**Subject**

**Geometry**

**Curriculum Guide**

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**LINDEN PUBLIC SCHOOLS**

**LINDEN, NEW JERSEY**

**DR. MARNIE HAZELTON**

 **SUPERINTENDENT**

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**The Linden Board of Education adopted the Curriculum Guide on:**

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|  **July 28, 2022** |  | **Education Report #22** |
| **Date** |  | **Agenda Item** |
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| **Rationale** |

**EDUCATION EQUITY:** The Linden Public School District guarantees each student equal educational opportunity regardless of age, race, color, creed, religion, gender, language, affectional or sexual orientation, ancestry, national origin, marital or economic status. For Information, contact District Educational Equity Officer Kevin Thurston at **(**908) 486-2800 x 8307**.**

**NONDISCRIMATION:** The Linden Public School District does not discriminate against handicapped persons in admission or access to or treatment or employment in its programs, activities, and vocational opportunities. For information contact District Public 504 Officer Annabell Louis at (908) 486-2800 x 8025.

**Linden Public Schools Vision**

The Linden Public School District is committed to developing respect for diversity, excellence in education, and a commitment to service, in order to promote global citizenship and ensure personal success for all students

**Linden Public Schools Mission**

The mission of the Linden Public School District is to promote distinction through the infinite resource that is Linden’s diversity, combined with our profound commitment to instructional excellence, so that each and every student achieves their maximum potential in an engaging, inspiring, and challenging learning environment.

**Math Department Vision**

To equip students with the understanding and application of mathematical skills and processes to foster a drive for advanced mathematics and higher-level thinking.

**Math Department Mission Statement**

To develop a community of learners who construct and communicate meaning from the mathematical world around them. Students will experience mathematics that encourage them to think critically, discover and apply concepts to solve problems strategically. Students will be encouraged to solve equations with accuracy, efficiency, and flexibility. Furthermore, students will have a multitude of opportunities to apply mathematical tools and practice standards to solve real-world and multi-step problems.

**Math Department Goals**

* Provide opportunities for student to develop computation skills, conceptual understanding, and problem-solving skills
* Require students to explain, justify or prove their thinking through mathematical reasoning, modeling, and speaking

Course Description

This is a one-year college preparatory course designed to develop reasoning and problem-solving skills. The topics studied include congruence, similarity, and properties of lines, triangles, quadrilaterals and circles. Study also focuses on problem solving skills using length, perimeter, area, circumference, surface area and volume in the setting of real-world situations. A summer math project is required for all students entering this course.

Course Instructional Materials

* LPS Adopted Textbooks and Programs
	+ Pearson EnVision Geometry
	+ Pearson Realize (Computer Based program supplementing Envision)
* Khan Academy
* Edmentum Exact Path

Standards and NJDOE Mandates Guiding Instruction

* 1. New Jersey Student Learning Standards

 <https://www.state.nj.us/education/cccs/>

General Interdisciplinary Connections / Materials

Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, and liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other.

Diversity, Equity, and Inclusion

* Use students’ interests in conceptualized tasks
* Expose students to a diverse group of mathematicians
* Design assessments and assignments with a variety of response types
* Use systematic grading and participation methods
* Encourage students to embrace a growth mindset

Pacing Guide

 Linden Public Schools

Pacing Guide

Geometry

2022-2023

Marking Period 1: September 6, 2022 to November 15, 2022

Topic 1 – Foundations of Geometry – Estimated Time: 19 Days

Topic 2 – Parallel and Perpendicular Lines – Estimated Time: 11 Days

Marking Period 2: November 16, 2022 to January 31, 2023

Topic 3 – Transformations – Estimated Time: 13 Days

Topic 4 – Triangle Congruence – Estimated Time: 15 Days

Topic 5 – Relationships in Triangles – Estimated Time: 13 Days

Marking Period 3: February 1, 2023 to April 5, 2023

Topic 6 – Quadrilaterals and Other Polygons – Estimated Time: 15 Days

Topic 7 – Similarity – Estimated Time: 13 Days

Marking Period 4: April 17, 2023 to June 22, 2023

Topic 8 – Right Triangles and Trigonometry – Estimated Time: 13 Days

Topic 9 – Coordinate Geometry – Estimated Time: 11 Days

Topic 10 – Circles – Estimated Time: 13 Days

Topic 11 – Two- and Three-Dimensional Models – Estimated Time: 11 Days

Topic 12 – Probability – Estimated Time – 15 Days (if time permits)

\***Assessment days are built into each chapter**.

1. Vertical Integration – Program Mapping

The standards in this unit were introduced in Algebra I. Geometry coursework focuses on preparing the students be proficient in Algebra II standards.

1. Accommodations, Modifications, and Teacher Strategies

(Specific recommendations are made in each unit)

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| **Instructional Strategies*** Teacher Presentation
* Student Presentation
* Class Discussion
* Reading for Meaning
* Inquiry Design Model
* Interactive Lecture
* Interactive Notetaking
* Compare and Contrast
* Research Based
* Problem Based
* Project Based

**504 Plans**Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:* walk, breathe, eat, or sleep
* communicate, see, hear, or speak
* read, concentrate, think, or learn
* stand, bend, lift, or work

Examples of accommodations in 504 plans include:* preferential seating
* extended time on tests and assignments
* reduced homework or classwork
* verbal, visual, or technology aids
* modified textbooks or audio-video materials
* behavior management support
* adjusted class schedules or grading
* verbal testing
* excused lateness, absence, or missed classwork
* pre-approved nurse's office visits and accompaniment to visits occupational or physical therapy
 | **Gifted and Talent Accommodations and Modifications*** Increase the level of complexity
* Decrease scaffolding
* Variety of finished products
* Allow for greater independence
* Learning stations, interest groups
* Varied texts and supplementary materials
* Use of technology
* Flexibility in assignments
* Varied questioning strategies
* Encourage research
* Strategy and flexible groups based on formative assessment or student choice
* Acceleration within a unit of study
* Exposure to more advanced or complex concepts, abstractions, and materials
* Encourage students to move through content areas at their own pace
* After mastery of a unit, provide students with more advanced learning activities, not more of the same activity
* Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas
 | **Special Education and At-Risk Accommodations and Modifications*** Remove unnecessary material, words, etc., that can distract from the content
* Use of off-grade level materials
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Time allowed
* Level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Varied homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Ability to work at their own pace
* Present ideas using auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment
* Differentiated checklists and rubrics, if available and appropriate
 | **English Language Learners Accommodations and Modifications*** Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials, including visuals
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Allow students to work at their own pace
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Role play
* Provide graphic organizers, highlighted materials
* Strategy and flexible groups based on formative assessment
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| **Unit 1**  |
| **Overview:**In unit 1, the following topics will be covered: Foundations of Geometry, Parallel & Perpendicular Lines, Transformations and Triangle Congruence. Time Period: **First / Second Marking Period** Length:  **11 Weeks** |

