| |  |  | | --- | --- | | MA.G-C.A | Understand and apply theorems about circles | | MA.G-C.A.1 | Prove that all circles are similar. | | MA.G-C.A.2 | Identify and describe relationships among inscribed angles, radii, and chords. | | MA.G-C.A.3 | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. | | MA.G-C.A.4 | Construct a tangent line from a point outside a given circle to the circle. | | MA.G-C.B | Find arc lengths and areas of sectors of circles | | MA.G-C.B.5 | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. | |

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| **Domain: Expressing Geometric Properties with Equations** |
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| **Domain: Geometric Measurement and Dimension** |
| |  |  | | --- | --- | | MA.G-GMD.B.4 | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. | | MA.G-GMD.A | Explain volume formulas and use them to solve problems | | MA.G-GMD.A.1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. | | MA.G-GMD.A.2 | Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures. | | MA.G-GMD.A.3 | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. | | MA.G-GMD.B | Visualize relationships between two-dimensional and three-dimensional objects | |

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| **Domain: Modeling with Geometry** |
| |  |  | | --- | --- | | MA.G-MG.A | Apply geometric concepts in modeling situations | | MA.G-MG.A.1 | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). | | MA.G-MG.A.2 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). | | MA.G-MG.A.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). | |

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| **Standard Set: Modeling** |
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| **Standard Set: Functions** |
| |  |  | | --- | --- | |  | Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. | |  | In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car’s speed in miles per hour, v; the rule T(v) = 100/v expresses this relationship algebraically and defines a function whose name is T. | |  | The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. | |  | A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, “I’ll give you a state, you give me the capital city;” by an algebraic expression like f(x) = a + bx; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function’s properties. | |  | Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships. | |  | A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions. | |  | Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology. | |

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| **Domain: Interpreting Functions** |
| |  |  | | --- | --- | | MA.F-IF.C.8b | Use the properties of exponents to interpret expressions for exponential functions. | | MA.F-IF.A | Understand the concept of a function and use function notation | | MA.F-IF.A.1 | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If 𝑓 is a function and 𝑥 is an element of its domain, then 𝑓(𝑥) denotes the output of 𝑓 corresponding to the input 𝑥. The graph of 𝑓 is the graph of the equation𝑦 = 𝑓(𝑥). | | MA.F-IF.A.2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | | MA.F-IF.C.9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). | | MA.F-IF.A.3 | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. | | MA.F-IF.B | Interpret functions that arise in applications in terms of the context | | MA.F-IF.B.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. | | MA.F-IF.B.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. | | MA.F-IF.B.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | | MA.F-IF.C | Analyze functions using different representations | | MA.F-IF.C.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | | MA.F-IF.C.7a | Graph linear and quadratic functions and show intercepts, maxima, and minima. | | MA.F-IF.C.7b | Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. | | MA.F-IF.C.7c | Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7d | Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7e | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | | MA.F-IF.C.8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. | | MA.F-IF.C.8a | Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | |

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| **Domain: Building Functions** |
| |  |  | | --- | --- | | MA.F-BF.B.4b | Verify by composition that one function is the inverse of another. | | MA.F-BF.B.4c | Read values of an inverse function from a graph or a table, given that the function has an inverse. | | MA.F-BF.B.4d | Produce an invertible function from a non-invertible function by restricting the domain. | | MA.F-BF.B.5 | Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents. | | MA.F-BF.A.1 | Write a function that describes a relationship between two quantities. | | MA.F-BF.A.1a | Determine an explicit expression, a recursive process, or steps for calculation from a context. | | MA.F-BF.A.1b | Combine standard function types using arithmetic operations. | | MA.F-BF.A | Build a function that models a relationship between two quantities | | MA.F-BF.A.1c | Compose functions. | | MA.F-BF.A.2 | Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. | | MA.F-BF.B | Build new functions from existing functions | | MA.F-BF.B.3 | Identify the effect on the graph of replacing 𝑓(𝑥) by 𝑓(𝑥) + 𝑘, 𝑘𝑓(𝑥), 𝑓(𝑘𝑥), and 𝑓(𝑥 + 𝑘) for specific values of 𝑘 (both positive and negative); find the value of 𝑘 given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. | | MA.F-BF.B.4 | Find inverse functions. | | MA.F-BF.B.4a | Solve an equation of the form 𝑓(𝑥) = 𝑐 for a simple function 𝑓 that has an inverse and write an expression for the inverse. | |

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| **Domain: Linear, Quadratic, and Exponential Models** |
| |  |  | | --- | --- | | MA.F-LE.A | Construct and compare linear and exponential models and solve problems | | MA.F-LE.A.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions. | | MA.F-LE.A.1a | Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. | | MA.F-LE.A.1b | Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. | | MA.F-LE.A.1c | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | | MA.F-LE.A.2 | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). | | MA.F-LE.A.3 | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | | MA.F-LE.A.4 | Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to 𝑎𝑏 to the 𝑐𝘵 power = 𝑑 where 𝑎, 𝑐, and 𝑑 are numbers and the base 𝑏 is 2, 10, or 𝑒; evaluate the logarithm using technology. | | MA.F-LE.B | Interpret expressions for functions in terms of the situation they model | | MA.F-LE.B.5 | Interpret the parameters in a linear or exponential function in terms of a context. | |

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| **Domain: Trigonometric Functions** |
| |  |  | | --- | --- | | MA.F-TF.A.3 | Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π – 𝑥, π + 𝑥, and 2π – 𝑥 in terms of their values for 𝑥, where 𝑥 is any real number. | | MA.F-TF.A.4 | Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. | | MA.F-TF.B | Model periodic phenomena with trigonometric functions | | MA.F-TF.B.5 | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. | | MA.F-TF.B.6 | Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. | | MA.F-TF.B.7 | Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. | | MA.F-TF.C | Prove and apply trigonometric identities | | MA.F-TF.C.8 | Prove the Pythagorean identity 𝑠𝑖𝑛²(θ) + 𝑐𝑜𝑠²(θ) = 1 and use it to find 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) given 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) and the quadrant of the angle. | | MA.F-TF.C.9 | Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. | | MA.F-TF.A | Extend the domain of trigonometric functions using the unit circle | | MA.F-TF.A.1 | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. | | MA.F-TF.A.2 | Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. | |

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| **Standard Set: Algebra** |
| |  |  | | --- | --- | |  | An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances. | |  | Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor. | |  | Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure. | |  | A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave. | |  | An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. | |  | The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system. | |  | An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. | |  | Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x² – 2 = 0 are real numbers, not rational numbers; and the solutions of x² + 2 = 0 are complex numbers, not real numbers. | |  | Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling. | |  | The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, 𝘈 = ((𝘣₁+𝘣₂)/2)𝘩, can be solved for 𝘩 using the same deductive process. | |  | Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them. | |

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| **Domain: Seeing Structures in Expressions** |
| |  |  | | --- | --- | | MA.A-SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. For example, see 𝑥⁴ – 𝑦⁴ as (𝑥²)² – (𝑦²)², thus recognizing it as a difference of squares that can be factored as (𝑥² – 𝑦²)(𝑥² + 𝑦²). | | MA.A-SSE.B | Write expressions in equivalent forms to solve problems | | MA.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. | | MA.A-SSE.B.3a | Factor a quadratic expression to reveal the zeros of the function it defines. | | MA.A-SSE.B.3b | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. | | MA.A-SSE.A | Interpret the structure of expressions | | MA.A-SSE.B.3c | Use the properties of exponents to transform expressions for exponential functions. | | MA.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context. | | MA.A-SSE.A.1a | Interpret parts of an expression, such as terms, factors, and coefficients. | | MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. | | MA.A-SSE.A.1b | Interpret complicated expressions by viewing one or more of their parts as a single entity. | |

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| **Domain: Arithmetic with Polynomials and Rational Functions** |
| |  |  | | --- | --- | | MA.A-APR.C.4 | Prove polynomial identities and use them to describe numerical relationships. | | MA.A-APR.A | Perform arithmetic operations on polynomials | | MA.A-APR.A.1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | | MA.A-APR.B | Understand the relationship between zeros and factors of polynomials | | MA.A-APR.B.2 | Know and apply the Remainder Theorem: For a polynomial 𝑝(𝑥) and a number 𝑎, the remainder on division by 𝑥 – 𝑎 is 𝑝(𝑎), so 𝑝(𝑎) = 0 if and only if (𝑥 – 𝑎) is a factor of 𝑝(𝑥). | | MA.A-APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. | | MA.A-APR.C.5 | Know and apply the Binomial Theorem for the expansion of (𝑥 + 𝑦)ⁿ in powers of 𝑥 and 𝑦 for a positive integer 𝑛, where 𝑥 and 𝑦 are any numbers, with coefficients determined for example by Pascal’s Triangle. | | MA.A-APR.D | Rewrite rational expressions | | MA.A-APR.D.6 | Rewrite simple rational expressions in different forms; write 𝑎(𝑥)/𝑏(𝑥) in the form 𝑞(𝑥) + 𝑟(𝑥)/𝑏(𝑥), where 𝑎(𝑥), 𝑏(𝑥), 𝑞(𝑥), and 𝑟(𝑥) are polynomials with the degree of 𝑟(𝑥) less than the degree of 𝑏(𝑥), using inspection, long division, or, for the more complicated examples, a computer algebra system. | | MA.A-APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. | | MA.A-APR.C | Use polynomial identities to solve problems | |

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| **Domain: Creating Equations** |
| |  |  | | --- | --- | | MA.A-CED.A | Create equations that describe numbers or relationships | | MA.A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems. | | MA.A-CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | | MA.A-CED.A.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. | | MA.A-CED.A.4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. | |

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| **Domain: Reasoning with Equations and Inequalities** |
| |  |  | | --- | --- | | MA.A-REI.B | Solve equations and inequalities in one variable | | MA.A-REI.B.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. | | MA.A-REI.B.4 | Solve quadratic equations in one variable. | | MA.A-REI.C | Solve systems of equations | | MA.A-REI.C.5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. | | MA.A-REI.C.6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. | | MA.A-REI.C.7 | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. | | MA.A-REI.C.8 | Represent a system of linear equations as a single matrix equation in a vector variable. | | MA.A-REI.C.9 | Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). | | MA.A-REI.D | Represent and solve equations and inequalities graphically | | MA.A-REI.B.4a | Use the method of completing the square to transform any quadratic equation in 𝑥 into an equation of the form (𝑥 – 𝑝)² = 𝑞 that has the same solutions. Derive the quadratic formula from this form. | | MA.A-REI.D.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). | | MA.A-REI.B.4b | Solve quadratic equations by inspection (e.g., for 𝑥² = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as 𝑎 ± 𝑏𝑖 for real numbers 𝑎 and 𝑏. | | MA.A-REI.A | Understand solving equations as a process of reasoning and explain the reasoning | | MA.A-REI.D.12 | Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | | MA.A-REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | | MA.A-REI.D.11 | Explain why the 𝑥-coordinates of the points where the graphs of the equations 𝑦 = 𝑓(𝑥) and 𝑦 = 𝑔(𝑥) intersect are the solutions of the equation𝑓(𝑥) = 𝑔(𝑥); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where 𝑓(𝑥) and/or 𝑔(𝑥) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | | MA.A-REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. | |

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| **Essential Questions** |
| How can you convert between degree and radian angle measure?  What are the domains, ranges, and periods of the six trigonometric functions?  How can standard transformations be applied to trigonometric graphs?  What is the unit circle and how is it used to determine the trigonometric values for key angles?  What are reference angles?  Why do the domains of the trigonometric functions need to be restricted before the inverse functions can be found?  How do you evaluate the compositions of trigonometric functions and their inverses? |

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| **Enduring Understanding** |
| Angles can be measured in revolutions, degrees or radians.  The six trigonometric functions can be extended to angles of any measure using the circular definition and periodic nature of trigonometric functions.  Trigonometric functions can be transformed by translation, reflection or non-rigid stretches or shrinks.  Non-rigid transformations result in period or amplitude changes whereas horizontal translations are called phase shifts. Restricting the domains of trigonometric functions allows us to define inverse trigonometric functions. |

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| **Students will know...** |
| 1. Trigonometric Functions 2. Graphs of Trigonometric Functions 3. Applications of Trigonometry |

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| **Students will be able to...** |
| 1. Understand angles as a measure of rotation and express in terms of degree and radian measures. 2. Evaluate trigonometric functions using the unit circle, right triangles, and any angle. 3. Construct graphs of sine, cosine, tangent, cotangent, secant, and cosecant functions. 4. Identify the period, amplitude, domain and range of the six trigonometric functions.  Identify whether a given trigonometric function is even or odd. 5. Apply transformations to trigonometric functions, and understand impact on period, amplitude, and other aspects. 6. Understand the meaning of function and inverse and how to apply them to trigonometric functions. 7. Apply trigonometry to solve real-world problems, including ones involving angle of elevation and angle of depression. |

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| **STAGE 2- EVIDENCE OF LEARNING** |
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| **Formative Assessment Suggestions** |
| |  |  | | --- | --- | | • 3- Minute Pause | **.** | | • A-B-C Summaries | **.** | | • Analogy Prompt | **.** | | • Choral Response | **.** | | • Debriefing | **.** | | • Exit Card / Ticket | **.** | | • Hand Signals | **.** | | • Idea Spinner | **.** | | • Index Card Summaries | **.** | | • Inside-Outside Circle Discussion (Fishbowl) | **.** | | • Journal Entry | **.** | | • Misconception Check | **.** | | • Observation | **.** | | • One Minute Essay | **.** | | • One Word Summary | **.** | | • Portfolio Check | **.** | | • Questions & Answers | **.** | | • Quiz | **.** | | • Self-Assessment | **.** | | • Student Conference | **.** | | • Think-Pair-Share | **.** | | • Web or Concept Map | **.** | |

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| **Authentic Assessments Suggestions** |
| Edementum Exact Path (BOY, MOY, EOY)  Class Discussions  Do Nows  Notebook Checks  Homework  Presentations  Webassign |

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| **Benchmark Assessments** |
| Edementum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3- LEARNING PLAN** |
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| **Instructional Map** |
| Topic #1 – Trigonometric Functions  Topic #2 – Graphs of Trigonometric Functions  Topic #3 – Applications of Trigonometry |

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| **Modifications/Differentiation of Instruction** |
| Differentiation Strategies for Special Education Students   * Remove unnecessary material, words, etc., that can distract from the content * Use of off-grade level materials * Provide appropriate scaffolding * Limit the number of steps required for completion * Time allowed * Level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Varied homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Ability to work at their own pace * Present ideas using auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment * Differentiated checklists and rubrics, if available and appropriate       Differentiation Strategies for Gifted and Talented Students   * Increase the level of complexity * Decrease scaffolding * Variety of finished products * Allow for greater independence * Learning stations, interest groups * Varied texts and supplementary materials * Use of technology * Flexibility in assignments * Varied questioning strategies * Encourage research * Strategy and flexible groups based on formative assessment or student choice * Acceleration within a unit of study * Exposure to more advanced or complex concepts, abstractions, and materials * Encourage students to move through content areas at their own pace * After mastery of a unit, provide students with more advanced learning activities, not more of the same activity * Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas       Differentiated Strategies for ELL Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials, including visuals * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Allow students to work at their own pace * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Role play * Provide graphic organizers, highlighted materials * Strategy and flexible groups based on formative assessment       Differentiation Strategies for At Risk Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment       **504 Plans**  Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:   * walk, breathe, eat, or sleep * communicate, see, hear, or speak * read, concentrate, think, or learn * stand, bend, lift, or work       Examples of accommodations in 504 plans include:   * preferential seating * extended time on tests and assignments * reduced homework or classwork * verbal, visual, or technology aids * modified textbooks or audio-video materials * behavior management support * adjusted class schedules or grading * verbal testing * excused lateness, absence, or missed classwork * pre-approved nurse's office visits and accompaniment to visits * occupational or physical therapy |

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| **Modification Strategies** |
| |  |  | | --- | --- | | • Extended Time | **.** | | • Frequent Breaks | **.** | | • Highlighted Text | **.** | | • Interactive Notebook | **.** | | • Modified Test | **.** | | • Oral Directions | **.** | | • Peer Tutoring | **.** | | • Preferential Seating | **.** | | • Re-Direct | **.** | | • Repeated Drill / Practice | **.** | | • Shortened Assignments | **.** | | • Teacher Notes | **.** | | • Tutorials | **.** | | • Use of Additional Reference Material | **.** | | • Use of Audio Resources | **.** | |

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| **High Preparation Differentiation** |
| |  |  | | --- | --- | | • Alternative Assessments | **.** | | • Choice Boards | **.** | | • Games and Tournaments | **.** | | • Group Investigations | **.** | | • Guided Reading | **.** | | • Independent Research / Project | **.** | | • Interest Groups | **.** | | • Learning Contracts | **.** | | • Leveled Rubrics | **.** | | • Literature Circles | **.** | | • Menu Assignments | **.** | | • Multiple Intelligence Options | **.** | | • Multiple Texts | **.** | | • Personal Agendas | **.** | | • Project Based Learning (PBL) | **.** | | • Stations / Centers | **.** | | • Think-Tac-Toe | **.** | | • Tiered Activities / Assignments | **.** | | • Varying Graphic Organizers | **.** | |

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| **Low Preparation Differentiation** |
| |  |  | | --- | --- | | • Choice of Book / Activity | **.** | | • Cubing Activities | **.** | | • Exploration by Interest (using interest inventories) | **.** | | • Flexible Grouping | **.** | | • Goal Setting With Student | **.** | | • Homework Options | **.** | | • Jigsaw | **.** | | • Mini Workshops to Extend Skills | **.** | | • Mini Workshops to Re-teach | **.** | | • Open-ended Activities | **.** | | • Think-Pair-Share by Interest | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Readiness | **.** | | • Use of Collaboration | **.** | | • Use of Reading Buddies | **.** | | • Varied Journal Prompts | **.** | | • Varied Product Choice | **.** | | • Varied Supplemental Materials | **.** | | • Work Alone / Together | **.** | |

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| **Horizontal Integration- Interdisciplinary Connections** |
| Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other. |

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| **Vertical Integration- Discipline Mapping** |
| The standards in this unit were introduced in Algebra I, Geometry and Algebra II. Precalculus coursework focuses on preparing the students be proficient in Calculus standards. |

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| **Additional Materials** |
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LPS Adopted Textbooks and Programs

* Cengage: Precalculus with Limits: A Graphing Approach
* WebAssign (Computer Based program supplementing Cengage)

Khan Academy

Edmentum Exact Path

**Unit 6: Solving Triangles**

Content Area: **Mathematics**  
Course(s): **Pre-Calculus**  
Time Period: **Third Marking Period**  
Length: **4 Weeks**  
Status: **Published**

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| **Unit Overview** |
| Trigonometry is extended using the Law of Sines and Law of Cosines to solve or find the area of oblique (non-right) triangles.   Because the sine values for acute and obtuse angles are positive, there can be ambiguity in applying the Law of Sines. Navigation and surveying are common applications of trigonometry. |

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| **STAGE 1- DESIRED RESULTS** |
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| **Educational Standards** |
| The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:  1. Make sense of problems and persevre in solving them.  2. Reason abstractly and quantitatively.  3. Construct viable arguments and critique the reasoning of others.  4. Model with mathematics.  5. Use appropriate tools strategically.  6. Attend to precision.  7. Look for and make use of structure.  8. Look for and express regularity in repeated reasoning. |

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| **New Jersey Student Learning Standards- Mathematics** |
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| **Standard Set: Number and Quantity** |
| |  |  | | --- | --- | |  | During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers. | |  | With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. | |  | Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that (5 to the 1/3 power)³ should be 5 to the (1/3)³ power = 5¹ = 5 and that 5 to the 1/3 power should be the cube root of 5. | |  | Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents. | |  | In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them. | |

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| **Domain: The Real Number System** |
| |  |  | | --- | --- | | MA.N-RN.B.3 | Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. | | MA.N-RN.A | Extend the properties of exponents to rational exponents. | | MA.N-RN.A.1 | Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. | | MA.N-RN.A.2 | Rewrite expressions involving radicals and rational exponents using the properties of exponents. | | MA.N-RN.B | Use properties of rational and irrational numbers. | |

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| **Domain: Quantities** |
| |  |  | | --- | --- | | MA.N-Q.A | Reason quantitatively and use units to solve problems. | | MA.N-Q.A.1 | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | | MA.N-Q.A.2 | Define appropriate quantities for the purpose of descriptive modeling. | | MA.N-Q.A.3 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | |

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| **Domain: The Complex Number System** |
| |  |  | | --- | --- | | MA.N-CN.B.6 | Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. | | MA.N-CN.A | Perform arithmetic operations with complex numbers. | | MA.N-CN.A.1 | Know there is a complex number 𝑖 such that 𝑖² = -1, and every complex number has the form 𝑎 + 𝑏𝑖 with 𝑎 and 𝑏 real. | | MA.N-CN.A.2 | Use the relation 𝑖² = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. | | MA.N-CN.A.3 | Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. | | MA.N-CN.B | Represent complex numbers and their operations on the complex plane. | | MA.N-CN.B.4 | Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. | | MA.N-CN.B.5 | Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. | | MA.N-CN.C | Use complex numbers in polynomial identities and equations. | | MA.N-CN.C.7 | Solve quadratic equations with real coefficients that have complex solutions. | | MA.N-CN.C.8 | Extend polynomial identities to the complex numbers. | | MA.N-CN.C.9 | Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. | |

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| **Domain: Vector and Matrix Quantities** |
| |  |  | | --- | --- | | MA.N-VM.B.5a | Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as 𝑐(𝒗ₓ, 𝒗 subscript 𝑦) = (𝑐𝒗ₓ, 𝑐𝒗 subscript 𝑦). | | MA.N-VM.B.4c | Understand vector subtraction 𝒗 − 𝑤 as 𝒗 + (−𝑤), where –𝑤 is the additive inverse of 𝑤, with the same magnitude as 𝑤 and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. | | MA.N-VM.B.5b | Compute the magnitude of a scalar multiple 𝑐𝒗 using ||𝑐𝒗|| = |𝑐|𝒗. Compute the direction of 𝑐𝒗 knowing that when |𝑐|𝒗 ≠ 0, the direction of 𝑐𝒗 is either along 𝒗 (for 𝑐 > 0) or against 𝒗 (for 𝑐 < 0). | | MA.N-VM.A | Represent and model with vector quantities. | | MA.N-VM.C | Perform operations on matrices and use matrices in applications. | | MA.N-VM.C.6 | Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. | | MA.N-VM.C.7 | Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. | | MA.N-VM.C.8 | Add, subtract, and multiply matrices of appropriate dimensions. | | MA.N-VM.C.9 | Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. | | MA.N-VM.C.10 | Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. | | MA.N-VM.C.11 | Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. | | MA.N-VM.C.12 | Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area. | | MA.N-VM.A.1 | Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., 𝒗, |𝒗|, ||𝒗||, 𝒗). | | MA.N-VM.A.2 | Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. | | MA.N-VM.A.3 | Solve problems involving velocity and other quantities that can be represented by vectors. | | MA.N-VM.B | Perform operations on vectors. | | MA.N-VM.B.4 | Add and subtract vectors. | | MA.N-VM.B.5 | Multiply a vector by a scalar. | | MA.N-VM.B.4a | Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. | | MA.N-VM.B.4b | Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. | |

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| **Standard Set: Statistics and Probability** |
| |  |  | | --- | --- | |  | Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account. | |  | Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. | |  | Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. | |  | Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. | |  | Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time. | |  | Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient. | |

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| **Domain: Interpreting Categorical and Quantitative Data** |
| |  |  | | --- | --- | | MA.S-ID.B.6c | Fit a linear function for a scatter plot that suggests a linear association. | | MA.S-ID.C | Interpret linear models | | MA.S-ID.C.7 | Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | | MA.S-ID.C.8 | Compute (using technology) and interpret the correlation coefficient of a linear fit. | | MA.S-ID.C.9 | Distinguish between correlation and causation. | | MA.S-ID.A | Summarize, represent, and interpret data on a single count or measurement variable | | MA.S-ID.A.1 | Represent data with plots on the real number line (dot plots, histograms, and box plots). | | MA.S-ID.A.2 | Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | | MA.S-ID.A.3 | Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | | MA.S-ID.A.4 | Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. | | MA.S-ID.B | Summarize, represent, and interpret data on two categorical and quantitative variables | | MA.S-ID.B.5 | Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. | | MA.S-ID.B.6 | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. | | MA.S-ID.B.6a | Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. | | MA.S-ID.B.6b | Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology. | |

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| **Domain: Making Inferences and Justifying Conclusions** |
| |  |  | | --- | --- | | MA.S-IC.A | Understand and evaluate random processes underlying statistical experiments | | MA.S-IC.A.1 | Understand statistics as a process for making inferences about population parameters based on a random sample from that population. | | MA.S-IC.A.2 | Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. | | MA.S-IC.B | Make inferences and justify conclusions from sample surveys, experiments, and observational studies | | MA.S-IC.B.3 | Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. | | MA.S-IC.B.4 | Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. | | MA.S-IC.B.5 | Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. | | MA.S-IC.B.6 | Evaluate reports based on data. | |

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| **Domain: Conditional Probability and the Rules of Probability** |
| |  |  | | --- | --- | | MA.S-CP.B | Use the rules of probability to compute probabilities of compound events in a uniform probability model | | MA.S-CP.B.6 | Find the conditional probability of 𝐴 given 𝐵 as the fraction of 𝐵’s outcomes that also belong to 𝐴, and interpret the answer in terms of the model. | | MA.S-CP.B.7 | Apply the Addition Rule, 𝑃(𝐴 𝑜𝑟 𝐵) = 𝑃(𝐴) + 𝑃(𝐵) – 𝑃(𝐴 𝑎𝑛𝑑 𝐵), and interpret the answer in terms of the model. | | MA.S-CP.B.8 | Apply the general Multiplication Rule in a uniform probability model, 𝑃(𝐴 𝑎𝑛𝑑 𝐵) = [𝑃(𝐴)] × [𝑃(𝐵|𝐴)] = [𝑃(𝐵)] × [𝑃(𝐴|𝐵)], and interpret the answer in terms of the model. | | MA.S-CP.B.9 | Use permutations and combinations to compute probabilities of compound events and solve problems. | | MA.S-CP.A | Understand independence and conditional probability and use them to interpret data | | MA.S-CP.A.1 | Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). | | MA.S-CP.A.2 | Understand that two events 𝐴 and 𝐵 are independent if the probability of 𝐴 and 𝐵 occurring together is the product of their probabilities, and use this characterization to determine if they are independent. | | MA.S-CP.A.3 | Understand the conditional probability of 𝐴 given 𝐵 as 𝑃(𝐴 𝑎𝑛𝑑 𝐵)/𝑃(𝐵), and interpret independence of 𝐴 and 𝐵 as saying that the conditional probability of 𝐴 given 𝐵 is the same as the probability of 𝐴, and the conditional probability of 𝐵 given 𝐴 is the same as the probability of 𝐵. | | MA.S-CP.A.4 | Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. | | MA.S-CP.A.5 | Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. | |

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| **Domain: Using Probability to Make Decisions** |
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| **Standard Set: Geometry** |
| |  |  | | --- | --- | |  | Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations. | |  | Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. | |  | The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. | |  | An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. | |  | Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) | |  | During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. | |  | The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. | |  | In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. | |  | Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. | |  | The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion. | |

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| **Domain: Congruence** |
| |  |  | | --- | --- | | MA.G-CO.A.1 | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. | | MA.G-CO.A.2 | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). | | MA.G-CO.A.3 | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. | | MA.G-CO.A.4 | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. | | MA.G-CO.A.5 | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. | | MA.G-CO.D.12 | Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). | | MA.G-CO.B | Understand congruence in terms of rigid motions | | MA.G-CO.D.13 | Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. | | MA.G-CO.B.6 | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. | | MA.G-CO.B.7 | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. | | MA.G-CO.B.8 | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. | | MA.G-CO.C | Prove geometric theorems | | MA.G-CO.C.9 | Prove theorems about lines and angles. | | MA.G-CO.C.10 | Prove theorems about triangles. | | MA.G-CO.C.11 | Prove theorems about parallelograms. | | MA.G-CO.D | Make geometric constructions | | MA.G-CO.A | Experiment with transformations in the plane | |

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| **Domain: Similarity, Right Triangles, and Trigonometry** |
| |  |  | | --- | --- | | MA.G-SRT.D.9 | Derive the formula 𝐴 = (1/2)𝑎𝑏 𝑠𝑖𝑛(𝐶) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. | | MA.G-SRT.D.10 | Prove the Laws of Sines and Cosines and use them to solve problems. | | MA.G-SRT.D.11 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). | | MA.G-SRT.A | Understand similarity in terms of similarity transformations | | MA.G-SRT.A.1 | Verify experimentally the properties of dilations given by a center and a scale factor: | | MA.G-SRT.A.1a | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. | | MA.G-SRT.A.1b | The dilation of a line segment is longer or shorter in the ratio given by the scale factor. | | MA.G-SRT.A.2 | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. | | MA.G-SRT.A.3 | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. | | MA.G-SRT.B | Prove theorems involving similarity | | MA.G-SRT.B.4 | Prove theorems about triangles. | | MA.G-SRT.B.5 | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | | MA.G-SRT.C | Define trigonometric ratios and solve problems involving right triangles | | MA.G-SRT.C.6 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. | | MA.G-SRT.C.7 | Explain and use the relationship between the sine and cosine of complementary angles. | | MA.G-SRT.C.8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. | | MA.G-SRT.D | Apply trigonometry to general triangles | |

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| **Domain: Circles** |
| |  |  | | --- | --- | | MA.G-C.A | Understand and apply theorems about circles | | MA.G-C.A.1 | Prove that all circles are similar. | | MA.G-C.A.2 | Identify and describe relationships among inscribed angles, radii, and chords. | | MA.G-C.A.3 | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. | | MA.G-C.A.4 | Construct a tangent line from a point outside a given circle to the circle. | | MA.G-C.B | Find arc lengths and areas of sectors of circles | | MA.G-C.B.5 | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. | |

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| **Domain: Expressing Geometric Properties with Equations** |
| |  |  | | --- | --- | | MA.G-GPE.B.4 | Use coordinates to prove simple geometric theorems algebraically. | | MA.G-GPE.B.5 | Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). | | MA.G-GPE.B.6 | Find the point on a directed line segment between two given points that partitions the segment in a given ratio. | | MA.G-GPE.B.7 | Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. | | MA.G-GPE.A | Translate between the geometric description and the equation for a conic section | | MA.G-GPE.A.1 | Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. | | MA.G-GPE.A.2 | Derive the equation of a parabola given a focus and directrix. | | MA.G-GPE.A.3 | Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. | | MA.G-GPE.B | Use coordinates to prove simple geometric theorems algebraically | |

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| **Domain: Geometric Measurement and Dimension** |
| |  |  | | --- | --- | | MA.G-GMD.B.4 | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. | | MA.G-GMD.A | Explain volume formulas and use them to solve problems | | MA.G-GMD.A.1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. | | MA.G-GMD.A.2 | Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures. | | MA.G-GMD.A.3 | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. | | MA.G-GMD.B | Visualize relationships between two-dimensional and three-dimensional objects | |

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| **Domain: Modeling with Geometry** |
| |  |  | | --- | --- | | MA.G-MG.A | Apply geometric concepts in modeling situations | | MA.G-MG.A.1 | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). | | MA.G-MG.A.2 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). | | MA.G-MG.A.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). | |

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| **Standard Set: Modeling** |
| |  |  | | --- | --- | |  | Designing the layout of the stalls in a school fair so as to raise as much money as possible. | |  | Analyzing stopping distance for a car. | |  | Modeling savings account balance, bacterial colony growth, or investment growth. | |  | Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport. | |  | Analyzing risk in situations such as extreme sports, pandemics, and terrorism. | |  | Relating population statistics to individual predictions. | |  | Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). | |  | Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. | |  | A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity. | |  | In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations. | |  | Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed. | |  | Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player. | |

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| **Standard Set: Functions** |
| |  |  | | --- | --- | |  | Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. | |  | In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car’s speed in miles per hour, v; the rule T(v) = 100/v expresses this relationship algebraically and defines a function whose name is T. | |  | The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. | |  | A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, “I’ll give you a state, you give me the capital city;” by an algebraic expression like f(x) = a + bx; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function’s properties. | |  | Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships. | |  | A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions. | |  | Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology. | |

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| **Domain: Interpreting Functions** |
| |  |  | | --- | --- | | MA.F-IF.C.8b | Use the properties of exponents to interpret expressions for exponential functions. | | MA.F-IF.A | Understand the concept of a function and use function notation | | MA.F-IF.A.1 | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If 𝑓 is a function and 𝑥 is an element of its domain, then 𝑓(𝑥) denotes the output of 𝑓 corresponding to the input 𝑥. The graph of 𝑓 is the graph of the equation𝑦 = 𝑓(𝑥). | | MA.F-IF.A.2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | | MA.F-IF.C.9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). | | MA.F-IF.A.3 | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. | | MA.F-IF.B | Interpret functions that arise in applications in terms of the context | | MA.F-IF.B.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. | | MA.F-IF.B.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. | | MA.F-IF.B.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | | MA.F-IF.C | Analyze functions using different representations | | MA.F-IF.C.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | | MA.F-IF.C.7a | Graph linear and quadratic functions and show intercepts, maxima, and minima. | | MA.F-IF.C.7b | Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. | | MA.F-IF.C.7c | Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7d | Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7e | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | | MA.F-IF.C.8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. | | MA.F-IF.C.8a | Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | |

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| **Domain: Building Functions** |
| |  |  | | --- | --- | | MA.F-BF.B.4b | Verify by composition that one function is the inverse of another. | | MA.F-BF.B.4c | Read values of an inverse function from a graph or a table, given that the function has an inverse. | | MA.F-BF.B.4d | Produce an invertible function from a non-invertible function by restricting the domain. | | MA.F-BF.B.5 | Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents. | | MA.F-BF.A.1 | Write a function that describes a relationship between two quantities. | | MA.F-BF.A.1a | Determine an explicit expression, a recursive process, or steps for calculation from a context. | | MA.F-BF.A.1b | Combine standard function types using arithmetic operations. | | MA.F-BF.A | Build a function that models a relationship between two quantities | | MA.F-BF.A.1c | Compose functions. | | MA.F-BF.A.2 | Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. | | MA.F-BF.B | Build new functions from existing functions | | MA.F-BF.B.3 | Identify the effect on the graph of replacing 𝑓(𝑥) by 𝑓(𝑥) + 𝑘, 𝑘𝑓(𝑥), 𝑓(𝑘𝑥), and 𝑓(𝑥 + 𝑘) for specific values of 𝑘 (both positive and negative); find the value of 𝑘 given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. | | MA.F-BF.B.4 | Find inverse functions. | | MA.F-BF.B.4a | Solve an equation of the form 𝑓(𝑥) = 𝑐 for a simple function 𝑓 that has an inverse and write an expression for the inverse. | |