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| **STAGE 1 – Desired Results** |

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| **Educational Standards**The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:1. Make sense of problems and persevere in solving them.2. Reason abstractly and quantitatively.3. Construct viable arguments and critique the reasoning of others.4. Model with mathematics.5. Use appropriate tools strategically.6. Attend to precision.7. Look for and make use of structure.8. Look for and express regularity in repeated reasoning.

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| **New Jersey Student Learning Standards- Mathematics** |
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| **Introduction- Geometry** |
| Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations. Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. |
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**Congruence**

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| MA.G-CO.A.1 | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.  |
| MA.G-CO.A.2 | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).  |
| MA.G-CO.A.3 | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.  |
| MA.G-CO.A.4 | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.  |
| MA.G-CO.A.5 | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.  |
| MA.G-CO.D.12 | Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).  |
| MA.G-CO.B | Understand congruence in terms of rigid motions  |
| MA.G-CO.D.13 | Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.  |
| MA.G-CO.B.6 | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.  |
| MA.G-CO.B.7 | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.  |
| MA.G-CO.B.8 | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.  |
| MA.G-CO.C | Prove geometric theorems  |
| MA.G-CO.C.9 | Prove theorems about lines and angles.  |
| MA.G-CO.C.10 | Prove theorems about triangles.  |
| MA.G-CO.C.11 | Prove theorems about parallelograms.  |
| MA.G-CO.D | Make geometric constructions  |
| MA.G-CO.A | Experiment with transformations in the plane  |

**Similarity, Right Triangles and Trigonometry**

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| MA.G-SRT.D.9 | Derive the formula 𝐴 = (1/2)𝑎𝑏 𝑠𝑖𝑛(𝐶) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.  |
| MA.G-SRT.D.10 | Prove the Laws of Sines and Cosines and use them to solve problems.  |
| MA.G-SRT.D.11 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).  |
| MA.G-SRT.A | Understand similarity in terms of similarity transformations  |
| MA.G-SRT.A.1 | Verify experimentally the properties of dilations given by a center and a scale factor:  |
| MA.G-SRT.A.1a | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.  |
| MA.G-SRT.A.1b | The dilation of a line segment is longer or shorter in the ratio given by the scale factor.  |
| MA.G-SRT.A.2 | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.  |
| MA.G-SRT.A.3 | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.  |
| MA.G-SRT.B | Prove theorems involving similarity  |
| MA.G-SRT.B.4 | Prove theorems about triangles.  |
| MA.G-SRT.B.5 | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.  |
| MA.G-SRT.C | Define trigonometric ratios and solve problems involving right triangles  |
| MA.G-SRT.C.6 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.  |
| MA.G-SRT.C.7 | Explain and use the relationship between the sine and cosine of complementary angles.  |
| MA.G-SRT.C.8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.  |
| MA.G-SRT.D | Apply trigonometry to general triangles  |

**Circles**

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| MA.G-C.A | Understand and apply theorems about circles  |
| MA.G-C.A.1 | Prove that all circles are similar.  |
| MA.G-C.A.2 | Identify and describe relationships among inscribed angles, radii, and chords.  |
| MA.G-C.A.3 | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.  |
| MA.G-C.A.4 | Construct a tangent line from a point outside a given circle to the circle.  |
| MA.G-C.B | Find arc lengths and areas of sectors of circles  |
| MA.G-C.B.5 | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.  |

**Expressing Geometric Properties with Equations**

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| MA.G-GPE.B.4 | Use coordinates to prove simple geometric theorems algebraically.  |
| MA.G-GPE.B.5 | Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).  |
| MA.G-GPE.B.6 | Find the point on a directed line segment between two given points that partitions the segment in a given ratio.  |
| MA.G-GPE.B.7 | Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.  |
| MA.G-GPE.A | Translate between the geometric description and the equation for a conic section  |
| MA.G-GPE.A.1 | Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.  |
| MA.G-GPE.A.2 | Derive the equation of a parabola given a focus and directrix.  |
| MA.G-GPE.A.3 | Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.  |
| MA.G-GPE.B | Use coordinates to prove simple geometric theorems algebraically  |

**Modeling with Geometry**

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| MA.G-MG.A | Apply geometric concepts in modeling situations  |
| MA.G-MG.A.1 | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).  |
| MA.G-MG.A.2 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).  |
| MA.G-MG.A.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).  |

**Career Ready Practices**CRP2.   Apply appropriate academic and technical skills.CRP4.   Communicate clearly and effectively and with reason.CRP6.   Demonstrate creativity and innovation.CRP8.   Utilize critical thinking to make sense of problems and persevere in solving them.CRP11.   Use technology to enhance productivity.CRP12.   Work productively in teams while using cultural global competence.**Essential Questions…** * How are the properties of reflection used to transform a figure?
* How can rigid motions be classified?
* What are the properties of a translation?
* What are the properties that identify a rotation?
* How can you tell whether a figure is symmetric?
* What is the relationship between rigid motions and congruence?

**Enduring Understanding…*** Reflections are rigid motions across a line of reflection.
* A translation is a rigid motion that moves all points of the preimage the same distance in the same direction.
* Rotation is a rigid motion described by its center of rotation and angle of rotation.
* Any composition of rigid motions can be represented by combination of at least two of the following: a translation, reflection, rotation or glide reflection.
* A figure that can be mapped onto itself using rigid motions is symmetric.
* Many real-world problem situations can be represented with a mathematical model, but that model might not represent the real – world situation exactly.