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| **Domain: Linear, Quadratic, and Exponential Models** |
| |  |  | | --- | --- | | MA.F-LE.A | Construct and compare linear and exponential models and solve problems | | MA.F-LE.A.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions. | | MA.F-LE.A.1a | Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. | | MA.F-LE.A.1b | Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. | | MA.F-LE.A.1c | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | | MA.F-LE.A.2 | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). | | MA.F-LE.A.3 | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | | MA.F-LE.A.4 | Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to 𝑎𝑏 to the 𝑐𝘵 power = 𝑑 where 𝑎, 𝑐, and 𝑑 are numbers and the base 𝑏 is 2, 10, or 𝑒; evaluate the logarithm using technology. | | MA.F-LE.B | Interpret expressions for functions in terms of the situation they model | | MA.F-LE.B.5 | Interpret the parameters in a linear or exponential function in terms of a context. | |

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| **Domain: Trigonometric Functions** |
| |  |  | | --- | --- | | MA.F-TF.A.3 | Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π – 𝑥, π + 𝑥, and 2π – 𝑥 in terms of their values for 𝑥, where 𝑥 is any real number. | | MA.F-TF.A.4 | Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. | | MA.F-TF.B | Model periodic phenomena with trigonometric functions | | MA.F-TF.B.5 | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. | | MA.F-TF.B.6 | Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. | | MA.F-TF.B.7 | Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. | | MA.F-TF.C | Prove and apply trigonometric identities | | MA.F-TF.C.8 | Prove the Pythagorean identity 𝑠𝑖𝑛²(θ) + 𝑐𝑜𝑠²(θ) = 1 and use it to find 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) given 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) and the quadrant of the angle. | | MA.F-TF.C.9 | Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. | | MA.F-TF.A | Extend the domain of trigonometric functions using the unit circle | | MA.F-TF.A.1 | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. | | MA.F-TF.A.2 | Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. | |

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| **Standard Set: Algebra** |
| |  |  | | --- | --- | |  | An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances. | |  | Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor. | |  | Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure. | |  | A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave. | |  | An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. | |  | The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system. | |  | An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. | |  | Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x² – 2 = 0 are real numbers, not rational numbers; and the solutions of x² + 2 = 0 are complex numbers, not real numbers. | |  | Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling. | |  | The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, 𝘈 = ((𝘣₁+𝘣₂)/2)𝘩, can be solved for 𝘩 using the same deductive process. | |  | Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them. | |

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| **Domain: Seeing Structures in Expressions** |
| |  |  | | --- | --- | | MA.A-SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. For example, see 𝑥⁴ – 𝑦⁴ as (𝑥²)² – (𝑦²)², thus recognizing it as a difference of squares that can be factored as (𝑥² – 𝑦²)(𝑥² + 𝑦²). | | MA.A-SSE.B | Write expressions in equivalent forms to solve problems | | MA.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. | | MA.A-SSE.B.3a | Factor a quadratic expression to reveal the zeros of the function it defines. | | MA.A-SSE.B.3b | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. | | MA.A-SSE.A | Interpret the structure of expressions | | MA.A-SSE.B.3c | Use the properties of exponents to transform expressions for exponential functions. | | MA.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context. | | MA.A-SSE.A.1a | Interpret parts of an expression, such as terms, factors, and coefficients. | | MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. | | MA.A-SSE.A.1b | Interpret complicated expressions by viewing one or more of their parts as a single entity. | |

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| **Domain: Arithmetic with Polynomials and Rational Functions** |
| |  |  | | --- | --- | | MA.A-APR.C.4 | Prove polynomial identities and use them to describe numerical relationships. | | MA.A-APR.A | Perform arithmetic operations on polynomials | | MA.A-APR.A.1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | | MA.A-APR.B | Understand the relationship between zeros and factors of polynomials | | MA.A-APR.B.2 | Know and apply the Remainder Theorem: For a polynomial 𝑝(𝑥) and a number 𝑎, the remainder on division by 𝑥 – 𝑎 is 𝑝(𝑎), so 𝑝(𝑎) = 0 if and only if (𝑥 – 𝑎) is a factor of 𝑝(𝑥). | | MA.A-APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. | | MA.A-APR.C.5 | Know and apply the Binomial Theorem for the expansion of (𝑥 + 𝑦)ⁿ in powers of 𝑥 and 𝑦 for a positive integer 𝑛, where 𝑥 and 𝑦 are any numbers, with coefficients determined for example by Pascal’s Triangle. | | MA.A-APR.D | Rewrite rational expressions | | MA.A-APR.D.6 | Rewrite simple rational expressions in different forms; write 𝑎(𝑥)/𝑏(𝑥) in the form 𝑞(𝑥) + 𝑟(𝑥)/𝑏(𝑥), where 𝑎(𝑥), 𝑏(𝑥), 𝑞(𝑥), and 𝑟(𝑥) are polynomials with the degree of 𝑟(𝑥) less than the degree of 𝑏(𝑥), using inspection, long division, or, for the more complicated examples, a computer algebra system. | | MA.A-APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. | | MA.A-APR.C | Use polynomial identities to solve problems | |

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| **Domain: Creating Equations** |
| |  |  | | --- | --- | | MA.A-CED.A | Create equations that describe numbers or relationships | | MA.A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems. | | MA.A-CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | | MA.A-CED.A.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. | | MA.A-CED.A.4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. | |

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| **Domain: Reasoning with Equations and Inequalities** |
| |  |  | | --- | --- | | MA.A-REI.B | Solve equations and inequalities in one variable | | MA.A-REI.B.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. | | MA.A-REI.B.4 | Solve quadratic equations in one variable. | | MA.A-REI.C | Solve systems of equations | | MA.A-REI.C.5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. | | MA.A-REI.C.6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. | | MA.A-REI.C.7 | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. | | MA.A-REI.C.8 | Represent a system of linear equations as a single matrix equation in a vector variable. | | MA.A-REI.C.9 | Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). | | MA.A-REI.D | Represent and solve equations and inequalities graphically | | MA.A-REI.B.4a | Use the method of completing the square to transform any quadratic equation in 𝑥 into an equation of the form (𝑥 – 𝑝)² = 𝑞 that has the same solutions. Derive the quadratic formula from this form. | | MA.A-REI.D.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). | | MA.A-REI.B.4b | Solve quadratic equations by inspection (e.g., for 𝑥² = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as 𝑎 ± 𝑏𝑖 for real numbers 𝑎 and 𝑏. | | MA.A-REI.A | Understand solving equations as a process of reasoning and explain the reasoning | | MA.A-REI.D.12 | Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | | MA.A-REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | | MA.A-REI.D.11 | Explain why the 𝑥-coordinates of the points where the graphs of the equations 𝑦 = 𝑓(𝑥) and 𝑦 = 𝑔(𝑥) intersect are the solutions of the equation𝑓(𝑥) = 𝑔(𝑥); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where 𝑓(𝑥) and/or 𝑔(𝑥) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | | MA.A-REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. | |

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| **Essential Questions** |
| What cases are insufficient or ambiguous for solving a triangle?  When do we use the Law of Sines to solve an oblique triangle?  When do we use Law of Cosines?  How are these laws applied in navigation and surveying applications?  How can we prove the Law of Sines and the Law of Cosines? |

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| **Enduring Understanding** |
| Trigonometry is extended using the Law of Sines and Law of Cosines to solve or find the area of oblique (non-right) triangles.   Because the sine values for acute and obtuse angles are positive, there can be ambiguity in applying the Law of Sines. Navigation and surveying are common applications of trigonometry. |

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| **Students will know...** |
| 1. Law of Sines and Law of Cosines 2. Area of a Triangle |

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| **Students will be able to...** |
| 1. Use the Law of Sines to find the unknown parts of an oblique triangle. 2. Use the Law of Cosines to find the unknown parts of an oblique triangle. 3. Use trigonometry to find the area of an oblique triangle (SAS and SSS). 4. Use trigonometry to solve navigation and surveying problems. |

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| **STAGE 2- EVIDENCE OF LEARNING** |
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| **Formative Assessment Suggestions** |
| |  |  | | --- | --- | | • 3- Minute Pause | **.** | | • A-B-C Summaries | **.** | | • Analogy Prompt | **.** | | • Choral Response | **.** | | • Debriefing | **.** | | • Exit Card / Ticket | **.** | | • Hand Signals | **.** | | • Idea Spinner | **.** | | • Index Card Summaries | **.** | | • Inside-Outside Circle Discussion (Fishbowl) | **.** | | • Journal Entry | **.** | | • Misconception Check | **.** | | • Observation | **.** | | • One Minute Essay | **.** | | • One Word Summary | **.** | | • Portfolio Check | **.** | | • Questions & Answers | **.** | | • Quiz | **.** | | • Self-Assessment | **.** | | • Student Conference | **.** | | • Think-Pair-Share | **.** | | • Web or Concept Map | **.** | |

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| **Authentic Assessments Suggestions** |
| Edementum Exact Path (BOY, MOY, EOY)  Class Discussions  Do Nows  Notebook Checks  Homework  Presentations  Webassign |

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| **Benchmark Assessments** |
| Edementum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3- LEARNING PLAN** |
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| **Instructional Map** |
| Topic #1 – Law of Sines and Law of Cosines  Topic #2 – Area of a Triangle |

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| **Modifications/Differentiation of Instruction** |
| Differentiation Strategies for Special Education Students   * Remove unnecessary material, words, etc., that can distract from the content * Use of off-grade level materials * Provide appropriate scaffolding * Limit the number of steps required for completion * Time allowed * Level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Varied homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Ability to work at their own pace * Present ideas using auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment * Differentiated checklists and rubrics, if available and appropriate       Differentiation Strategies for Gifted and Talented Students   * Increase the level of complexity * Decrease scaffolding * Variety of finished products * Allow for greater independence * Learning stations, interest groups * Varied texts and supplementary materials * Use of technology * Flexibility in assignments * Varied questioning strategies * Encourage research * Strategy and flexible groups based on formative assessment or student choice * Acceleration within a unit of study * Exposure to more advanced or complex concepts, abstractions, and materials * Encourage students to move through content areas at their own pace * After mastery of a unit, provide students with more advanced learning activities, not more of the same activity * Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas       Differentiated Strategies for ELL Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials, including visuals * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Allow students to work at their own pace * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Role play * Provide graphic organizers, highlighted materials * Strategy and flexible groups based on formative assessment       Differentiation Strategies for At Risk Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment       **504 Plans**  Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:   * walk, breathe, eat, or sleep * communicate, see, hear, or speak * read, concentrate, think, or learn * stand, bend, lift, or work       Examples of accommodations in 504 plans include:   * preferential seating * extended time on tests and assignments * reduced homework or classwork * verbal, visual, or technology aids * modified textbooks or audio-video materials * behavior management support * adjusted class schedules or grading * verbal testing * excused lateness, absence, or missed classwork * pre-approved nurse's office visits and accompaniment to visits * occupational or physical therapy |

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| **Modification Strategies** |
| |  |  | | --- | --- | | • Extended Time | **.** | | • Frequent Breaks | **.** | | • Highlighted Text | **.** | | • Interactive Notebook | **.** | | • Modified Test | **.** | | • Oral Directions | **.** | | • Peer Tutoring | **.** | | • Preferential Seating | **.** | | • Re-Direct | **.** | | • Repeated Drill / Practice | **.** | | • Shortened Assignments | **.** | | • Teacher Notes | **.** | | • Tutorials | **.** | | • Use of Additional Reference Material | **.** | | • Use of Audio Resources | **.** | |

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| **High Preparation Differentiation** |
| |  |  | | --- | --- | | • Alternative Assessments | **.** | | • Choice Boards | **.** | | • Games and Tournaments | **.** | | • Group Investigations | **.** | | • Guided Reading | **.** | | • Independent Research / Project | **.** | | • Interest Groups | **.** | | • Learning Contracts | **.** | | • Leveled Rubrics | **.** | | • Literature Circles | **.** | | • Menu Assignments | **.** | | • Multiple Intelligence Options | **.** | | • Multiple Texts | **.** | | • Personal Agendas | **.** | | • Project Based Learning (PBL) | **.** | | • Stations / Centers | **.** | | • Think-Tac-Toe | **.** | | • Tiered Activities / Assignments | **.** | | • Varying Graphic Organizers | **.** | |

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| **Low Preparation Differentiation** |
| |  |  | | --- | --- | | • Choice of Book / Activity | **.** | | • Cubing Activities | **.** | | • Exploration by Interest (using interest inventories) | **.** | | • Flexible Grouping | **.** | | • Goal Setting With Student | **.** | | • Homework Options | **.** | | • Jigsaw | **.** | | • Mini Workshops to Extend Skills | **.** | | • Mini Workshops to Re-teach | **.** | | • Open-ended Activities | **.** | | • Think-Pair-Share by Interest | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Readiness | **.** | | • Use of Collaboration | **.** | | • Use of Reading Buddies | **.** | | • Varied Journal Prompts | **.** | | • Varied Product Choice | **.** | | • Varied Supplemental Materials | **.** | | • Work Alone / Together | **.** | |

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| **Horizontal Integration- Interdisciplinary Connections** |
| Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other. |

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| **Vertical Integration- Discipline Mapping** |
| The standards in this unit were introduced in Algebra I, Geometry and Algebra II. Precalculus coursework focuses on preparing the students be proficient in Calculus standards. |

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| **Additional Materials** |
| LPS Adopted Textbooks and Programs   * Cengage: Precalculus with Limits: A Graphing Approach, * WebAssign (Computer Based program supplementing Cengage)   Khan Academy  Edmentum Exact Path |

**Unit 7: Analytic Geometry**

Content Area: **Mathematics**  
Course(s): **Pre-Calculus**  
Time Period: **Third Marking Period**  
Length: **4 Weeks**  
Status: **Published**

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| **Unit Overview** |
| Fundamental trigonometric identities, sum and difference formulas together with algebraic techniques are used to simplify expressions, prove identities and solve equations. When trigonometric equations are solved an angle measure of the result. |

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| **STAGE 1- DESIRED RESULTS** |
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| **Educational Standards** |
| The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:  1. Make sense of problems and persevre in solving them.  2. Reason abstractly and quantitatively.  3. Construct viable arguments and critique the reasoning of others.  4. Model with mathematics.  5. Use appropriate tools strategically.  6. Attend to precision.  7. Look for and make use of structure.  8. Look for and express regularity in repeated reasoning. |

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| **New Jersey Student Learning Standards- Mathematics** |
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| **Standard Set: Number and Quantity** |
| |  |  | | --- | --- | |  | During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers. | |  | With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. | |  | Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that (5 to the 1/3 power)³ should be 5 to the (1/3)³ power = 5¹ = 5 and that 5 to the 1/3 power should be the cube root of 5. | |  | Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents. | |  | In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them. | |

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| **Domain: The Real Number System** |
| |  |  | | --- | --- | | MA.N-RN.B.3 | Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. | | MA.N-RN.A | Extend the properties of exponents to rational exponents. | | MA.N-RN.A.1 | Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. | | MA.N-RN.A.2 | Rewrite expressions involving radicals and rational exponents using the properties of exponents. | | MA.N-RN.B | Use properties of rational and irrational numbers. | |

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| **Domain: Quantities** |
| |  |  | | --- | --- | | MA.N-Q.A | Reason quantitatively and use units to solve problems. | | MA.N-Q.A.1 | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | | MA.N-Q.A.2 | Define appropriate quantities for the purpose of descriptive modeling. | | MA.N-Q.A.3 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | |

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| **Domain: The Complex Number System** |
| |  |  | | --- | --- | | MA.N-CN.B.6 | Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. | | MA.N-CN.A | Perform arithmetic operations with complex numbers. | | MA.N-CN.A.1 | Know there is a complex number 𝑖 such that 𝑖² = -1, and every complex number has the form 𝑎 + 𝑏𝑖 with 𝑎 and 𝑏 real. | | MA.N-CN.A.2 | Use the relation 𝑖² = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. | | MA.N-CN.A.3 | Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. | | MA.N-CN.B | Represent complex numbers and their operations on the complex plane. | | MA.N-CN.B.4 | Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. | | MA.N-CN.B.5 | Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. | | MA.N-CN.C | Use complex numbers in polynomial identities and equations. | | MA.N-CN.C.7 | Solve quadratic equations with real coefficients that have complex solutions. | | MA.N-CN.C.8 | Extend polynomial identities to the complex numbers. | | MA.N-CN.C.9 | Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. | |