**Students will know...*** Experiment with transformations in the plane
* Understand congruence in terms of rigid motions
* Prove geometric theorems
* Make geometric constructions

**Students will be able to...*** Use the undefined notion of a point, line, distance along a line and distance around a circular arc to develop definitions for angles, circles, parallel lines, perpendicular lines, and line segments.
* Apply the definitions of angles, circles, parallel lines, perpendicular lines, and line segments to describe rotations, reflections, and translations.
* Develop and perform rigid transformations that include reflections, rotations, and translations using geometric software, graph paper, tracing paper, and geometric tools, and compare them to non-rigid transformations.
* Use rigid transformations to determine, explain and prove congruence of geometric figures.
* Create proofs of theorems involving lines, angles, triangles, and parallelograms
* Generate formal constructions with paper folding, geometric software and geometric tools to include, but not limited to, the construction of regular polygons inscribed in a circle.
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| **STAGE 2 – Evidence of Learning** |

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| **Formative Assessment Suggestions**

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| • 3- Minute Pause |
| • A-B-C Summaries |
| • Analogy Prompt |
| • Choral Response |
| • Debriefing |
| • Exit Card / Ticket |
| • Hand Signals |
| • Idea Spinner |
| • Index Card Summaries |
| • Inside-Outside Circle Discussion (Fishbowl) |
| • Journal Entry |
| • Misconception Check |
| • Observation |
| • One Minute Essay |
| • One Word Summary |
| • Portfolio Check |
| • Questions & Answers |
| • Quiz |
| • Self-Assessment |
| • Student Conference |
| • Think-Pair-Share |
| • Web or Concept Map |

**Authentic Assessment Suggestions**Through the following authentic assessments, students will develop traits regarding thinking and reasoning, settings, mathematical tools and attitudes and dispositions:* Performance Assessments
* Short Investigations
* Open Ended Response Questions
* Portfolios
* Self-Assessments

**Benchmark Assessments*** Edementum Exact Path (BOY, MOY, EOY)
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| **STAGE 3 – Learning Plan** |

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| **Instructional Map**

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| **Modifications/Differentiation of Instruction** |
| **Differentiation Strategies for Special Education Students*** Remove unnecessary material, words, etc., that can distract from the content
* Use of off-grade level materials
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Time allowed
* Level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Varied homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Ability to work at their own pace
* Present ideas using auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment
* Differentiated checklists and rubrics, if available and appropriate

**Differentiation Strategies for Gifted and Talented Students*** Increase the level of complexity
* Decrease scaffolding
* Variety of finished products
* Allow for greater independence
* Learning stations, interest groups
* Varied texts and supplementary materials
* Use of technology
* Flexibility in assignments
* Varied questioning strategies
* Encourage research
* Strategy and flexible groups based on formative assessment or student choice
* Acceleration within a unit of study
* Exposure to more advanced or complex concepts, abstractions, and materials
* Encourage students to move through content areas at their own pace
* After mastery of a unit, provide students with more advanced learning activities, not more of the same activity
* Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas

**Differentiated Strategies for ELL Students*** Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials, including visuals
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Allow students to work at their own pace
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Role play
* Provide graphic organizers, highlighted materials
* Strategy and flexible groups based on formative assessment

**Differentiation Strategies for At Risk Students*** Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessments

**504 Plans**Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:* walk, breathe, eat, or sleep
* communicate, see, hear, or speak
* read, concentrate, think, or learn
* stand, bend, lift, or work

Examples of accommodations in 504 plans include:* preferential seating
* extended time on tests and assignments
* reduced homework or classwork
* verbal, visual, or technology aids
* modified textbooks or audio-video materials
* behavior management support
* adjusted class schedules or grading
* verbal testing
* excused lateness, absence, or missed classwork
* pre-approved nurse's office visits and accompaniment to visits
* occupational or physical therapy
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| **Modification Strategies** |
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| • Extended Time |  .  |
| • Frequent Breaks |  .  |
| • Highlighted Text |  .  |
| • Interactive Notebook |  .  |
| • Modified Test |  .  |
| • Oral Directions |  .  |
| • Peer Tutoring |  .  |
| • Preferential Seating |  .  |
| • Re-Direct |  .  |
| • Repeated Drill / Practice |  .  |
| • Shortened Assignments |  .  |
| • Teacher Notes |  .  |
| • Tutorials |  .  |
| • Use of Additional Reference Material |  .  |
| • Use of Audio Resources |  .  |

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| **High Preparation Differentiation** |
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| • Alternative Assessments |  .  |
| • Choice Boards |  .  |
| • Games and Tournaments |  .  |
| • Group Investigations |  .  |
| • Guided Reading |  .  |
| • Independent Research / Project |  .  |
| • Interest Groups |  .  |
| • Learning Contracts |  .  |
| • Leveled Rubrics |  .  |
| • Literature Circles |  .  |
| • Menu Assignments |  .  |
| • Multiple Intelligence Options |  .  |
| • Multiple Texts |  .  |
| • Personal Agendas |  .  |
| • Project Based Learning (PBL) |  .  |
| • Stations / Centers |  .  |
| • Think-Tac-Toe |  .  |
| • Tiered Activities / Assignments |  .  |
| • Varying Graphic Organizers |  .  |

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| **Low Preparation Differentiation** |
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| • Choice of Book / Activity |  .  |
| • Cubing Activities |  .  |
| • Exploration by Interest (using interest inventories) |  .  |
| • Flexible Grouping |  .  |
| • Goal Setting With Student |  .  |
| • Homework Options |  .  |
| • Jigsaw |  .  |
| • Mini Workshops to Extend Skills |  .  |
| • Mini Workshops to Re-teach |  .  |
| • Open-ended Activities |  .  |
| • Think-Pair-Share by Interest |  .  |
| • Think-Pair-Share by Learning Style |  .  |
| • Think-Pair-Share by Learning Style |  .  |
| • Think-Pair-Share by Readiness |  .  |
| • Use of Collaboration |  .  |
| • Use of Reading Buddies |  .  |
| • Varied Journal Prompts |  .  |
| • Varied Product Choice |  .  |
| • Varied Supplemental Materials |  .  |
| • Work Alone / Together |  .  |

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**Unit Specific Interdisciplinary Connections / Materials**With interdisciplinary instruction, the subject areas are woven together and explored through an overarching theme or concept. We use math to help us solve everyday problems in the kitchen, in the garden, and for many of us at our jobs.Brain research has shown that information in our brains is organized in schematic structures. These structures are made up of interconnected bits of information and serve as a framework for the knowledge we acquire. When a learner’s knowledge is connected, it is much more likely that they will apply the prior knowledge to a wide variety of new situations. They will acquire new information in a way that is more accessible and will be better able to relate it to previously acquired knowledge.Students learn about patterns in math, science, social studies, and even literature. Because of this, they are much more likely to “see” these patterns when they encounter new situations. Since patterns are not only studied in math, they are able to make the connection and gain the understanding that patterns can be found in many areas of their lives.  Interdisciplinary instruction allows students to understand the interconnectedness of the disciplines and makes learning more meaningful and relevant as fascinating connections are made across the subject areas. |

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| **Additional Materials** |

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| LPS Adopted Textbooks and Programs * Pearson EnVision Geometry
* Pearson Realize (Computer Based program supplementing Envision)

Khan AcademyEdmentum Exact Path  |

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| **Unit 2** |
| **Overview:**In unit 2, the following topics will be covered: Relationships in Triangles, Quadrilaterals & Other Polygons, Similarity, and Right Triangles & Trigonometry.Time Period: **Second / Third / Fourth Marking Periods**Length: **11 Weeks** |

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| **STAGE 1 – Desired Results** |

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| **Educational Standards**The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:1. Make sense of problems and persevere in solving them.2. Reason abstractly and quantitatively.3. Construct viable arguments and critique the reasoning of others.4. Model with mathematics.5. Use appropriate tools strategically.6. Attend to precision.7. Look for and make use of structure.8. Look for and express regularity in repeated reasoning.**Introduction – Geometry**

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|  | Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations. |
|  | Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. |
|  | The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. |
|  | Connections to Equations. |
|  | An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. |
|  | Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) |
|  | During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. |
|  | The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. |
|  | In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. |
|  | Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. |

**Congruence**

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| MA.G-CO.A.1 | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.  |
| MA.G-CO.A.2 | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).  |
| MA.G-CO.A.3 | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.  |
| MA.G-CO.A.4 | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.  |
| MA.G-CO.A.5 | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.  |
| MA.G-CO.D.12 | Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).  |
| MA.G-CO.B | Understand congruence in terms of rigid motions  |
| MA.G-CO.D.13 | Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.  |
| MA.G-CO.B.6 | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.  |
| MA.G-CO.B.7 | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.  |
| MA.G-CO.B.8 | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.  |
| MA.G-CO.C | Prove geometric theorems  |
| MA.G-CO.C.9 | Prove theorems about lines and angles.  |
| MA.G-CO.C.10 | Prove theorems about triangles.  |
| MA.G-CO.C.11 | Prove theorems about parallelograms.  |
| MA.G-CO.D | Make geometric constructions  |
| MA.G-CO.A | Experiment with transformations in the plane  |

**Similarity, Right Triangles and Trigonometry**

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| MA.G-SRT.D.9 | Derive the formula 𝐴 = (1/2)𝑎𝑏 𝑠𝑖𝑛(𝐶) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.  |
| MA.G-SRT.D.10 | Prove the Laws of Sines and Cosines and use them to solve problems.  |
| MA.G-SRT.D.11 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).  |
| MA.G-SRT.A | Understand similarity in terms of similarity transformations  |
| MA.G-SRT.A.1 | Verify experimentally the properties of dilations given by a center and a scale factor:  |
| MA.G-SRT.A.1a | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.  |
| MA.G-SRT.A.1b | The dilation of a line segment is longer or shorter in the ratio given by the scale factor.  |
| MA.G-SRT.A.2 | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.  |
| MA.G-SRT.A.3 | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.  |
| MA.G-SRT.B | Prove theorems involving similarity  |
| MA.G-SRT.B.4 | Prove theorems about triangles.  |
| MA.G-SRT.B.5 | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.  |
| MA.G-SRT.C | Define trigonometric ratios and solve problems involving right triangles  |
| MA.G-SRT.C.6 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.  |
| MA.G-SRT.C.7 | Explain and use the relationship between the sine and cosine of complementary angles.  |
| MA.G-SRT.C.8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.  |
| MA.G-SRT.D | Apply trigonometry to general triangles  |

**Circles**

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| MA.G-C.A | Understand and apply theorems about circles  |
| MA.G-C.A.1 | Prove that all circles are similar.  |
| MA.G-C.A.2 | Identify and describe relationships among inscribed angles, radii, and chords.  |
| MA.G-C.A.3 | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.  |
| MA.G-C.A.4 | Construct a tangent line from a point outside a given circle to the circle.  |
| MA.G-C.B | Find arc lengths and areas of sectors of circles  |
| MA.G-C.B.5 | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.  |

**Expressing Geometric Properties with Equations**

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| MA.G-GPE.B.4 | Use coordinates to prove simple geometric theorems algebraically.  |
| MA.G-GPE.B.5 | Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).  |
| MA.G-GPE.B.6 | Find the point on a directed line segment between two given points that partitions the segment in a given ratio.  |
| MA.G-GPE.B.7 | Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.  |
| MA.G-GPE.A | Translate between the geometric description and the equation for a conic section  |
| MA.G-GPE.A.1 | Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.  |
| MA.G-GPE.A.2 | Derive the equation of a parabola given a focus and directrix.  |
| MA.G-GPE.A.3 | Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.  |
| MA.G-GPE.B | Use coordinates to prove simple geometric theorems algebraically  |

**Geometric Measurement and Dimension**

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| MA.G-GMD.B.4 | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.  |
| MA.G-GMD.A | Explain volume formulas and use them to solve problems  |
| MA.G-GMD.A.1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.  |
| MA.G-GMD.A.2 | Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures.  |
| MA.G-GMD.A.3 | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.  |
| MA.G-GMD.B | Visualize relationships between two-dimensional and three-dimensional objects  |

**Modeling with Geometry**

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| MA.G-MG.A | Apply geometric concepts in modeling situations  |
| MA.G-MG.A.1 | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).  |
| MA.G-MG.A.2 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).  |
| MA.G-MG.A.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).  |

**Career Ready Practices**CRP2.   Apply appropriate academic and technical skills.CRP4.   Communicate clearly and effectively and with reason.CRP6.   Demonstrate creativity and innovation.CRP8.   Utilize critical thinking to make sense of problems and persevere in solving them.CRP11.   Use technology to enhance productivity.CRP12.   Work productively in teams while using cultural global competence.**Essential Questions…** * What makes a transformation a similarity transformation? What is the relationship between a preimage and the image resulting from a similarity transformation?
* How can you use the angles and sides of two triangles to determine whether they are similar?
* In a right triangle, what is the relationship between the altitude to the hypotenuse, triangle similarity, and the geometric mean?