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| **Domain: Vector and Matrix Quantities** |
| |  |  | | --- | --- | | MA.N-VM.B.5a | Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as 𝑐(𝒗ₓ, 𝒗 subscript 𝑦) = (𝑐𝒗ₓ, 𝑐𝒗 subscript 𝑦). | | MA.N-VM.B.4c | Understand vector subtraction 𝒗 − 𝑤 as 𝒗 + (−𝑤), where –𝑤 is the additive inverse of 𝑤, with the same magnitude as 𝑤 and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. | | MA.N-VM.B.5b | Compute the magnitude of a scalar multiple 𝑐𝒗 using ||𝑐𝒗|| = |𝑐|𝒗. Compute the direction of 𝑐𝒗 knowing that when |𝑐|𝒗 ≠ 0, the direction of 𝑐𝒗 is either along 𝒗 (for 𝑐 > 0) or against 𝒗 (for 𝑐 < 0). | | MA.N-VM.A | Represent and model with vector quantities. | | MA.N-VM.C | Perform operations on matrices and use matrices in applications. | | MA.N-VM.C.6 | Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. | | MA.N-VM.C.7 | Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. | | MA.N-VM.C.8 | Add, subtract, and multiply matrices of appropriate dimensions. | | MA.N-VM.C.9 | Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. | | MA.N-VM.C.10 | Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. | | MA.N-VM.C.11 | Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. | | MA.N-VM.C.12 | Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area. | | MA.N-VM.A.1 | Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., 𝒗, |𝒗|, ||𝒗||, 𝒗). | | MA.N-VM.A.2 | Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. | | MA.N-VM.A.3 | Solve problems involving velocity and other quantities that can be represented by vectors. | | MA.N-VM.B | Perform operations on vectors. | | MA.N-VM.B.4 | Add and subtract vectors. | | MA.N-VM.B.5 | Multiply a vector by a scalar. | | MA.N-VM.B.4a | Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. | | MA.N-VM.B.4b | Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. | |

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| **Standard Set: Statistics and Probability** |
| |  |  | | --- | --- | |  | Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account. | |  | Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. | |  | Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. | |  | Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. | |  | Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time. | |  | Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient. | |

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| **Domain: Interpreting Categorical and Quantitative Data** |
| |  |  | | --- | --- | | MA.S-ID.B.6c | Fit a linear function for a scatter plot that suggests a linear association. | | MA.S-ID.C | Interpret linear models | | MA.S-ID.C.7 | Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | | MA.S-ID.C.8 | Compute (using technology) and interpret the correlation coefficient of a linear fit. | | MA.S-ID.C.9 | Distinguish between correlation and causation. | | MA.S-ID.A | Summarize, represent, and interpret data on a single count or measurement variable | | MA.S-ID.A.1 | Represent data with plots on the real number line (dot plots, histograms, and box plots). | | MA.S-ID.A.2 | Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | | MA.S-ID.A.3 | Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | | MA.S-ID.A.4 | Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. | | MA.S-ID.B | Summarize, represent, and interpret data on two categorical and quantitative variables | | MA.S-ID.B.5 | Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. | | MA.S-ID.B.6 | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. | | MA.S-ID.B.6a | Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. | | MA.S-ID.B.6b | Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology. | |

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| **Domain: Making Inferences and Justifying Conclusions** |
| |  |  | | --- | --- | | MA.S-IC.A | Understand and evaluate random processes underlying statistical experiments | | MA.S-IC.A.1 | Understand statistics as a process for making inferences about population parameters based on a random sample from that population. | | MA.S-IC.A.2 | Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. | | MA.S-IC.B | Make inferences and justify conclusions from sample surveys, experiments, and observational studies | | MA.S-IC.B.3 | Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. | | MA.S-IC.B.4 | Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. | | MA.S-IC.B.5 | Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. | | MA.S-IC.B.6 | Evaluate reports based on data. | |

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| **Domain: Conditional Probability and the Rules of Probability** |
| |  |  | | --- | --- | | MA.S-CP.B | Use the rules of probability to compute probabilities of compound events in a uniform probability model | | MA.S-CP.B.6 | Find the conditional probability of 𝐴 given 𝐵 as the fraction of 𝐵’s outcomes that also belong to 𝐴, and interpret the answer in terms of the model. | | MA.S-CP.B.7 | Apply the Addition Rule, 𝑃(𝐴 𝑜𝑟 𝐵) = 𝑃(𝐴) + 𝑃(𝐵) – 𝑃(𝐴 𝑎𝑛𝑑 𝐵), and interpret the answer in terms of the model. | | MA.S-CP.B.8 | Apply the general Multiplication Rule in a uniform probability model, 𝑃(𝐴 𝑎𝑛𝑑 𝐵) = [𝑃(𝐴)] × [𝑃(𝐵|𝐴)] = [𝑃(𝐵)] × [𝑃(𝐴|𝐵)], and interpret the answer in terms of the model. | | MA.S-CP.B.9 | Use permutations and combinations to compute probabilities of compound events and solve problems. | | MA.S-CP.A | Understand independence and conditional probability and use them to interpret data | | MA.S-CP.A.1 | Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). | | MA.S-CP.A.2 | Understand that two events 𝐴 and 𝐵 are independent if the probability of 𝐴 and 𝐵 occurring together is the product of their probabilities, and use this characterization to determine if they are independent. | | MA.S-CP.A.3 | Understand the conditional probability of 𝐴 given 𝐵 as 𝑃(𝐴 𝑎𝑛𝑑 𝐵)/𝑃(𝐵), and interpret independence of 𝐴 and 𝐵 as saying that the conditional probability of 𝐴 given 𝐵 is the same as the probability of 𝐴, and the conditional probability of 𝐵 given 𝐴 is the same as the probability of 𝐵. | | MA.S-CP.A.4 | Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. | | MA.S-CP.A.5 | Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. | |

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| **Domain: Using Probability to Make Decisions** |
| |  |  | | --- | --- | | MA.S-MD.B.5 | Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. | | MA.S-MD.B.5a | Find the expected payoff for a game of chance. | | MA.S-MD.B.5b | Evaluate and compare strategies on the basis of expected values. | | MA.S-MD.A | Calculate expected values and use them to solve problems | | MA.S-MD.B.6 | Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). | | MA.S-MD.A.1 | Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. | | MA.S-MD.B.7 | Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). | | MA.S-MD.A.2 | Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. | | MA.S-MD.A.3 | Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. | | MA.S-MD.A.4 | Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. | | MA.S-MD.B | Use probability to evaluate outcomes of decisions | |

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| **Standard Set: Geometry** |
| |  |  | | --- | --- | |  | Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations. | |  | Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. | |  | The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. | |  | An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. | |  | Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) | |  | During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. | |  | The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. | |  | In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. | |  | Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. | |  | The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion. | |

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| **Domain: Congruence** |
| |  |  | | --- | --- | | MA.G-CO.A.1 | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. | | MA.G-CO.A.2 | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). | | MA.G-CO.A.3 | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. | | MA.G-CO.A.4 | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. | | MA.G-CO.A.5 | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. | | MA.G-CO.D.12 | Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). | | MA.G-CO.B | Understand congruence in terms of rigid motions | | MA.G-CO.D.13 | Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. | | MA.G-CO.B.6 | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. | | MA.G-CO.B.7 | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. | | MA.G-CO.B.8 | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. | | MA.G-CO.C | Prove geometric theorems | | MA.G-CO.C.9 | Prove theorems about lines and angles. | | MA.G-CO.C.10 | Prove theorems about triangles. | | MA.G-CO.C.11 | Prove theorems about parallelograms. | | MA.G-CO.D | Make geometric constructions | | MA.G-CO.A | Experiment with transformations in the plane | |

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| **Domain: Similarity, Right Triangles, and Trigonometry** |
| |  |  | | --- | --- | | MA.G-SRT.D.9 | Derive the formula 𝐴 = (1/2)𝑎𝑏 𝑠𝑖𝑛(𝐶) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. | | MA.G-SRT.D.10 | Prove the Laws of Sines and Cosines and use them to solve problems. | | MA.G-SRT.D.11 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). | | MA.G-SRT.A | Understand similarity in terms of similarity transformations | | MA.G-SRT.A.1 | Verify experimentally the properties of dilations given by a center and a scale factor: | | MA.G-SRT.A.1a | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. | | MA.G-SRT.A.1b | The dilation of a line segment is longer or shorter in the ratio given by the scale factor. | | MA.G-SRT.A.2 | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. | | MA.G-SRT.A.3 | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. | | MA.G-SRT.B | Prove theorems involving similarity | | MA.G-SRT.B.4 | Prove theorems about triangles. | | MA.G-SRT.B.5 | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | | MA.G-SRT.C | Define trigonometric ratios and solve problems involving right triangles | | MA.G-SRT.C.6 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. | | MA.G-SRT.C.7 | Explain and use the relationship between the sine and cosine of complementary angles. | | MA.G-SRT.C.8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. | | MA.G-SRT.D | Apply trigonometry to general triangles | |

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| **Domain: Circles** |
| |  |  | | --- | --- | | MA.G-C.A | Understand and apply theorems about circles | | MA.G-C.A.1 | Prove that all circles are similar. | | MA.G-C.A.2 | Identify and describe relationships among inscribed angles, radii, and chords. | | MA.G-C.A.3 | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. | | MA.G-C.A.4 | Construct a tangent line from a point outside a given circle to the circle. | | MA.G-C.B | Find arc lengths and areas of sectors of circles | | MA.G-C.B.5 | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. | |

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| **Domain: Expressing Geometric Properties with Equations** |
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| **Domain: Geometric Measurement and Dimension** |
| |  |  | | --- | --- | | MA.G-GMD.B.4 | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. | | MA.G-GMD.A | Explain volume formulas and use them to solve problems | | MA.G-GMD.A.1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. | | MA.G-GMD.A.2 | Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures. | | MA.G-GMD.A.3 | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. | | MA.G-GMD.B | Visualize relationships between two-dimensional and three-dimensional objects | |

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| **Domain: Modeling with Geometry** |
| |  |  | | --- | --- | | MA.G-MG.A | Apply geometric concepts in modeling situations | | MA.G-MG.A.1 | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). | | MA.G-MG.A.2 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). | | MA.G-MG.A.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). | |

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| **Standard Set: Modeling** |
| |  |  | | --- | --- | |  | Designing the layout of the stalls in a school fair so as to raise as much money as possible. | |  | Analyzing stopping distance for a car. | |  | Modeling savings account balance, bacterial colony growth, or investment growth. | |  | Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport. | |  | Analyzing risk in situations such as extreme sports, pandemics, and terrorism. | |  | Relating population statistics to individual predictions. | |  | Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). | |  | Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. | |  | A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity. | |  | In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations. | |  | Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed. | |  | Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player. | |

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| **Standard Set: Functions** |
| |  |  | | --- | --- | |  | Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. | |  | In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car’s speed in miles per hour, v; the rule T(v) = 100/v expresses this relationship algebraically and defines a function whose name is T. | |  | The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. | |  | A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, “I’ll give you a state, you give me the capital city;” by an algebraic expression like f(x) = a + bx; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function’s properties. | |  | Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships. | |  | A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions. | |  | Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology. | |

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| **Domain: Interpreting Functions** |
| |  |  | | --- | --- | | MA.F-IF.C.8b | Use the properties of exponents to interpret expressions for exponential functions. | | MA.F-IF.A | Understand the concept of a function and use function notation | | MA.F-IF.A.1 | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If 𝑓 is a function and 𝑥 is an element of its domain, then 𝑓(𝑥) denotes the output of 𝑓 corresponding to the input 𝑥. The graph of 𝑓 is the graph of the equation𝑦 = 𝑓(𝑥). | | MA.F-IF.A.2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | | MA.F-IF.C.9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). | | MA.F-IF.A.3 | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. | | MA.F-IF.B | Interpret functions that arise in applications in terms of the context | | MA.F-IF.B.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. | | MA.F-IF.B.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. | | MA.F-IF.B.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | | MA.F-IF.C | Analyze functions using different representations | | MA.F-IF.C.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | | MA.F-IF.C.7a | Graph linear and quadratic functions and show intercepts, maxima, and minima. | | MA.F-IF.C.7b | Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. | | MA.F-IF.C.7c | Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7d | Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7e | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | | MA.F-IF.C.8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. | | MA.F-IF.C.8a | Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | |

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| **Domain: Building Functions** |
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| **Domain: Linear, Quadratic, and Exponential Models** |
| |  |  | | --- | --- | | MA.F-LE.A | Construct and compare linear and exponential models and solve problems | | MA.F-LE.A.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions. | | MA.F-LE.A.1a | Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. | | MA.F-LE.A.1b | Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. | | MA.F-LE.A.1c | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | | MA.F-LE.A.2 | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). | | MA.F-LE.A.3 | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | | MA.F-LE.A.4 | Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to 𝑎𝑏 to the 𝑐𝘵 power = 𝑑 where 𝑎, 𝑐, and 𝑑 are numbers and the base 𝑏 is 2, 10, or 𝑒; evaluate the logarithm using technology. | | MA.F-LE.B | Interpret expressions for functions in terms of the situation they model | | MA.F-LE.B.5 | Interpret the parameters in a linear or exponential function in terms of a context. | |

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| **Domain: Trigonometric Functions** |
| |  |  | | --- | --- | | MA.F-TF.A.3 | Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π – 𝑥, π + 𝑥, and 2π – 𝑥 in terms of their values for 𝑥, where 𝑥 is any real number. | | MA.F-TF.A.4 | Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. | | MA.F-TF.B | Model periodic phenomena with trigonometric functions | | MA.F-TF.B.5 | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. | | MA.F-TF.B.6 | Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. | | MA.F-TF.B.7 | Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. | | MA.F-TF.C | Prove and apply trigonometric identities | | MA.F-TF.C.8 | Prove the Pythagorean identity 𝑠𝑖𝑛²(θ) + 𝑐𝑜𝑠²(θ) = 1 and use it to find 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) given 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) and the quadrant of the angle. | | MA.F-TF.C.9 | Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. | | MA.F-TF.A | Extend the domain of trigonometric functions using the unit circle | | MA.F-TF.A.1 | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. | | MA.F-TF.A.2 | Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. | |

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| **Standard Set: Algebra** |
| |  |  | | --- | --- | |  | An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances. | |  | Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor. | |  | Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure. | |  | A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave. | |  | An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. | |  | The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system. | |  | An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. | |  | Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x² – 2 = 0 are real numbers, not rational numbers; and the solutions of x² + 2 = 0 are complex numbers, not real numbers. | |  | Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling. | |  | The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, 𝘈 = ((𝘣₁+𝘣₂)/2)𝘩, can be solved for 𝘩 using the same deductive process. | |  | Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them. | |

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| **Domain: Seeing Structures in Expressions** |
| |  |  | | --- | --- | | MA.A-SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. For example, see 𝑥⁴ – 𝑦⁴ as (𝑥²)² – (𝑦²)², thus recognizing it as a difference of squares that can be factored as (𝑥² – 𝑦²)(𝑥² + 𝑦²). | | MA.A-SSE.B | Write expressions in equivalent forms to solve problems | | MA.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. | | MA.A-SSE.B.3a | Factor a quadratic expression to reveal the zeros of the function it defines. | | MA.A-SSE.B.3b | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. | | MA.A-SSE.A | Interpret the structure of expressions | | MA.A-SSE.B.3c | Use the properties of exponents to transform expressions for exponential functions. | | MA.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context. | | MA.A-SSE.A.1a | Interpret parts of an expression, such as terms, factors, and coefficients. | | MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. | | MA.A-SSE.A.1b | Interpret complicated expressions by viewing one or more of their parts as a single entity. | |

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| **Domain: Arithmetic with Polynomials and Rational Functions** |
| |  |  | | --- | --- | | MA.A-APR.C.4 | Prove polynomial identities and use them to describe numerical relationships. | | MA.A-APR.A | Perform arithmetic operations on polynomials | | MA.A-APR.A.1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | | MA.A-APR.B | Understand the relationship between zeros and factors of polynomials | | MA.A-APR.B.2 | Know and apply the Remainder Theorem: For a polynomial 𝑝(𝑥) and a number 𝑎, the remainder on division by 𝑥 – 𝑎 is 𝑝(𝑎), so 𝑝(𝑎) = 0 if and only if (𝑥 – 𝑎) is a factor of 𝑝(𝑥). | | MA.A-APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. | | MA.A-APR.C.5 | Know and apply the Binomial Theorem for the expansion of (𝑥 + 𝑦)ⁿ in powers of 𝑥 and 𝑦 for a positive integer 𝑛, where 𝑥 and 𝑦 are any numbers, with coefficients determined for example by Pascal’s Triangle. | | MA.A-APR.D | Rewrite rational expressions | | MA.A-APR.D.6 | Rewrite simple rational expressions in different forms; write 𝑎(𝑥)/𝑏(𝑥) in the form 𝑞(𝑥) + 𝑟(𝑥)/𝑏(𝑥), where 𝑎(𝑥), 𝑏(𝑥), 𝑞(𝑥), and 𝑟(𝑥) are polynomials with the degree of 𝑟(𝑥) less than the degree of 𝑏(𝑥), using inspection, long division, or, for the more complicated examples, a computer algebra system. | | MA.A-APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. | | MA.A-APR.C | Use polynomial identities to solve problems | |

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| **Domain: Creating Equations** |
| |  |  | | --- | --- | | MA.A-CED.A | Create equations that describe numbers or relationships | | MA.A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems. | | MA.A-CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | | MA.A-CED.A.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. | | MA.A-CED.A.4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. | |