**Enduring Understanding…*** A dilation is a transformation that preserves angle measure but not length.
* A similarity transformation is a dilation combined with one or more rigid motions.
* Two triangles are similar if a composition rigid motions and dilation will map one triangle onto the other.
* The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the two segments into which the altitude divides the hypotenuse.
* Many real-world situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly.
* A segment parallel to one side of a triangle divides the triangle into two similar triangles.

**Students will know...*** Use coordinates to prove simple geometric theorems algebraically.
* Understand similarity in terms of similarity transformations.
* Prove geometric theorems
* Prove theorems involving similarity
* Define trigonometric ratios and solve problems involving right triangles
* Apply trigonometry to general triangles.

**Students will be able to...*** Use the undefined notion of a point, line, distance along a line and distance around a circular arc to develop definitions for angles, circles, parallel lines, perpendicular lines, and line segments.
* Apply the definitions of angles, circles, parallel lines, perpendicular lines, and line segments to describe rotations, reflections, and translations.
* Develop and perform rigid transformations that include reflections, rotations, and translations using geometric software, graph paper, tracing paper, and geometric tools, and compare them to non-rigid transformations.
* Use rigid transformations to determine, explain and prove congruence of geometric figures.
* Create proofs of theorems involving lines, angles, triangles, and parallelograms
* Generate formal constructions with paper folding, geometric software and geometric tools to include, but not limited to, the construction of regular polygons inscribed in a circle.
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| **STAGE 2 – Evidence of Learning** |

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| **Formative Assessment Suggestions**

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| • 3- Minute Pause |
| • A-B-C Summaries |
| • Analogy Prompt |
| • Choral Response |
| • Debriefing |
| • Exit Card / Ticket |
| • Hand Signals |
| • Idea Spinner |
| • Index Card Summaries |
| • Inside-Outside Circle Discussion (Fishbowl) |
| • Journal Entry |
| • Misconception Check |
| • Observation |
| • One Minute Essay |
| • One Word Summary |
| • Portfolio Check |
| • Questions & Answers |
| • Quiz |
| • Self-Assessment |
| • Student Conference |
| • Think-Pair-Share |
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**Authentic Assessment Suggestions**Through the following authentic assessments, students will develop traits regarding thinking and reasoning, settings, mathematical tools and attitudes and dispositions:* Performance Assessments
* Short Investigations
* Open Ended Response Questions
* Portfolios
* Self-Assessments

**Benchmark Assessments**Edementum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3 – Learning Plan** |

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| **Instructional Map**

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| **Modifications/Differentiation of Instruction** |
| **Differentiation Strategies for Special Education Students*** Remove unnecessary material, words, etc., that can distract from the content
* Use of off-grade level materials
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Time allowed
* Level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Varied homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Ability to work at their own pace
* Present ideas using auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment
* Differentiated checklists and rubrics, if available and appropriate

**Differentiation Strategies for Gifted and Talented Students*** Increase the level of complexity
* Decrease scaffolding
* Variety of finished products
* Allow for greater independence
* Learning stations, interest groups
* Varied texts and supplementary materials
* Use of technology
* Flexibility in assignments
* Varied questioning strategies
* Encourage research
* Strategy and flexible groups based on formative assessment or student choice
* Acceleration within a unit of study
* Exposure to more advanced or complex concepts, abstractions, and materials
* Encourage students to move through content areas at their own pace
* After mastery of a unit, provide students with more advanced learning activities, not more of the same activity
* Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas

**Differentiated Strategies for ELL Students*** Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials, including visuals
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* Provide cross-content application of concepts
* Allow students to work at their own pace
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Role play
* Provide graphic organizers, highlighted materials
* Strategy and flexible groups based on formative assessments

**Differentiation Strategies for At Risk Students*** Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessments

**504 Plans**Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:* walk, breathe, eat, or sleep
* communicate, see, hear, or speak
* read, concentrate, think, or learn
* stand, bend, lift, or work

Examples of accommodations in 504 plans include:* preferential seating
* extended time on tests and assignments
* reduced homework or classwork
* verbal, visual, or technology aids
* modified textbooks or audio-video materials
* behavior management support
* adjusted class schedules or grading
* verbal testing
* excused lateness, absence, or missed classwork
* pre-approved nurse's office visits and accompaniment to visits
* occupational or physical therapy
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| **Modification Strategies** |
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| • Extended Time |  .  |
| • Frequent Breaks |  .  |
| • Highlighted Text |  .  |
| • Interactive Notebook |  .  |
| • Modified Test |  .  |
| • Oral Directions |  .  |
| • Peer Tutoring |  .  |
| • Preferential Seating |  .  |
| • Re-Direct |  .  |
| • Repeated Drill / Practice |  .  |
| • Shortened Assignments |  .  |
| • Teacher Notes |  .  |
| • Tutorials |  .  |
| • Use of Additional Reference Material |  .  |
| • Use of Audio Resources |  .  |

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| **High Preparation Differentiation** |
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| • Alternative Assessments |  .  |
| • Choice Boards |  .  |
| • Games and Tournaments |  .  |
| • Group Investigations |  .  |
| • Guided Reading |  .  |
| • Independent Research / Project |  .  |
| • Interest Groups |  .  |
| • Learning Contracts |  .  |
| • Leveled Rubrics |  .  |
| • Literature Circles |  .  |
| • Menu Assignments |  .  |
| • Multiple Intelligence Options |  .  |
| • Multiple Texts |  .  |
| • Personal Agendas |  .  |
| • Project Based Learning (PBL) |  .  |
| • Stations / Centers |  .  |
| • Think-Tac-Toe |  .  |
| • Tiered Activities / Assignments |  .  |
| • Varying Graphic Organizers |  .  |

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| **Low Preparation Differentiation** |
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| • Choice of Book / Activity |  .  |
| • Cubing Activities |  .  |
| • Exploration by Interest (using interest inventories) |  .  |
| • Flexible Grouping |  .  |
| • Goal Setting With Student |  .  |
| • Homework Options |  .  |
| • Jigsaw |  .  |
| • Mini Workshops to Extend Skills |  .  |
| • Mini Workshops to Re-teach |  .  |
| • Open-ended Activities |  .  |
| • Think-Pair-Share by Interest |  .  |
| • Think-Pair-Share by Learning Style |  .  |
| • Think-Pair-Share by Learning Style |  .  |
| • Think-Pair-Share by Readiness |  .  |
| • Use of Collaboration |  .  |
| • Use of Reading Buddies |  .  |
| • Varied Journal Prompts |  .  |
| • Varied Product Choice |  .  |
| • Varied Supplemental Materials |  .  |
| • Work Alone / Together |  .  |

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**Unit Specific Interdisciplinary Connections / Materials**With interdisciplinary instruction, the subject areas are woven together and explored through an overarching theme or concept. We use math to help us solve everyday problems in the kitchen, in the garden, and for many of us at our jobs.Brain research has shown that information in our brains is organized in schematic structures. These structures are made up of interconnected bits of information and serve as a framework for the knowledge we acquire. When a learner’s knowledge is connected, it is much more likely that they will apply the prior knowledge to a wide variety of new situations. They will acquire new information in a way that is more accessible and will be better able to relate it to previously acquired knowledge.Students learn about patterns in math, science, social studies, and even literature. Because of this, they are much more likely to “see” these patterns when they encounter new situations. Since patterns are not only studied in math, they are able to make the connection and gain the understanding that patterns can be found in many areas of their lives.  Interdisciplinary instruction allows students to understand the interconnectedness of the disciplines and makes learning more meaningful and relevant as fascinating connections are made across the subject areas. |

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| **Additional Materials** |

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| LPS Adopted Textbooks and Programs * Pearson EnVision Geometry
* Pearson Realize (Computer Based program supplementing Envision)

Khan AcademyEdmentum Exact Path |

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| **Unit 3** |
| Overview:In unit 3, the following topics will be covered: Coordinate Geometry and Circles.Time Period: **Fourth Marking Period**Length: 4 **Weeks** |

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| **STAGE 1 – Desired Results** |

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| **Educational Standards**The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:1. Make sense of problems and persevere in solving them.2. Reason abstractly and quantitatively.3. Construct viable arguments and critique the reasoning of others.4. Model with mathematics.5. Use appropriate tools strategically.6. Attend to precision.7. Look for and make use of structure.8. Look for and express regularity in repeated reasoning.

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| **Introduction- Geometry** |
| Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations. Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. |
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**Congruence**

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| MA.G-CO.A.1 | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.  |
| MA.G-CO.A.2 | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).  |
| MA.G-CO.A.3 | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.  |
| MA.G-CO.A.4 | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.  |
| MA.G-CO.A.5 | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.  |
| MA.G-CO.D.12 | Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).  |
| MA.G-CO.B | Understand congruence in terms of rigid motions  |
| MA.G-CO.D.13 | Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.  |
| MA.G-CO.B.6 | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.  |
| MA.G-CO.B.7 | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.  |
| MA.G-CO.B.8 | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.  |
| MA.G-CO.C | Prove geometric theorems  |
| MA.G-CO.C.9 | Prove theorems about lines and angles.  |
| MA.G-CO.C.10 | Prove theorems about triangles.  |
| MA.G-CO.C.11 | Prove theorems about parallelograms.  |
| MA.G-CO.D | Make geometric constructions  |
| MA.G-CO.A | Experiment with transformations in the plane  |

**Similarity, Right Triangles and Trigonometry**

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| MA.G-SRT.D.9 | Derive the formula 𝐴 = (1/2)𝑎𝑏 𝑠𝑖𝑛(𝐶) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.  |
| MA.G-SRT.D.10 | Prove the Laws of Sines and Cosines and use them to solve problems.  |
| MA.G-SRT.D.11 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).  |
| MA.G-SRT.A | Understand similarity in terms of similarity transformations  |
| MA.G-SRT.A.1 | Verify experimentally the properties of dilations given by a center and a scale factor:  |
| MA.G-SRT.A.1a | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.  |
| MA.G-SRT.A.1b | The dilation of a line segment is longer or shorter in the ratio given by the scale factor.  |
| MA.G-SRT.A.2 | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.  |
| MA.G-SRT.A.3 | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.  |
| MA.G-SRT.B | Prove theorems involving similarity  |
| MA.G-SRT.B.4 | Prove theorems about triangles.  |
| MA.G-SRT.B.5 | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.  |
| MA.G-SRT.C | Define trigonometric ratios and solve problems involving right triangles  |
| MA.G-SRT.C.6 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.  |
| MA.G-SRT.C.7 | Explain and use the relationship between the sine and cosine of complementary angles.  |
| MA.G-SRT.C.8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.  |
| MA.G-SRT.D | Apply trigonometry to general triangles  |

**Circles**

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| MA.G-C.A | Understand and apply theorems about circles  |
| MA.G-C.A.1 | Prove that all circles are similar.  |
| MA.G-C.A.2 | Identify and describe relationships among inscribed angles, radii, and chords.  |
| MA.G-C.A.3 | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.  |
| MA.G-C.A.4 | Construct a tangent line from a point outside a given circle to the circle.  |
| MA.G-C.B | Find arc lengths and areas of sectors of circles  |
| MA.G-C.B.5 | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.  |

**Expressing Geometric Properties with Equations**

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| MA.G-GPE.B.4 | Use coordinates to prove simple geometric theorems algebraically.  |
| MA.G-GPE.B.5 | Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).  |
| MA.G-GPE.B.6 | Find the point on a directed line segment between two given points that partitions the segment in a given ratio.  |
| MA.G-GPE.B.7 | Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.  |
| MA.G-GPE.A | Translate between the geometric description and the equation for a conic section  |
| MA.G-GPE.A.1 | Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.  |
| MA.G-GPE.A.2 | Derive the equation of a parabola given a focus and directrix.  |
| MA.G-GPE.A.3 | Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.  |
| MA.G-GPE.B | Use coordinates to prove simple geometric theorems algebraically  |