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| **Domain: Reasoning with Equations and Inequalities** |
| |  |  | | --- | --- | | MA.A-REI.B | Solve equations and inequalities in one variable | | MA.A-REI.B.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. | | MA.A-REI.B.4 | Solve quadratic equations in one variable. | | MA.A-REI.C | Solve systems of equations | | MA.A-REI.C.5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. | | MA.A-REI.C.6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. | | MA.A-REI.C.7 | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. | | MA.A-REI.C.8 | Represent a system of linear equations as a single matrix equation in a vector variable. | | MA.A-REI.C.9 | Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). | | MA.A-REI.D | Represent and solve equations and inequalities graphically | | MA.A-REI.B.4a | Use the method of completing the square to transform any quadratic equation in 𝑥 into an equation of the form (𝑥 – 𝑝)² = 𝑞 that has the same solutions. Derive the quadratic formula from this form. | | MA.A-REI.D.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). | | MA.A-REI.B.4b | Solve quadratic equations by inspection (e.g., for 𝑥² = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as 𝑎 ± 𝑏𝑖 for real numbers 𝑎 and 𝑏. | | MA.A-REI.A | Understand solving equations as a process of reasoning and explain the reasoning | | MA.A-REI.D.12 | Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | | MA.A-REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | | MA.A-REI.D.11 | Explain why the 𝑥-coordinates of the points where the graphs of the equations 𝑦 = 𝑓(𝑥) and 𝑦 = 𝑔(𝑥) intersect are the solutions of the equation𝑓(𝑥) = 𝑔(𝑥); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where 𝑓(𝑥) and/or 𝑔(𝑥) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | | MA.A-REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. | |

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| **Essential Questions** |
| How can new trigonometric identities be proven using a set of fundamental trigonometric identities or simplifying trigonometric expressions? How can quadratic techniques be used to solve a trigonometric equation? How can trigonometric equations be solved within a specific interval or across all real numbers? |

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| **Enduring Understanding** |
| Fundamental trigonometric identities, sum and difference formulas together with algebraic techniques are used to simplify expressions, prove identities and solve equations. When trigonometric equations are solved an angle measure is the result. |

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| **Students will know...** |
| 1. Fundamental Trigonometric Identities 2. Solving Trigonometric Equations 3. Sum and Difference Formulas |

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| **Students will be able to...** |
| 1. Use Fundamental Trigonometric Identities to simplify trigonometric expressions. 2. Use Fundamental Trigonometric Identities to verify trigonometric identities. 3. Use Fundamental Trigonometric Identities to solve equations. Use quadratic techniques to solve trigonometric equations. 4. Simplify and evaluate expressions using sum, difference, and double angle formulas. 5. Prove identities and solve equations using sum, difference, and double angle formulas. |

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| **STAGE 2- EVIDENCE OF LEARNING** |
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| **Formative Assessment Suggestions** |
| |  |  | | --- | --- | | • 3- Minute Pause | **.** | | • A-B-C Summaries | **.** | | • Analogy Prompt | **.** | | • Choral Response | **.** | | • Debriefing | **.** | | • Exit Card / Ticket | **.** | | • Hand Signals | **.** | | • Idea Spinner | **.** | | • Index Card Summaries | **.** | | • Inside-Outside Circle Discussion (Fishbowl) | **.** | | • Journal Entry | **.** | | • Misconception Check | **.** | | • Observation | **.** | | • One Minute Essay | **.** | | • One Word Summary | **.** | | • Portfolio Check | **.** | | • Questions & Answers | **.** | | • Quiz | **.** | | • Self-Assessment | **.** | | • Student Conference | **.** | | • Think-Pair-Share | **.** | | • Web or Concept Map | **.** | |

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| **Authentic Assessments Suggestions** |
| Edementum Exact Path (BOY, MOY, EOY)  Class Discussions  Do Nows  Notebook Checks  Homework  Presentations  Webassign |

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| **Benchmark Assessments** |
| Edementum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3- LEARNING PLAN** |
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| **Instructional Map** |
| Topic #1 – Fundamental Trigonometric Identities  Topic #2 – Solving Trigonometric Equations  Topic #3 – Sum and Difference Formulas |

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| **Modifications/Differentiation of Instruction** |
| Differentiation Strategies for Special Education Students   * Remove unnecessary material, words, etc., that can distract from the content * Use of off-grade level materials * Provide appropriate scaffolding * Limit the number of steps required for completion * Time allowed * Level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Varied homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Ability to work at their own pace * Present ideas using auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment * Differentiated checklists and rubrics, if available and appropriate       Differentiation Strategies for Gifted and Talented Students   * Increase the level of complexity * Decrease scaffolding * Variety of finished products * Allow for greater independence * Learning stations, interest groups * Varied texts and supplementary materials * Use of technology * Flexibility in assignments * Varied questioning strategies * Encourage research * Strategy and flexible groups based on formative assessment or student choice * Acceleration within a unit of study * Exposure to more advanced or complex concepts, abstractions, and materials * Encourage students to move through content areas at their own pace * After mastery of a unit, provide students with more advanced learning activities, not more of the same activity * Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas       Differentiated Strategies for ELL Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials, including visuals * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Allow students to work at their own pace * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Role play * Provide graphic organizers, highlighted materials * Strategy and flexible groups based on formative assessment       Differentiation Strategies for At Risk Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment       **504 Plans**  Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:   * walk, breathe, eat, or sleep * communicate, see, hear, or speak * read, concentrate, think, or learn * stand, bend, lift, or work       Examples of accommodations in 504 plans include:   * preferential seating * extended time on tests and assignments * reduced homework or classwork * verbal, visual, or technology aids * modified textbooks or audio-video materials * behavior management support * adjusted class schedules or grading * verbal testing * excused lateness, absence, or missed classwork * pre-approved nurse's office visits and accompaniment to visits * occupational or physical therapy |

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| **Modification Strategies** |
| |  |  | | --- | --- | | • Extended Time | **.** | | • Frequent Breaks | **.** | | • Highlighted Text | **.** | | • Interactive Notebook | **.** | | • Modified Test | **.** | | • Oral Directions | **.** | | • Peer Tutoring | **.** | | • Preferential Seating | **.** | | • Re-Direct | **.** | | • Repeated Drill / Practice | **.** | | • Shortened Assignments | **.** | | • Teacher Notes | **.** | | • Tutorials | **.** | | • Use of Additional Reference Material | **.** | | • Use of Audio Resources | **.** | |

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| **High Preparation Differentiation** |
| |  |  | | --- | --- | | • Alternative Assessments | **.** | | • Choice Boards | **.** | | • Games and Tournaments | **.** | | • Group Investigations | **.** | | • Guided Reading | **.** | | • Independent Research / Project | **.** | | • Interest Groups | **.** | | • Learning Contracts | **.** | | • Leveled Rubrics | **.** | | • Literature Circles | **.** | | • Menu Assignments | **.** | | • Multiple Intelligence Options | **.** | | • Multiple Texts | **.** | | • Personal Agendas | **.** | | • Project Based Learning (PBL) | **.** | | • Stations / Centers | **.** | | • Think-Tac-Toe | **.** | | • Tiered Activities / Assignments | **.** | | • Varying Graphic Organizers | **.** | |

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| **Low Preparation Differentiation** |
| |  |  | | --- | --- | | • Choice of Book / Activity | **.** | | • Cubing Activities | **.** | | • Exploration by Interest (using interest inventories) | **.** | | • Flexible Grouping | **.** | | • Goal Setting With Student | **.** | | • Homework Options | **.** | | • Jigsaw | **.** | | • Mini Workshops to Extend Skills | **.** | | • Mini Workshops to Re-teach | **.** | | • Open-ended Activities | **.** | | • Think-Pair-Share by Interest | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Readiness | **.** | | • Use of Collaboration | **.** | | • Use of Reading Buddies | **.** | | • Varied Journal Prompts | **.** | | • Varied Product Choice | **.** | | • Varied Supplemental Materials | **.** | | • Work Alone / Together | **.** | |

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| **Horizontal Integration- Interdisciplinary Connections** |
| Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other. |

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| **Vertical Integration- Discipline Mapping** |
| The standards in this unit were introduced in Algebra I, Geometry and Algebra II. Precalculus coursework focuses on preparing the students be proficient in Calculus standards. |

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| **Additional Materials** |
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LPS Adopted Textbooks and Programs

* Cengage: Precalculus with Limits: A Graphing Approach
* WebAssign (Computer Based program supplementing Cengage)

Khan Academy

Edmentum Exact Path

**Unit 8: Sequences and Series, Counting Principles and Probability**

Content Area: **Mathematics**  
Course(s): **Pre-Calculus**  
Time Period: **Fourth Marking Period**  
Length: **5 Weeks**  
Status: **Published**

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| **Unit Overview** |
| A sequence implies a pattern or an order.  In mathematics, a sequence is a special function whose domain is the positive integers, and whose range is the discrete set of terms that make up the sequence. A series is the summation of the terms of a sequence.  Mathematical induction can be used to prove statements that are incrementally defined.  Basic counting theory and probability have many applications in the real world.  Permutations and combinations are two common ways to enumerate choices.  Binomial coefficients are linked to combinations and can be used to expand binomials raised to a power. |

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| **STAGE 1- DESIRED RESULTS** |
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| **Educational Standards** |
| The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:  1. Make sense of problems and persevre in solving them.  2. Reason abstractly and quantitatively.  3. Construct viable arguments and critique the reasoning of others.  4. Model with mathematics.  5. Use appropriate tools strategically.  6. Attend to precision.  7. Look for and make use of structure.  8. Look for and express regularity in repeated reasoning. |

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| **New Jersey Student Learning Standards- Mathematics** |
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| **Standard Set: Number and Quantity** |
| |  |  | | --- | --- | |  | During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers. | |  | With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. | |  | Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that (5 to the 1/3 power)³ should be 5 to the (1/3)³ power = 5¹ = 5 and that 5 to the 1/3 power should be the cube root of 5. | |  | Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents. | |  | In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them. | |

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| **Domain: The Real Number System** |
| |  |  | | --- | --- | | MA.N-RN.B.3 | Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. | | MA.N-RN.A | Extend the properties of exponents to rational exponents. | | MA.N-RN.A.1 | Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. | | MA.N-RN.A.2 | Rewrite expressions involving radicals and rational exponents using the properties of exponents. | | MA.N-RN.B | Use properties of rational and irrational numbers. | |

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| **Domain: Quantities** |
| |  |  | | --- | --- | | MA.N-Q.A | Reason quantitatively and use units to solve problems. | | MA.N-Q.A.1 | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | | MA.N-Q.A.2 | Define appropriate quantities for the purpose of descriptive modeling. | | MA.N-Q.A.3 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | |

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| **Domain: The Complex Number System** |
| |  |  | | --- | --- | | MA.N-CN.B.6 | Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. | | MA.N-CN.A | Perform arithmetic operations with complex numbers. | | MA.N-CN.A.1 | Know there is a complex number 𝑖 such that 𝑖² = -1, and every complex number has the form 𝑎 + 𝑏𝑖 with 𝑎 and 𝑏 real. | | MA.N-CN.A.2 | Use the relation 𝑖² = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. | | MA.N-CN.A.3 | Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. | | MA.N-CN.B | Represent complex numbers and their operations on the complex plane. | | MA.N-CN.B.4 | Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. | | MA.N-CN.B.5 | Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. | | MA.N-CN.C | Use complex numbers in polynomial identities and equations. | | MA.N-CN.C.7 | Solve quadratic equations with real coefficients that have complex solutions. | | MA.N-CN.C.8 | Extend polynomial identities to the complex numbers. | | MA.N-CN.C.9 | Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. | |

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| **Domain: Vector and Matrix Quantities** |
| |  |  | | --- | --- | | MA.N-VM.B.5a | Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as 𝑐(𝒗ₓ, 𝒗 subscript 𝑦) = (𝑐𝒗ₓ, 𝑐𝒗 subscript 𝑦). | | MA.N-VM.B.4c | Understand vector subtraction 𝒗 − 𝑤 as 𝒗 + (−𝑤), where –𝑤 is the additive inverse of 𝑤, with the same magnitude as 𝑤 and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. | | MA.N-VM.B.5b | Compute the magnitude of a scalar multiple 𝑐𝒗 using ||𝑐𝒗|| = |𝑐|𝒗. Compute the direction of 𝑐𝒗 knowing that when |𝑐|𝒗 ≠ 0, the direction of 𝑐𝒗 is either along 𝒗 (for 𝑐 > 0) or against 𝒗 (for 𝑐 < 0). | | MA.N-VM.A | Represent and model with vector quantities. | | MA.N-VM.C | Perform operations on matrices and use matrices in applications. | | MA.N-VM.C.6 | Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. | | MA.N-VM.C.7 | Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. | | MA.N-VM.C.8 | Add, subtract, and multiply matrices of appropriate dimensions. | | MA.N-VM.C.9 | Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. | | MA.N-VM.C.10 | Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. | | MA.N-VM.C.11 | Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. | | MA.N-VM.C.12 | Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area. | | MA.N-VM.A.1 | Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., 𝒗, |𝒗|, ||𝒗||, 𝒗). | | MA.N-VM.A.2 | Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. | | MA.N-VM.A.3 | Solve problems involving velocity and other quantities that can be represented by vectors. | | MA.N-VM.B | Perform operations on vectors. | | MA.N-VM.B.4 | Add and subtract vectors. | | MA.N-VM.B.5 | Multiply a vector by a scalar. | | MA.N-VM.B.4a | Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. | | MA.N-VM.B.4b | Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. | |

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| **Standard Set: Statistics and Probability** |
| |  |  | | --- | --- | |  | Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account. | |  | Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. | |  | Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. | |  | Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. | |  | Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time. | |  | Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient. | |

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| **Domain: Interpreting Categorical and Quantitative Data** |
| |  |  | | --- | --- | | MA.S-ID.B.6c | Fit a linear function for a scatter plot that suggests a linear association. | | MA.S-ID.C | Interpret linear models | | MA.S-ID.C.7 | Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | | MA.S-ID.C.8 | Compute (using technology) and interpret the correlation coefficient of a linear fit. | | MA.S-ID.C.9 | Distinguish between correlation and causation. | | MA.S-ID.A | Summarize, represent, and interpret data on a single count or measurement variable | | MA.S-ID.A.1 | Represent data with plots on the real number line (dot plots, histograms, and box plots). | | MA.S-ID.A.2 | Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | | MA.S-ID.A.3 | Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | | MA.S-ID.A.4 | Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. | | MA.S-ID.B | Summarize, represent, and interpret data on two categorical and quantitative variables | | MA.S-ID.B.5 | Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. | | MA.S-ID.B.6 | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. | | MA.S-ID.B.6a | Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. | | MA.S-ID.B.6b | Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology. | |

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| **Domain: Making Inferences and Justifying Conclusions** |
| |  |  | | --- | --- | | MA.S-IC.A | Understand and evaluate random processes underlying statistical experiments | | MA.S-IC.A.1 | Understand statistics as a process for making inferences about population parameters based on a random sample from that population. | | MA.S-IC.A.2 | Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. | | MA.S-IC.B | Make inferences and justify conclusions from sample surveys, experiments, and observational studies | | MA.S-IC.B.3 | Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. | | MA.S-IC.B.4 | Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. | | MA.S-IC.B.5 | Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. | | MA.S-IC.B.6 | Evaluate reports based on data. | |

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| **Domain: Conditional Probability and the Rules of Probability** |
| |  |  | | --- | --- | | MA.S-CP.B | Use the rules of probability to compute probabilities of compound events in a uniform probability model | | MA.S-CP.B.6 | Find the conditional probability of 𝐴 given 𝐵 as the fraction of 𝐵’s outcomes that also belong to 𝐴, and interpret the answer in terms of the model. | | MA.S-CP.B.7 | Apply the Addition Rule, 𝑃(𝐴 𝑜𝑟 𝐵) = 𝑃(𝐴) + 𝑃(𝐵) – 𝑃(𝐴 𝑎𝑛𝑑 𝐵), and interpret the answer in terms of the model. | | MA.S-CP.B.8 | Apply the general Multiplication Rule in a uniform probability model, 𝑃(𝐴 𝑎𝑛𝑑 𝐵) = [𝑃(𝐴)] × [𝑃(𝐵|𝐴)] = [𝑃(𝐵)] × [𝑃(𝐴|𝐵)], and interpret the answer in terms of the model. | | MA.S-CP.B.9 | Use permutations and combinations to compute probabilities of compound events and solve problems. | | MA.S-CP.A | Understand independence and conditional probability and use them to interpret data | | MA.S-CP.A.1 | Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). | | MA.S-CP.A.2 | Understand that two events 𝐴 and 𝐵 are independent if the probability of 𝐴 and 𝐵 occurring together is the product of their probabilities, and use this characterization to determine if they are independent. | | MA.S-CP.A.3 | Understand the conditional probability of 𝐴 given 𝐵 as 𝑃(𝐴 𝑎𝑛𝑑 𝐵)/𝑃(𝐵), and interpret independence of 𝐴 and 𝐵 as saying that the conditional probability of 𝐴 given 𝐵 is the same as the probability of 𝐴, and the conditional probability of 𝐵 given 𝐴 is the same as the probability of 𝐵. | | MA.S-CP.A.4 | Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. | | MA.S-CP.A.5 | Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. | |

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| **Domain: Using Probability to Make Decisions** |
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| **Standard Set: Geometry** |
| |  |  | | --- | --- | |  | Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations. | |  | Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. | |  | The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. | |  | An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. | |  | Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) | |  | During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. | |  | The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. | |  | In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. | |  | Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. | |  | The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion. | |

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| **Domain: Congruence** |
| |  |  | | --- | --- | | MA.G-CO.A.1 | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. | | MA.G-CO.A.2 | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). | | MA.G-CO.A.3 | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. | | MA.G-CO.A.4 | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. | | MA.G-CO.A.5 | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. | | MA.G-CO.D.12 | Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). | | MA.G-CO.B | Understand congruence in terms of rigid motions | | MA.G-CO.D.13 | Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. | | MA.G-CO.B.6 | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. | | MA.G-CO.B.7 | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. | | MA.G-CO.B.8 | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. | | MA.G-CO.C | Prove geometric theorems | | MA.G-CO.C.9 | Prove theorems about lines and angles. | | MA.G-CO.C.10 | Prove theorems about triangles. | | MA.G-CO.C.11 | Prove theorems about parallelograms. | | MA.G-CO.D | Make geometric constructions | | MA.G-CO.A | Experiment with transformations in the plane | |

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| **Domain: Similarity, Right Triangles, and Trigonometry** |
| |  |  | | --- | --- | | MA.G-SRT.D.9 | Derive the formula 𝐴 = (1/2)𝑎𝑏 𝑠𝑖𝑛(𝐶) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. | | MA.G-SRT.D.10 | Prove the Laws of Sines and Cosines and use them to solve problems. | | MA.G-SRT.D.11 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). | | MA.G-SRT.A | Understand similarity in terms of similarity transformations | | MA.G-SRT.A.1 | Verify experimentally the properties of dilations given by a center and a scale factor: | | MA.G-SRT.A.1a | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. | | MA.G-SRT.A.1b | The dilation of a line segment is longer or shorter in the ratio given by the scale factor. | | MA.G-SRT.A.2 | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. | | MA.G-SRT.A.3 | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. | | MA.G-SRT.B | Prove theorems involving similarity | | MA.G-SRT.B.4 | Prove theorems about triangles. | | MA.G-SRT.B.5 | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | | MA.G-SRT.C | Define trigonometric ratios and solve problems involving right triangles | | MA.G-SRT.C.6 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. | | MA.G-SRT.C.7 | Explain and use the relationship between the sine and cosine of complementary angles. | | MA.G-SRT.C.8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. | | MA.G-SRT.D | Apply trigonometry to general triangles | |

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| **Domain: Circles** |
| |  |  | | --- | --- | | MA.G-C.A | Understand and apply theorems about circles | | MA.G-C.A.1 | Prove that all circles are similar. | | MA.G-C.A.2 | Identify and describe relationships among inscribed angles, radii, and chords. | | MA.G-C.A.3 | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. | | MA.G-C.A.4 | Construct a tangent line from a point outside a given circle to the circle. | | MA.G-C.B | Find arc lengths and areas of sectors of circles | | MA.G-C.B.5 | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. | |

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| **Domain: Expressing Geometric Properties with Equations** |
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| **Domain: Geometric Measurement and Dimension** |
| |  |  | | --- | --- | | MA.G-GMD.B.4 | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. | | MA.G-GMD.A | Explain volume formulas and use them to solve problems | | MA.G-GMD.A.1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. | | MA.G-GMD.A.2 | Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures. | | MA.G-GMD.A.3 | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. | | MA.G-GMD.B | Visualize relationships between two-dimensional and three-dimensional objects | |

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| **Domain: Modeling with Geometry** |
| |  |  | | --- | --- | | MA.G-MG.A | Apply geometric concepts in modeling situations | | MA.G-MG.A.1 | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). | | MA.G-MG.A.2 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). | | MA.G-MG.A.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). | |

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| **Standard Set: Modeling** |
| |  |  | | --- | --- | |  | Designing the layout of the stalls in a school fair so as to raise as much money as possible. | |  | Analyzing stopping distance for a car. | |  | Modeling savings account balance, bacterial colony growth, or investment growth. | |  | Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport. | |  | Analyzing risk in situations such as extreme sports, pandemics, and terrorism. | |  | Relating population statistics to individual predictions. | |  | Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). | |  | Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. | |  | A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity. | |  | In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations. | |  | Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed. | |  | Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player. | |

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| **Standard Set: Functions** |
| |  |  | | --- | --- | |  | Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. | |  | In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car’s speed in miles per hour, v; the rule T(v) = 100/v expresses this relationship algebraically and defines a function whose name is T. | |  | The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. | |  | A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, “I’ll give you a state, you give me the capital city;” by an algebraic expression like f(x) = a + bx; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function’s properties. | |  | Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships. | |  | A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions. | |  | Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology. | |

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| **Domain: Interpreting Functions** |
| |  |  | | --- | --- | | MA.F-IF.C.8b | Use the properties of exponents to interpret expressions for exponential functions. | | MA.F-IF.A | Understand the concept of a function and use function notation | | MA.F-IF.A.1 | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If 𝑓 is a function and 𝑥 is an element of its domain, then 𝑓(𝑥) denotes the output of 𝑓 corresponding to the input 𝑥. The graph of 𝑓 is the graph of the equation𝑦 = 𝑓(𝑥). | | MA.F-IF.A.2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | | MA.F-IF.C.9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). | | MA.F-IF.A.3 | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. | | MA.F-IF.B | Interpret functions that arise in applications in terms of the context | | MA.F-IF.B.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. | | MA.F-IF.B.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. | | MA.F-IF.B.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | | MA.F-IF.C | Analyze functions using different representations | | MA.F-IF.C.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | | MA.F-IF.C.7a | Graph linear and quadratic functions and show intercepts, maxima, and minima. | | MA.F-IF.C.7b | Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. | | MA.F-IF.C.7c | Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7d | Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7e | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | | MA.F-IF.C.8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. | | MA.F-IF.C.8a | Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | |

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| **Domain: Building Functions** |
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| **Domain: Linear, Quadratic, and Exponential Models** |
| |  |  | | --- | --- | | MA.F-LE.A | Construct and compare linear and exponential models and solve problems | | MA.F-LE.A.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions. | | MA.F-LE.A.1a | Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. | | MA.F-LE.A.1b | Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. | | MA.F-LE.A.1c | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | | MA.F-LE.A.2 | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). | | MA.F-LE.A.3 | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | | MA.F-LE.A.4 | Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to 𝑎𝑏 to the 𝑐𝘵 power = 𝑑 where 𝑎, 𝑐, and 𝑑 are numbers and the base 𝑏 is 2, 10, or 𝑒; evaluate the logarithm using technology. | | MA.F-LE.B | Interpret expressions for functions in terms of the situation they model | | MA.F-LE.B.5 | Interpret the parameters in a linear or exponential function in terms of a context. | |

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| **Domain: Trigonometric Functions** |
| |  |  | | --- | --- | | MA.F-TF.A.3 | Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π – 𝑥, π + 𝑥, and 2π – 𝑥 in terms of their values for 𝑥, where 𝑥 is any real number. | | MA.F-TF.A.4 | Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. | | MA.F-TF.B | Model periodic phenomena with trigonometric functions | | MA.F-TF.B.5 | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. | | MA.F-TF.B.6 | Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. | | MA.F-TF.B.7 | Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. | | MA.F-TF.C | Prove and apply trigonometric identities | | MA.F-TF.C.8 | Prove the Pythagorean identity 𝑠𝑖𝑛²(θ) + 𝑐𝑜𝑠²(θ) = 1 and use it to find 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) given 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) and the quadrant of the angle. | | MA.F-TF.C.9 | Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. | | MA.F-TF.A | Extend the domain of trigonometric functions using the unit circle | | MA.F-TF.A.1 | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. | | MA.F-TF.A.2 | Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. | |

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| **Standard Set: Algebra** |
| |  |  | | --- | --- | |  | An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances. | |  | Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor. | |  | Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure. | |  | A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave. | |  | An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. | |  | The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system. | |  | An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. | |  | Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x² – 2 = 0 are real numbers, not rational numbers; and the solutions of x² + 2 = 0 are complex numbers, not real numbers. | |  | Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling. | |  | The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, 𝘈 = ((𝘣₁+𝘣₂)/2)𝘩, can be solved for 𝘩 using the same deductive process. | |  | Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them. | |

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| **Domain: Seeing Structures in Expressions** |
| |  |  | | --- | --- | | MA.A-SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. For example, see 𝑥⁴ – 𝑦⁴ as (𝑥²)² – (𝑦²)², thus recognizing it as a difference of squares that can be factored as (𝑥² – 𝑦²)(𝑥² + 𝑦²). | | MA.A-SSE.B | Write expressions in equivalent forms to solve problems | | MA.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. | | MA.A-SSE.B.3a | Factor a quadratic expression to reveal the zeros of the function it defines. | | MA.A-SSE.B.3b | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. | | MA.A-SSE.A | Interpret the structure of expressions | | MA.A-SSE.B.3c | Use the properties of exponents to transform expressions for exponential functions. | | MA.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context. | | MA.A-SSE.A.1a | Interpret parts of an expression, such as terms, factors, and coefficients. | | MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. | | MA.A-SSE.A.1b | Interpret complicated expressions by viewing one or more of their parts as a single entity. | |

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| **Domain: Arithmetic with Polynomials and Rational Functions** |
| |  |  | | --- | --- | | MA.A-APR.C.4 | Prove polynomial identities and use them to describe numerical relationships. | | MA.A-APR.A | Perform arithmetic operations on polynomials | | MA.A-APR.A.1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | | MA.A-APR.B | Understand the relationship between zeros and factors of polynomials | | MA.A-APR.B.2 | Know and apply the Remainder Theorem: For a polynomial 𝑝(𝑥) and a number 𝑎, the remainder on division by 𝑥 – 𝑎 is 𝑝(𝑎), so 𝑝(𝑎) = 0 if and only if (𝑥 – 𝑎) is a factor of 𝑝(𝑥). | | MA.A-APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. | | MA.A-APR.C.5 | Know and apply the Binomial Theorem for the expansion of (𝑥 + 𝑦)ⁿ in powers of 𝑥 and 𝑦 for a positive integer 𝑛, where 𝑥 and 𝑦 are any numbers, with coefficients determined for example by Pascal’s Triangle. | | MA.A-APR.D | Rewrite rational expressions | | MA.A-APR.D.6 | Rewrite simple rational expressions in different forms; write 𝑎(𝑥)/𝑏(𝑥) in the form 𝑞(𝑥) + 𝑟(𝑥)/𝑏(𝑥), where 𝑎(𝑥), 𝑏(𝑥), 𝑞(𝑥), and 𝑟(𝑥) are polynomials with the degree of 𝑟(𝑥) less than the degree of 𝑏(𝑥), using inspection, long division, or, for the more complicated examples, a computer algebra system. | | MA.A-APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. | | MA.A-APR.C | Use polynomial identities to solve problems | |

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| **Domain: Creating Equations** |
| |  |  | | --- | --- | | MA.A-CED.A | Create equations that describe numbers or relationships | | MA.A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems. | | MA.A-CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | | MA.A-CED.A.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. | | MA.A-CED.A.4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. | |

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| **Domain: Reasoning with Equations and Inequalities** |
| |  |  | | --- | --- | | MA.A-REI.B | Solve equations and inequalities in one variable | | MA.A-REI.B.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. | | MA.A-REI.B.4 | Solve quadratic equations in one variable. | | MA.A-REI.C | Solve systems of equations | | MA.A-REI.C.5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. | | MA.A-REI.C.6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. | | MA.A-REI.C.7 | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. | | MA.A-REI.C.8 | Represent a system of linear equations as a single matrix equation in a vector variable. | | MA.A-REI.C.9 | Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). | | MA.A-REI.D | Represent and solve equations and inequalities graphically | | MA.A-REI.B.4a | Use the method of completing the square to transform any quadratic equation in 𝑥 into an equation of the form (𝑥 – 𝑝)² = 𝑞 that has the same solutions. Derive the quadratic formula from this form. | | MA.A-REI.D.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). | | MA.A-REI.B.4b | Solve quadratic equations by inspection (e.g., for 𝑥² = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as 𝑎 ± 𝑏𝑖 for real numbers 𝑎 and 𝑏. | | MA.A-REI.A | Understand solving equations as a process of reasoning and explain the reasoning | | MA.A-REI.D.12 | Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | | MA.A-REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | | MA.A-REI.D.11 | Explain why the 𝑥-coordinates of the points where the graphs of the equations 𝑦 = 𝑓(𝑥) and 𝑦 = 𝑔(𝑥) intersect are the solutions of the equation𝑓(𝑥) = 𝑔(𝑥); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where 𝑓(𝑥) and/or 𝑔(𝑥) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | | MA.A-REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. | |

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| **Essential Questions** |
| What is the difference between a sequence and a series? What is sigma notation? What formulas are available to assist in determining the sum of finite or infinite series? How can induction be used to prove summation and other types of statements?  What counting techniques can be used to answer “how many” type questions? What is the Binomial Theorem?  How can a decision tree help to answer questions about probability? |

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| **Enduring Understanding** |
| A sequence implies a pattern or an order.  In mathematics, a sequence is a special function whose domain is the positive integers, and whose range is the discrete set of terms that make up the sequence. A series is the summation of the terms of a sequence.  Mathematical induction can be used to prove statements that are incrementally defined.  Basic counting theory and probability have many applications in the real world.  Permutations and combinations are two common ways to enumerate choices.  Binomial coefficients are linked to combinations and can be used to expand binomials raised to a power. |

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| **Students will know...** |
| 1. Review Sequences and Series (optional) 2. Mathematical Induction 3. Binomial Theorem 4. Counting Principles and Probability |

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| **Students will be able to...** |
| 1. Find the nth term a sequence. 2. Use sigma notation to express a sum. 3. Identify and work with arithmetic and geometric sequences and series. 4. Use mathematical induction to prove a statement is true. 5. Use the Binomial theorem to expand binomials of find a specified term in a binomial expansion. 6. Apply counting principles, permutations and combinations to answer “how many” questions. 7. Use counting techniques and properties of probability to solve probability problems. |

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| **STAGE 2- EVIDENCE OF LEARNING** |
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| **Formative Assessment Suggestions** |
| |  |  | | --- | --- | | • 3- Minute Pause | **.** | | • A-B-C Summaries | **.** | | • Analogy Prompt | **.** | | • Choral Response | **.** | | • Debriefing | **.** | | • Exit Card / Ticket | **.** | | • Hand Signals | **.** | | • Idea Spinner | **.** | | • Index Card Summaries | **.** | | • Inside-Outside Circle Discussion (Fishbowl) | **.** | | • Journal Entry | **.** | | • Misconception Check | **.** | | • Observation | **.** | | • One Minute Essay | **.** | | • One Word Summary | **.** | | • Portfolio Check | **.** | | • Questions & Answers | **.** | | • Quiz | **.** | | • Self-Assessment | **.** | | • Student Conference | **.** | | • Think-Pair-Share | **.** | | • Web or Concept Map | **.** | |

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| **Authentic Assessments Suggestions** |
| Edementum Exact Path (BOY, MOY, EOY)  Class Discussions  Do Nows  Notebook Checks  Homework  Presentations  Webassign |

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| **Benchmark Assessments** |
| Edementum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3- LEARNING PLAN** |
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| **Instructional Map** |
| Topic #1 – Review of Sequences and Series  Topic #2 – Mathematical Induction  Topic #3 – Binomial Theorem  Topic #4 – Counting and Probability |

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| **Modifications/Differentiation of Instruction** |
| Differentiation Strategies for Special Education Students   * Remove unnecessary material, words, etc., that can distract from the content * Use of off-grade level materials * Provide appropriate scaffolding * Limit the number of steps required for completion * Time allowed * Level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Varied homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Ability to work at their own pace * Present ideas using auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment * Differentiated checklists and rubrics, if available and appropriate       Differentiation Strategies for Gifted and Talented Students   * Increase the level of complexity * Decrease scaffolding * Variety of finished products * Allow for greater independence * Learning stations, interest groups * Varied texts and supplementary materials * Use of technology * Flexibility in assignments * Varied questioning strategies * Encourage research * Strategy and flexible groups based on formative assessment or student choice * Acceleration within a unit of study * Exposure to more advanced or complex concepts, abstractions, and materials * Encourage students to move through content areas at their own pace * After mastery of a unit, provide students with more advanced learning activities, not more of the same activity * Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas       Differentiated Strategies for ELL Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials, including visuals * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Allow students to work at their own pace * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Role play * Provide graphic organizers, highlighted materials * Strategy and flexible groups based on formative assessment       Differentiation Strategies for At Risk Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment       **504 Plans**  Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:   * walk, breathe, eat, or sleep * communicate, see, hear, or speak * read, concentrate, think, or learn * stand, bend, lift, or work       Examples of accommodations in 504 plans include:   * preferential seating * extended time on tests and assignments * reduced homework or classwork * verbal, visual, or technology aids * modified textbooks or audio-video materials * behavior management support * adjusted class schedules or grading * verbal testing * excused lateness, absence, or missed classwork * pre-approved nurse's office visits and accompaniment to visits * occupational or physical therapy |

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| **Modification Strategies** |
| |  |  | | --- | --- | | • Extended Time | **.** | | • Frequent Breaks | **.** | | • Highlighted Text | **.** | | • Interactive Notebook | **.** | | • Modified Test | **.** | | • Oral Directions | **.** | | • Peer Tutoring | **.** | | • Preferential Seating | **.** | | • Re-Direct | **.** | | • Repeated Drill / Practice | **.** | | • Shortened Assignments | **.** | | • Teacher Notes | **.** | | • Tutorials | **.** | | • Use of Additional Reference Material | **.** | | • Use of Audio Resources | **.** | |