**Geometric Measurement and Dimension**

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| MA.G-GMD.B.4 | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.  |
| MA.G-GMD.A | Explain volume formulas and use them to solve problems  |
| MA.G-GMD.A.1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.  |
| MA.G-GMD.A.2 | Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures.  |
| MA.G-GMD.A.3 | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.  |
| MA.G-GMD.B | Visualize relationships between two-dimensional and three-dimensional objects  |

**Modeling with Geometry**

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| MA.G-MG.A | Apply geometric concepts in modeling situations  |
| MA.G-MG.A.1 | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).  |
| MA.G-MG.A.2 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).  |
| MA.G-MG.A.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).  |

**Career Ready Practices**CRP2.   Apply appropriate academic and technical skills.CRP4.   Communicate clearly and effectively and with reason.CRP6.   Demonstrate creativity and innovation.CRP8.   Utilize critical thinking to make sense of problems and persevere in solving them.CRP11.   Use technology to enhance productivity.CRP12.   Work productively in teams while using cultural global competence.**Essential Questions…** * How are arc length and sector area related to circumference and area of a circle?
* How is a tangent line related to the radius of a circle at the point of tangency?
* How are chords related to their central angles and intercepted arcs?
* How is the measure of an inscribed angle related to its intercepted arc?
* How are the measures of angles, arcs, and segments formed by intersecting secant lines related?

**Enduring Understanding…*** Arcs are classified as minor arcs or major arcs depending on whether they are smaller or larger than a semicircle.
* A line that is tangent to a circle intersects the circle at exactly one point and is perpendicular to the radius to that point.
* Many real-world situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly.
* In a circle or congruent circles, two chords are congruent if the corresponding central angles are congruent arcs.
* In a circle, the measure of an inscribed angle is one-half of the measure of its intercepted arc.
* When two secants intersect inside a circle, the measure of the angle formed is half the sum of the intercepted arcs.

**Students will know...*** Understand and apply theorems about circles.
* Find arc lengths and areas of sectors of circles.
* Translate between the geometric description and the equations of a conic section.
* Prove geometric theorems.
* Use coordinates to prove simple geometric theorems algebraically.
* Apply geometric concepts in modeling situations

**Students will be able to...*** Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circ le is perpendicular to the tangent where the radius intersects the circle
* Generate proofs that demonstrate that all circles are similar.
* Prove the properties of angles for a quadrilateral inscribed in a circle and construct inscribed and circumscribed circles of a triangle, and a tangent line to a circle from a point outside a circle, using geometric tools and geometric software.
* Use similarity to show that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality.
* Derive the formula for the area of a circular sector, the equation of a circle (given the center and radius using the Pythagorean Theorem), and the equation of a parabola (given the focus and the directrix)
* Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems. (e.g. find the equation of a line parallel or perpendicular to a given line that passes through a given point.)
* Construct formal proofs using theorems, postulates, and definitions involving parallelograms
* Use the coordinate system to generate simple geometric proofs algebraically and to compute perimeters and areas of geometric figures using the distance formula.
* Use geometric shapes, measures, and properties to model multi-dimensional concepts in the context of real-world applications.
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| **STAGE 2 – Evidence of Learning** |

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| **Formative Assessment Suggestions**

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| • 3- Minute Pause |
| • A-B-C Summaries |
| • Analogy Prompt |
| • Choral Response |
| • Debriefing |
| • Exit Card / Ticket |
| • Hand Signals |
| • Idea Spinner |
| • Index Card Summaries |
| • Inside-Outside Circle Discussion (Fishbowl) |
| • Journal Entry |
| • Misconception Check |
| • Observation |
| • One Minute Essay |
| • One Word Summary |
| • Portfolio Check |
| • Questions & Answers |
| • Quiz |
| • Self-Assessment |
| • Student Conference |
| • Think-Pair-Share |
| • Web or Concept Map |

**Authentic Assessment Suggestions**Through the following authentic assessments, students will develop traits regarding thinking and reasoning, settings, mathematical tools and attitudes and dispositions:* Performance Assessments
* Short Investigations
* Open Ended Response Questions
* Portfolios
* Self-Assessments

**Benchmark Assessments**Edementum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3 – Learning Plan** |

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| **Instructional Map**

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| **Modifications/Differentiation of Instruction** |
| **Differentiation Strategies for Special Education Students*** Remove unnecessary material, words, etc., that can distract from the content
* Use of off-grade level materials
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Time allowed
* Level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Varied homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Ability to work at their own pace
* Present ideas using auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment
* Differentiated checklists and rubrics, if available and appropriate

**Differentiation Strategies for Gifted and Talented Students*** Increase the level of complexity
* Decrease scaffolding
* Variety of finished products
* Allow for greater independence
* Learning stations, interest groups
* Varied texts and supplementary materials
* Use of technology
* Flexibility in assignments
* Varied questioning strategies
* Encourage research
* Strategy and flexible groups based on formative assessment or student choice
* Acceleration within a unit of study
* Exposure to more advanced or complex concepts, abstractions, and materials
* Encourage students to move through content areas at their own pace
* After mastery of a unit, provide students with more advanced learning activities, not more of the same activity
* Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas

 **Differentiated Strategies for ELL Students*** Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials, including visuals
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Allow students to work at their own pace
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Role play
* Provide graphic organizers, highlighted materials
* Strategy and flexible groups based on formative assessment

**Differentiation Strategies for At Risk Students*** Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessments

**504 Plans**Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:* walk, breathe, eat, or sleep
* communicate, see, hear, or speak
* read, concentrate, think, or learn
* stand, bend, lift, or work

Examples of accommodations in 504 plans include:* preferential seating
* extended time on tests and assignments
* reduced homework or classwork
* verbal, visual, or technology aids
* modified textbooks or audio-video materials
* behavior management support
* adjusted class schedules or grading
* verbal testing
* excused lateness, absence, or missed classwork
* pre-approved nurse's office visits and accompaniment to visits
* occupational or physical therapy
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| **Modification Strategies** |
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| • Extended Time |  .  |
| • Frequent Breaks |  .  |
| • Highlighted Text |  .  |
| • Interactive Notebook |  .  |
| • Modified Test |  .  |
| • Oral Directions |  .  |
| • Peer Tutoring |  .  |
| • Preferential Seating |  .  |
| • Re-Direct |  .  |
| • Repeated Drill / Practice |  .  |
| • Shortened Assignments |  .  |
| • Teacher Notes |  .  |
| • Tutorials |  .  |
| • Use of Additional Reference Material |  .  |
| • Use of Audio Resources |  .  |