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| **High Preparation Differentiation** |
| |  |  | | --- | --- | | • Alternative Assessments | **.** | | • Choice Boards | **.** | | • Games and Tournaments | **.** | | • Group Investigations | **.** | | • Guided Reading | **.** | | • Independent Research / Project | **.** | | • Interest Groups | **.** | | • Learning Contracts | **.** | | • Leveled Rubrics | **.** | | • Literature Circles | **.** | | • Menu Assignments | **.** | | • Multiple Intelligence Options | **.** | | • Multiple Texts | **.** | | • Personal Agendas | **.** | | • Project Based Learning (PBL) | **.** | | • Stations / Centers | **.** | | • Think-Tac-Toe | **.** | | • Tiered Activities / Assignments | **.** | | • Varying Graphic Organizers | **.** | |

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| **Low Preparation Differentiation** |
| |  |  | | --- | --- | | • Choice of Book / Activity | **.** | | • Cubing Activities | **.** | | • Exploration by Interest (using interest inventories) | **.** | | • Flexible Grouping | **.** | | • Goal Setting With Student | **.** | | • Homework Options | **.** | | • Jigsaw | **.** | | • Mini Workshops to Extend Skills | **.** | | • Mini Workshops to Re-teach | **.** | | • Open-ended Activities | **.** | | • Think-Pair-Share by Interest | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Readiness | **.** | | • Use of Collaboration | **.** | | • Use of Reading Buddies | **.** | | • Varied Journal Prompts | **.** | | • Varied Product Choice | **.** | | • Varied Supplemental Materials | **.** | | • Work Alone / Together | **.** | |

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| **Horizontal Integration- Interdisciplinary Connections** |
| Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other. |

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| **Vertical Integration- Discipline Mapping** |
| The standards in this unit were introduced in Algebra I, Geometry and Algebra II. Precalculus coursework focuses on preparing the students be proficient in Calculus standards. |

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| **Additional Materials** |
| LPS Adopted Textbooks and Programs   * Cengage: Precalculus with Limits: A Graphing Approach * WebAssign (Computer Based program supplementing Cengage)   Khan Academy  Edmentum Exact Path |

**Unit 9: Limits**

Content Area: **Mathematics**  
Course(s): **Pre-Calculus**  
Time Period: **Fourth Marking Period**  
Length: **5 Weeks**  
Status: **Published**

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| **Unit Overview** |
| Calculus is the mathematics of change.  Limits are the fundamental process that converts Pre-calculus mathematics to Calculus. A limit is taken at a specific input value and, if defined, is a real number.  As a result, limits behave like real numbers. Limits can be evaluated graphically, numerically, or analytically. |

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| **STAGE 1- DESIRED RESULTS** |
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| **Educational Standards** |
| The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:  1. Make sense of problems and persevre in solving them.  2. Reason abstractly and quantitatively.  3. Construct viable arguments and critique the reasoning of others.  4. Model with mathematics.  5. Use appropriate tools strategically.  6. Attend to precision.  7. Look for and make use of structure.  8. Look for and express regularity in repeated reasoning. |

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| **New Jersey Student Learning Standards- Mathematics** |
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| **Standard Set: Number and Quantity** |
| |  |  | | --- | --- | |  | During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers. | |  | With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. | |  | Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that (5 to the 1/3 power)³ should be 5 to the (1/3)³ power = 5¹ = 5 and that 5 to the 1/3 power should be the cube root of 5. | |  | Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents. | |  | In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them. | |

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| **Domain: The Real Number System** |
| |  |  | | --- | --- | | MA.N-RN.B.3 | Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. | | MA.N-RN.A | Extend the properties of exponents to rational exponents. | | MA.N-RN.A.1 | Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. | | MA.N-RN.A.2 | Rewrite expressions involving radicals and rational exponents using the properties of exponents. | | MA.N-RN.B | Use properties of rational and irrational numbers. | |

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| **Domain: Quantities** |
| |  |  | | --- | --- | | MA.N-Q.A | Reason quantitatively and use units to solve problems. | | MA.N-Q.A.1 | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | | MA.N-Q.A.2 | Define appropriate quantities for the purpose of descriptive modeling. | | MA.N-Q.A.3 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | |

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| **Domain: The Complex Number System** |
| |  |  | | --- | --- | | MA.N-CN.B.6 | Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. | | MA.N-CN.A | Perform arithmetic operations with complex numbers. | | MA.N-CN.A.1 | Know there is a complex number 𝑖 such that 𝑖² = -1, and every complex number has the form 𝑎 + 𝑏𝑖 with 𝑎 and 𝑏 real. | | MA.N-CN.A.2 | Use the relation 𝑖² = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. | | MA.N-CN.A.3 | Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. | | MA.N-CN.B | Represent complex numbers and their operations on the complex plane. | | MA.N-CN.B.4 | Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. | | MA.N-CN.B.5 | Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. | | MA.N-CN.C | Use complex numbers in polynomial identities and equations. | | MA.N-CN.C.7 | Solve quadratic equations with real coefficients that have complex solutions. | | MA.N-CN.C.8 | Extend polynomial identities to the complex numbers. | | MA.N-CN.C.9 | Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. | |

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| **Domain: Vector and Matrix Quantities** |
| |  |  | | --- | --- | | MA.N-VM.B.5a | Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as 𝑐(𝒗ₓ, 𝒗 subscript 𝑦) = (𝑐𝒗ₓ, 𝑐𝒗 subscript 𝑦). | | MA.N-VM.B.4c | Understand vector subtraction 𝒗 − 𝑤 as 𝒗 + (−𝑤), where –𝑤 is the additive inverse of 𝑤, with the same magnitude as 𝑤 and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. | | MA.N-VM.B.5b | Compute the magnitude of a scalar multiple 𝑐𝒗 using ||𝑐𝒗|| = |𝑐|𝒗. Compute the direction of 𝑐𝒗 knowing that when |𝑐|𝒗 ≠ 0, the direction of 𝑐𝒗 is either along 𝒗 (for 𝑐 > 0) or against 𝒗 (for 𝑐 < 0). | | MA.N-VM.A | Represent and model with vector quantities. | | MA.N-VM.C | Perform operations on matrices and use matrices in applications. | | MA.N-VM.C.6 | Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. | | MA.N-VM.C.7 | Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. | | MA.N-VM.C.8 | Add, subtract, and multiply matrices of appropriate dimensions. | | MA.N-VM.C.9 | Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. | | MA.N-VM.C.10 | Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. | | MA.N-VM.C.11 | Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. | | MA.N-VM.C.12 | Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area. | | MA.N-VM.A.1 | Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., 𝒗, |𝒗|, ||𝒗||, 𝒗). | | MA.N-VM.A.2 | Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. | | MA.N-VM.A.3 | Solve problems involving velocity and other quantities that can be represented by vectors. | | MA.N-VM.B | Perform operations on vectors. | | MA.N-VM.B.4 | Add and subtract vectors. | | MA.N-VM.B.5 | Multiply a vector by a scalar. | | MA.N-VM.B.4a | Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. | | MA.N-VM.B.4b | Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. | |

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| **Standard Set: Statistics and Probability** |
| |  |  | | --- | --- | |  | Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account. | |  | Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. | |  | Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. | |  | Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. | |  | Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time. | |  | Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient. | |

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| **Domain: Interpreting Categorical and Quantitative Data** |
| |  |  | | --- | --- | | MA.S-ID.B.6c | Fit a linear function for a scatter plot that suggests a linear association. | | MA.S-ID.C | Interpret linear models | | MA.S-ID.C.7 | Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | | MA.S-ID.C.8 | Compute (using technology) and interpret the correlation coefficient of a linear fit. | | MA.S-ID.C.9 | Distinguish between correlation and causation. | | MA.S-ID.A | Summarize, represent, and interpret data on a single count or measurement variable | | MA.S-ID.A.1 | Represent data with plots on the real number line (dot plots, histograms, and box plots). | | MA.S-ID.A.2 | Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | | MA.S-ID.A.3 | Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | | MA.S-ID.A.4 | Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. | | MA.S-ID.B | Summarize, represent, and interpret data on two categorical and quantitative variables | | MA.S-ID.B.5 | Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. | | MA.S-ID.B.6 | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. | | MA.S-ID.B.6a | Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. | | MA.S-ID.B.6b | Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology. | |

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| **Domain: Making Inferences and Justifying Conclusions** |
| |  |  | | --- | --- | | MA.S-IC.A | Understand and evaluate random processes underlying statistical experiments | | MA.S-IC.A.1 | Understand statistics as a process for making inferences about population parameters based on a random sample from that population. | | MA.S-IC.A.2 | Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. | | MA.S-IC.B | Make inferences and justify conclusions from sample surveys, experiments, and observational studies | | MA.S-IC.B.3 | Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. | | MA.S-IC.B.4 | Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. | | MA.S-IC.B.5 | Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. | | MA.S-IC.B.6 | Evaluate reports based on data. | |

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| **Domain: Conditional Probability and the Rules of Probability** |
| |  |  | | --- | --- | | MA.S-CP.B | Use the rules of probability to compute probabilities of compound events in a uniform probability model | | MA.S-CP.B.6 | Find the conditional probability of 𝐴 given 𝐵 as the fraction of 𝐵’s outcomes that also belong to 𝐴, and interpret the answer in terms of the model. | | MA.S-CP.B.7 | Apply the Addition Rule, 𝑃(𝐴 𝑜𝑟 𝐵) = 𝑃(𝐴) + 𝑃(𝐵) – 𝑃(𝐴 𝑎𝑛𝑑 𝐵), and interpret the answer in terms of the model. | | MA.S-CP.B.8 | Apply the general Multiplication Rule in a uniform probability model, 𝑃(𝐴 𝑎𝑛𝑑 𝐵) = [𝑃(𝐴)] × [𝑃(𝐵|𝐴)] = [𝑃(𝐵)] × [𝑃(𝐴|𝐵)], and interpret the answer in terms of the model. | | MA.S-CP.B.9 | Use permutations and combinations to compute probabilities of compound events and solve problems. | | MA.S-CP.A | Understand independence and conditional probability and use them to interpret data | | MA.S-CP.A.1 | Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). | | MA.S-CP.A.2 | Understand that two events 𝐴 and 𝐵 are independent if the probability of 𝐴 and 𝐵 occurring together is the product of their probabilities, and use this characterization to determine if they are independent. | | MA.S-CP.A.3 | Understand the conditional probability of 𝐴 given 𝐵 as 𝑃(𝐴 𝑎𝑛𝑑 𝐵)/𝑃(𝐵), and interpret independence of 𝐴 and 𝐵 as saying that the conditional probability of 𝐴 given 𝐵 is the same as the probability of 𝐴, and the conditional probability of 𝐵 given 𝐴 is the same as the probability of 𝐵. | | MA.S-CP.A.4 | Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. | | MA.S-CP.A.5 | Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. | |

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| **Domain: Using Probability to Make Decisions** |
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| **Standard Set: Geometry** |
| |  |  | | --- | --- | |  | Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations. | |  | Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. | |  | The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. | |  | An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. | |  | Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) | |  | During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. | |  | The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. | |  | In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. | |  | Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. | |  | The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion. | |

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| **Domain: Congruence** |
| |  |  | | --- | --- | | MA.G-CO.A.1 | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. | | MA.G-CO.A.2 | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). | | MA.G-CO.A.3 | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. | | MA.G-CO.A.4 | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. | | MA.G-CO.A.5 | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. | | MA.G-CO.D.12 | Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). | | MA.G-CO.B | Understand congruence in terms of rigid motions | | MA.G-CO.D.13 | Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. | | MA.G-CO.B.6 | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. | | MA.G-CO.B.7 | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. | | MA.G-CO.B.8 | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. | | MA.G-CO.C | Prove geometric theorems | | MA.G-CO.C.9 | Prove theorems about lines and angles. | | MA.G-CO.C.10 | Prove theorems about triangles. | | MA.G-CO.C.11 | Prove theorems about parallelograms. | | MA.G-CO.D | Make geometric constructions | | MA.G-CO.A | Experiment with transformations in the plane | |

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| **Domain: Similarity, Right Triangles, and Trigonometry** |
| |  |  | | --- | --- | | MA.G-SRT.D.9 | Derive the formula 𝐴 = (1/2)𝑎𝑏 𝑠𝑖𝑛(𝐶) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. | | MA.G-SRT.D.10 | Prove the Laws of Sines and Cosines and use them to solve problems. | | MA.G-SRT.D.11 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). | | MA.G-SRT.A | Understand similarity in terms of similarity transformations | | MA.G-SRT.A.1 | Verify experimentally the properties of dilations given by a center and a scale factor: | | MA.G-SRT.A.1a | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. | | MA.G-SRT.A.1b | The dilation of a line segment is longer or shorter in the ratio given by the scale factor. | | MA.G-SRT.A.2 | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. | | MA.G-SRT.A.3 | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. | | MA.G-SRT.B | Prove theorems involving similarity | | MA.G-SRT.B.4 | Prove theorems about triangles. | | MA.G-SRT.B.5 | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | | MA.G-SRT.C | Define trigonometric ratios and solve problems involving right triangles | | MA.G-SRT.C.6 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. | | MA.G-SRT.C.7 | Explain and use the relationship between the sine and cosine of complementary angles. | | MA.G-SRT.C.8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. | | MA.G-SRT.D | Apply trigonometry to general triangles | |

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| **Domain: Circles** |
| |  |  | | --- | --- | | MA.G-C.A | Understand and apply theorems about circles | | MA.G-C.A.1 | Prove that all circles are similar. | | MA.G-C.A.2 | Identify and describe relationships among inscribed angles, radii, and chords. | | MA.G-C.A.3 | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. | | MA.G-C.A.4 | Construct a tangent line from a point outside a given circle to the circle. | | MA.G-C.B | Find arc lengths and areas of sectors of circles | | MA.G-C.B.5 | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. | |

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| **Domain: Expressing Geometric Properties with Equations** |
| |  |  | | --- | --- | | MA.G-GPE.B.4 | Use coordinates to prove simple geometric theorems algebraically. | | MA.G-GPE.B.5 | Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). | | MA.G-GPE.B.6 | Find the point on a directed line segment between two given points that partitions the segment in a given ratio. | | MA.G-GPE.B.7 | Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. | | MA.G-GPE.A | Translate between the geometric description and the equation for a conic section | | MA.G-GPE.A.1 | Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. | | MA.G-GPE.A.2 | Derive the equation of a parabola given a focus and directrix. | | MA.G-GPE.A.3 | Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. | | MA.G-GPE.B | Use coordinates to prove simple geometric theorems algebraically | |

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| **Domain: Geometric Measurement and Dimension** |
| |  |  | | --- | --- | | MA.G-GMD.B.4 | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. | | MA.G-GMD.A | Explain volume formulas and use them to solve problems | | MA.G-GMD.A.1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. | | MA.G-GMD.A.2 | Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures. | | MA.G-GMD.A.3 | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. | | MA.G-GMD.B | Visualize relationships between two-dimensional and three-dimensional objects | |

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| **Domain: Modeling with Geometry** |
| |  |  | | --- | --- | | MA.G-MG.A | Apply geometric concepts in modeling situations | | MA.G-MG.A.1 | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). | | MA.G-MG.A.2 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). | | MA.G-MG.A.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). | |

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| **Standard Set: Modeling** |
| |  |  | | --- | --- | |  | Designing the layout of the stalls in a school fair so as to raise as much money as possible. | |  | Analyzing stopping distance for a car. | |  | Modeling savings account balance, bacterial colony growth, or investment growth. | |  | Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport. | |  | Analyzing risk in situations such as extreme sports, pandemics, and terrorism. | |  | Relating population statistics to individual predictions. | |  | Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). | |  | Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. | |  | A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity. | |  | In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations. | |  | Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed. | |  | Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player. | |

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| **Standard Set: Functions** |
| |  |  | | --- | --- | |  | Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. | |  | In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car’s speed in miles per hour, v; the rule T(v) = 100/v expresses this relationship algebraically and defines a function whose name is T. | |  | The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. | |  | A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, “I’ll give you a state, you give me the capital city;” by an algebraic expression like f(x) = a + bx; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function’s properties. | |  | Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships. | |  | A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions. | |  | Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology. | |

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| **Domain: Interpreting Functions** |
| |  |  | | --- | --- | | MA.F-IF.C.8b | Use the properties of exponents to interpret expressions for exponential functions. | | MA.F-IF.A | Understand the concept of a function and use function notation | | MA.F-IF.A.1 | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If 𝑓 is a function and 𝑥 is an element of its domain, then 𝑓(𝑥) denotes the output of 𝑓 corresponding to the input 𝑥. The graph of 𝑓 is the graph of the equation𝑦 = 𝑓(𝑥). | | MA.F-IF.A.2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | | MA.F-IF.C.9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). | | MA.F-IF.A.3 | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. | | MA.F-IF.B | Interpret functions that arise in applications in terms of the context | | MA.F-IF.B.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. | | MA.F-IF.B.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. | | MA.F-IF.B.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | | MA.F-IF.C | Analyze functions using different representations | | MA.F-IF.C.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | | MA.F-IF.C.7a | Graph linear and quadratic functions and show intercepts, maxima, and minima. | | MA.F-IF.C.7b | Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. | | MA.F-IF.C.7c | Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7d | Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. | | MA.F-IF.C.7e | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | | MA.F-IF.C.8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. | | MA.F-IF.C.8a | Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | |

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| **Domain: Building Functions** |
| |  |  | | --- | --- | | MA.F-BF.B.4b | Verify by composition that one function is the inverse of another. | | MA.F-BF.B.4c | Read values of an inverse function from a graph or a table, given that the function has an inverse. | | MA.F-BF.B.4d | Produce an invertible function from a non-invertible function by restricting the domain. | | MA.F-BF.B.5 | Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents. | | MA.F-BF.A.1 | Write a function that describes a relationship between two quantities. | | MA.F-BF.A.1a | Determine an explicit expression, a recursive process, or steps for calculation from a context. | | MA.F-BF.A.1b | Combine standard function types using arithmetic operations. | | MA.F-BF.A | Build a function that models a relationship between two quantities | | MA.F-BF.A.1c | Compose functions. | | MA.F-BF.A.2 | Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. | | MA.F-BF.B | Build new functions from existing functions | | MA.F-BF.B.3 | Identify the effect on the graph of replacing 𝑓(𝑥) by 𝑓(𝑥) + 𝑘, 𝑘𝑓(𝑥), 𝑓(𝑘𝑥), and 𝑓(𝑥 + 𝑘) for specific values of 𝑘 (both positive and negative); find the value of 𝑘 given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. | | MA.F-BF.B.4 | Find inverse functions. | | MA.F-BF.B.4a | Solve an equation of the form 𝑓(𝑥) = 𝑐 for a simple function 𝑓 that has an inverse and write an expression for the inverse. | |

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| **Domain: Linear, Quadratic, and Exponential Models** |
| |  |  | | --- | --- | | MA.F-LE.A | Construct and compare linear and exponential models and solve problems | | MA.F-LE.A.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions. | | MA.F-LE.A.1a | Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. | | MA.F-LE.A.1b | Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. | | MA.F-LE.A.1c | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | | MA.F-LE.A.2 | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). | | MA.F-LE.A.3 | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | | MA.F-LE.A.4 | Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to 𝑎𝑏 to the 𝑐𝘵 power = 𝑑 where 𝑎, 𝑐, and 𝑑 are numbers and the base 𝑏 is 2, 10, or 𝑒; evaluate the logarithm using technology. | | MA.F-LE.B | Interpret expressions for functions in terms of the situation they model | | MA.F-LE.B.5 | Interpret the parameters in a linear or exponential function in terms of a context. | |

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| **Domain: Trigonometric Functions** |
| |  |  | | --- | --- | | MA.F-TF.A.3 | Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π – 𝑥, π + 𝑥, and 2π – 𝑥 in terms of their values for 𝑥, where 𝑥 is any real number. | | MA.F-TF.A.4 | Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. | | MA.F-TF.B | Model periodic phenomena with trigonometric functions | | MA.F-TF.B.5 | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. | | MA.F-TF.B.6 | Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. | | MA.F-TF.B.7 | Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. | | MA.F-TF.C | Prove and apply trigonometric identities | | MA.F-TF.C.8 | Prove the Pythagorean identity 𝑠𝑖𝑛²(θ) + 𝑐𝑜𝑠²(θ) = 1 and use it to find 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) given 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) and the quadrant of the angle. | | MA.F-TF.C.9 | Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. | | MA.F-TF.A | Extend the domain of trigonometric functions using the unit circle | | MA.F-TF.A.1 | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. | | MA.F-TF.A.2 | Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. | |

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| **Standard Set: Algebra** |
| |  |  | | --- | --- | |  | An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances. | |  | Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor. | |  | Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure. | |  | A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave. | |  | An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. | |  | The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system. | |  | An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. | |  | Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x² – 2 = 0 are real numbers, not rational numbers; and the solutions of x² + 2 = 0 are complex numbers, not real numbers. | |  | Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling. | |  | The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, 𝘈 = ((𝘣₁+𝘣₂)/2)𝘩, can be solved for 𝘩 using the same deductive process. | |  | Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them. | |

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| **Domain: Seeing Structures in Expressions** |
| |  |  | | --- | --- | | MA.A-SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. For example, see 𝑥⁴ – 𝑦⁴ as (𝑥²)² – (𝑦²)², thus recognizing it as a difference of squares that can be factored as (𝑥² – 𝑦²)(𝑥² + 𝑦²). | | MA.A-SSE.B | Write expressions in equivalent forms to solve problems | | MA.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. | | MA.A-SSE.B.3a | Factor a quadratic expression to reveal the zeros of the function it defines. | | MA.A-SSE.B.3b | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. | | MA.A-SSE.A | Interpret the structure of expressions | | MA.A-SSE.B.3c | Use the properties of exponents to transform expressions for exponential functions. | | MA.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context. | | MA.A-SSE.A.1a | Interpret parts of an expression, such as terms, factors, and coefficients. | | MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. | | MA.A-SSE.A.1b | Interpret complicated expressions by viewing one or more of their parts as a single entity. | |

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| **Domain: Arithmetic with Polynomials and Rational Functions** |
| |  |  | | --- | --- | | MA.A-APR.C.4 | Prove polynomial identities and use them to describe numerical relationships. | | MA.A-APR.A | Perform arithmetic operations on polynomials | | MA.A-APR.A.1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | | MA.A-APR.B | Understand the relationship between zeros and factors of polynomials | | MA.A-APR.B.2 | Know and apply the Remainder Theorem: For a polynomial 𝑝(𝑥) and a number 𝑎, the remainder on division by 𝑥 – 𝑎 is 𝑝(𝑎), so 𝑝(𝑎) = 0 if and only if (𝑥 – 𝑎) is a factor of 𝑝(𝑥). | | MA.A-APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. | | MA.A-APR.C.5 | Know and apply the Binomial Theorem for the expansion of (𝑥 + 𝑦)ⁿ in powers of 𝑥 and 𝑦 for a positive integer 𝑛, where 𝑥 and 𝑦 are any numbers, with coefficients determined for example by Pascal’s Triangle. | | MA.A-APR.D | Rewrite rational expressions | | MA.A-APR.D.6 | Rewrite simple rational expressions in different forms; write 𝑎(𝑥)/𝑏(𝑥) in the form 𝑞(𝑥) + 𝑟(𝑥)/𝑏(𝑥), where 𝑎(𝑥), 𝑏(𝑥), 𝑞(𝑥), and 𝑟(𝑥) are polynomials with the degree of 𝑟(𝑥) less than the degree of 𝑏(𝑥), using inspection, long division, or, for the more complicated examples, a computer algebra system. | | MA.A-APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. | | MA.A-APR.C | Use polynomial identities to solve problems | |

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| **Domain: Creating Equations** |
| |  |  | | --- | --- | | MA.A-CED.A | Create equations that describe numbers or relationships | | MA.A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems. | | MA.A-CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | | MA.A-CED.A.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. | | MA.A-CED.A.4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. | |

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| **Domain: Reasoning with Equations and Inequalities** |
| |  |  | | --- | --- | | MA.A-REI.B | Solve equations and inequalities in one variable | | MA.A-REI.B.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. | | MA.A-REI.B.4 | Solve quadratic equations in one variable. | | MA.A-REI.C | Solve systems of equations | | MA.A-REI.C.5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. | | MA.A-REI.C.6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. | | MA.A-REI.C.7 | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. | | MA.A-REI.C.8 | Represent a system of linear equations as a single matrix equation in a vector variable. | | MA.A-REI.C.9 | Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). | | MA.A-REI.D | Represent and solve equations and inequalities graphically | | MA.A-REI.B.4a | Use the method of completing the square to transform any quadratic equation in 𝑥 into an equation of the form (𝑥 – 𝑝)² = 𝑞 that has the same solutions. Derive the quadratic formula from this form. | | MA.A-REI.D.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). | | MA.A-REI.B.4b | Solve quadratic equations by inspection (e.g., for 𝑥² = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as 𝑎 ± 𝑏𝑖 for real numbers 𝑎 and 𝑏. | | MA.A-REI.A | Understand solving equations as a process of reasoning and explain the reasoning | | MA.A-REI.D.12 | Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | | MA.A-REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | | MA.A-REI.D.11 | Explain why the 𝑥-coordinates of the points where the graphs of the equations 𝑦 = 𝑓(𝑥) and 𝑦 = 𝑔(𝑥) intersect are the solutions of the equation𝑓(𝑥) = 𝑔(𝑥); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where 𝑓(𝑥) and/or 𝑔(𝑥) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | | MA.A-REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. | |

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| **Essential Questions** |
| How can a limit be found graphically? Numerically? Analytically?  When does a limit not exist?  What are the properties of limits?  How is continuity defined in terms of limits?  How can one-sided limits be used to extend continuity to a closed interval?  What is the Intermediate Value Theorem?  What are infinite limits? |

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| **Enduring Understanding** |
| Calculus is the mathematics of change.  Limits are the fundamental process that converts Pre-calculus mathematics to Calculus. A limit is taken at a specific input value and, if defined, is a real number.  As a result, limits behave like real numbers. Limits can be evaluated graphically, numerically, or analytically. |

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| **Students will know...** |
| 1. Limits 2. Continuity |

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| **Students will be able to...** |
| 1. Understand how the limit process is the foundation of Calculus. 2. Develop basic properties of limits. 3. Find a limit both graphically and numerically. 4. Evaluate a limit analytically. 5. Find limits at infinity (x → ±∞). 6. Find infinite limits (y → ±∞) [optional]. 7. Explore the definition and properties of continuity and their relationship to limits. 8. Apply the Intermediate Value Theorem. |

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| **STAGE 2- EVIDENCE OF LEARNING** |
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| **Formative Assessment Suggestions** |
| |  |  | | --- | --- | | • 3- Minute Pause | **.** | | • A-B-C Summaries | **.** | | • Analogy Prompt | **.** | | • Choral Response | **.** | | • Debriefing | **.** | | • Exit Card / Ticket | **.** | | • Hand Signals | **.** | | • Idea Spinner | **.** | | • Index Card Summaries | **.** | | • Inside-Outside Circle Discussion (Fishbowl) | **.** | | • Journal Entry | **.** | | • Misconception Check | **.** | | • Observation | **.** | | • One Minute Essay | **.** | | • One Word Summary | **.** | | • Portfolio Check | **.** | | • Questions & Answers | **.** | | • Quiz | **.** | | • Self-Assessment | **.** | | • Student Conference | **.** | | • Think-Pair-Share | **.** | | • Web or Concept Map | **.** | |

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| **Authentic Assessments Suggestions** |
| Edementum Exact Path (BOY, MOY, EOY)  Class Discussions  Do Nows  Notebook Checks  Homework  Presentations  Webassign |

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| **Benchmark Assessments** |
| Edementum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3- LEARNING PLAN** |
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| **Instructional Map** |
| Topic #1 – Limits  Topic #2 – Continuity |

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| **Modifications/Differentiation of Instruction** |
| Differentiation Strategies for Special Education Students   * Remove unnecessary material, words, etc., that can distract from the content * Use of off-grade level materials * Provide appropriate scaffolding * Limit the number of steps required for completion * Time allowed * Level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Varied homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Ability to work at their own pace * Present ideas using auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment * Differentiated checklists and rubrics, if available and appropriate       Differentiation Strategies for Gifted and Talented Students   * Increase the level of complexity * Decrease scaffolding * Variety of finished products * Allow for greater independence * Learning stations, interest groups * Varied texts and supplementary materials * Use of technology * Flexibility in assignments * Varied questioning strategies * Encourage research * Strategy and flexible groups based on formative assessment or student choice * Acceleration within a unit of study * Exposure to more advanced or complex concepts, abstractions, and materials * Encourage students to move through content areas at their own pace * After mastery of a unit, provide students with more advanced learning activities, not more of the same activity * Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas       Differentiated Strategies for ELL Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials, including visuals * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language. * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Allow students to work at their own pace * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Role play * Provide graphic organizers, highlighted materials * Strategy and flexible groups based on formative assessment       Differentiation Strategies for At Risk Students   * Remove unnecessary materials, words, etc., that can distract from the content * Provide appropriate scaffolding * Limit the number of steps required for completion * Gradually increase the level of independence required * Tiered centers, assignments, lessons, or products * Provide appropriate leveled reading materials * Deliver the content in “chunks” * Varied texts and supplementary materials * Use technology, if available and appropriate * Differentiate homework and products * Varied questioning strategies * Provide background knowledge * Define key vocabulary, multiple-meaning words, and figurative language * Use audio and visual supports, if available and appropriate * Provide multiple learning opportunities to reinforce key concepts and vocabulary * Meet with small groups to reteach idea/skill * Provide cross-content application of concepts * Presenting ideas through auditory, visual, kinesthetic, & tactile means * Provide graphic organizers and/or highlighted materials * Strategy and flexible groups based on formative assessment       **504 Plans**  Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:   * walk, breathe, eat, or sleep * communicate, see, hear, or speak * read, concentrate, think, or learn * stand, bend, lift, or work       Examples of accommodations in 504 plans include:   * preferential seating * extended time on tests and assignments * reduced homework or classwork * verbal, visual, or technology aids * modified textbooks or audio-video materials * behavior management support * adjusted class schedules or grading * verbal testing * excused lateness, absence, or missed classwork * pre-approved nurse's office visits and accompaniment to visits * occupational or physical therapy |

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| **Modification Strategies** |
| |  |  | | --- | --- | | • Extended Time | **.** | | • Frequent Breaks | **.** | | • Highlighted Text | **.** | | • Interactive Notebook | **.** | | • Modified Test | **.** | | • Oral Directions | **.** | | • Peer Tutoring | **.** | | • Preferential Seating | **.** | | • Re-Direct | **.** | | • Repeated Drill / Practice | **.** | | • Shortened Assignments | **.** | | • Teacher Notes | **.** | | • Tutorials | **.** | | • Use of Additional Reference Material | **.** | | • Use of Audio Resources | **.** | |

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| **High Preparation Differentiation** |
| |  |  | | --- | --- | | • Alternative Assessments | **.** | | • Choice Boards | **.** | | • Games and Tournaments | **.** | | • Group Investigations | **.** | | • Guided Reading | **.** | | • Independent Research / Project | **.** | | • Interest Groups | **.** | | • Learning Contracts | **.** | | • Leveled Rubrics | **.** | | • Literature Circles | **.** | | • Menu Assignments | **.** | | • Multiple Intelligence Options | **.** | | • Multiple Texts | **.** | | • Personal Agendas | **.** | | • Project Based Learning (PBL) | **.** | | • Stations / Centers | **.** | | • Think-Tac-Toe | **.** | | • Tiered Activities / Assignments | **.** | | • Varying Graphic Organizers | **.** | |

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| **Low Preparation Differentiation** |
| |  |  | | --- | --- | | • Choice of Book / Activity | **.** | | • Cubing Activities | **.** | | • Exploration by Interest (using interest inventories) | **.** | | • Flexible Grouping | **.** | | • Goal Setting With Student | **.** | | • Homework Options | **.** | | • Jigsaw | **.** | | • Mini Workshops to Extend Skills | **.** | | • Mini Workshops to Re-teach | **.** | | • Open-ended Activities | **.** | | • Think-Pair-Share by Interest | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Learning Style | **.** | | • Think-Pair-Share by Readiness | **.** | | • Use of Collaboration | **.** | | • Use of Reading Buddies | **.** | | • Varied Journal Prompts | **.** | | • Varied Product Choice | **.** | | • Varied Supplemental Materials | **.** | | • Work Alone / Together | **.** | |

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| **Horizontal Integration- Interdisciplinary Connections** |
| Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other. |

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| **Vertical Integration- Discipline Mapping** |
| The standards in this unit were introduced in Algebra I, Geometry and Algebra II. Precalculus coursework focuses on preparing the students be proficient in Calculus standards. |

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| **Additional Materials** |
| LPS Adopted Textbooks and Programs   * Cengage: Precalculus with Limits: A Graphing Approach * WebAssign (Computer Based program supplementing Cengage)   Khan Academy  Edmentum Exact Path  **Interdisciplinary Connections & Standards**  With interdisciplinary instruction, the subject areas are woven together and explored through an overarching theme or concept. We use math to help us solve everyday problems in the kitchen, in the garden, and for many of us at our jobs.  Brain research has shown that information in our brains is organized in schematic structures. These structures are made up of interconnected bits of information and serve as a framework for the knowledge we acquire. When a learner’s knowledge is connected, it is much more likely that they will apply the prior knowledge to a wide variety of new situations. They will acquire new information in a way that is more accessible and will be better able to relate it to previously acquired knowledge.  Students learn about patterns in math, science, social studies, and even literature. Because of this, they are much more likely to “see” these patterns when they encounter new situations. Since patterns are not only studied in math they are able to make the connection and gain the understanding that patterns can be found in many areas of their lives. Interdisciplinary instruction allows students to understand the interconnectedness of the disciplines and makes learning more meaningful and relevant as fascinating connections are made across the subject areas.  **Science**  HS-PS3-1 Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other components(s) and energy flows in and out of the system.  HS-PS3-3 Design, build, and refine a device that woirks within given constraints to covert one form of energy into another form of energy.  HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.  **Language Arts**  RL.11-12.1. Cite strong and thorough textual evidence and make relevant connections to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain.  RI.11-12.1. Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.  RI.11-12.2. Determine two or more central ideas of a text, and analyze their development and how they interact to provide a complex analysis; provide an objective summary of the text.  **Social Studies**  6.1.12.EconEM.2 Analyze how technological developments transformed the economy, created international markets, and affected the environment in New Jersey and the nation.  6.1.12.EconGE.16 Use quantitative data and other sources to assess the impact of international, global business organizations, and oversees competition on the United States economy and workforce. |