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| **High Preparation Differentiation** |
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| • Alternative Assessments |  .  |
| • Choice Boards |  .  |
| • Games and Tournaments |  .  |
| • Group Investigations |  .  |
| • Guided Reading |  .  |
| • Independent Research / Project |  .  |
| • Interest Groups |  .  |
| • Learning Contracts |  .  |
| • Leveled Rubrics |  .  |
| • Literature Circles |  .  |
| • Menu Assignments |  .  |
| • Multiple Intelligence Options |  .  |
| • Multiple Texts |  .  |
| • Personal Agendas |  .  |
| • Project Based Learning (PBL) |  .  |
| • Stations / Centers |  .  |
| • Think-Tac-Toe |  .  |
| • Tiered Activities / Assignments |  .  |
| • Varying Graphic Organizers |  .  |

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| **Low Preparation Differentiation** |
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| • Choice of Book / Activity |  .  |
| • Cubing Activities |  .  |
| • Exploration by Interest (using interest inventories) |  .  |
| • Flexible Grouping |  .  |
| • Goal Setting With Student |  .  |
| • Homework Options |  .  |
| • Jigsaw |  .  |
| • Mini Workshops to Extend Skills |  .  |
| • Mini Workshops to Re-teach |  .  |
| • Open-ended Activities |  .  |
| • Think-Pair-Share by Interest |  .  |
| • Think-Pair-Share by Learning Style |  .  |
| • Think-Pair-Share by Learning Style |  .  |
| • Think-Pair-Share by Readiness |  .  |
| • Use of Collaboration |  .  |
| • Use of Reading Buddies |  .  |
| • Varied Journal Prompts |  .  |
| • Varied Product Choice |  .  |
| • Varied Supplemental Materials |  .  |
| • Work Alone / Together |  .  |

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**Unit Specific Interdisciplinary Connections / Materials**With interdisciplinary instruction, the subject areas are woven together and explored through an overarching theme or concept. We use math to help us solve everyday problems in the kitchen, in the garden, and for many of us at our jobs.Brain research has shown that information in our brains is organized in schematic structures. These structures are made up of interconnected bits of information and serve as a framework for the knowledge we acquire. When a learner’s knowledge is connected, it is much more likely that they will apply the prior knowledge to a wide variety of new situations. They will acquire new information in a way that is more accessible and will be better able to relate it to previously acquired knowledge.Students learn about patterns in math, science, social studies, and even literature. Because of this, they are much more likely to “see” these patterns when they encounter new situations. Since patterns are not only studied in math, they are able to make the connection and gain the understanding that patterns can be found in many areas of their lives.  Interdisciplinary instruction allows students to understand the interconnectedness of the disciplines and makes learning more meaningful and relevant as fascinating connections are made across the subject areas.In Topic 6, students will learn about how much a rhinoceros beetle and the world’s largest cargo can each lift. Their task will be to design their own Quadrilateral Lift for this STEM Task. |

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| **Additional Materials** |

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| LPS Adopted Textbooks and Programs * Pearson EnVision Geometry
* Pearson Realize (Computer Based program supplementing Envision)

Khan AcademyEdmentum Exact Path |

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| **Unit 4** |
| **Overview:**In unit 4, the following topics will be covered: Two and Three Dimensional ModelsTime Period: **Fourth Marking Period**Length: 8 **Weeks** |

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| **STAGE 1 – Desired Results** |

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| **Educational Standards**The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:1. Make sense of problems and persevere in solving them.2. Reason abstractly and quantitatively.3. Construct viable arguments and critique the reasoning of others.4. Model with mathematics.5. Use appropriate tools strategically.6. Attend to precision.7. Look for and make use of structure.8. Look for and express regularity in repeated reasoning.

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| **Introduction- Geometry** |
| Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations. Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. |
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**Congruence**

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| MA.G-CO.A.1 | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.  |
| MA.G-CO.A.2 | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).  |
| MA.G-CO.A.3 | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.  |
| MA.G-CO.A.4 | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.  |
| MA.G-CO.A.5 | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.  |
| MA.G-CO.D.12 | Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).  |
| MA.G-CO.B | Understand congruence in terms of rigid motions  |
| MA.G-CO.D.13 | Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.  |
| MA.G-CO.B.6 | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.  |
| MA.G-CO.B.7 | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.  |
| MA.G-CO.B.8 | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.  |
| MA.G-CO.C | Prove geometric theorems  |
| MA.G-CO.C.9 | Prove theorems about lines and angles.  |
| MA.G-CO.C.10 | Prove theorems about triangles.  |
| MA.G-CO.C.11 | Prove theorems about parallelograms.  |
| MA.G-CO.D | Make geometric constructions  |
| MA.G-CO.A | Experiment with transformations in the plane  |

**Similarity, Right Triangles and Trigonometry**

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| MA.G-SRT.D.9 | Derive the formula 𝐴 = (1/2)𝑎𝑏 𝑠𝑖𝑛(𝐶) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.  |
| MA.G-SRT.D.10 | Prove the Laws of Sines and Cosines and use them to solve problems.  |
| MA.G-SRT.D.11 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).  |
| MA.G-SRT.A | Understand similarity in terms of similarity transformations  |
| MA.G-SRT.A.1 | Verify experimentally the properties of dilations given by a center and a scale factor:  |
| MA.G-SRT.A.1a | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.  |
| MA.G-SRT.A.1b | The dilation of a line segment is longer or shorter in the ratio given by the scale factor.  |
| MA.G-SRT.A.2 | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.  |
| MA.G-SRT.A.3 | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.  |
| MA.G-SRT.B | Prove theorems involving similarity  |
| MA.G-SRT.B.4 | Prove theorems about triangles.  |
| MA.G-SRT.B.5 | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.  |
| MA.G-SRT.C | Define trigonometric ratios and solve problems involving right triangles  |
| MA.G-SRT.C.6 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.  |
| MA.G-SRT.C.7 | Explain and use the relationship between the sine and cosine of complementary angles.  |
| MA.G-SRT.C.8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.  |
| MA.G-SRT.D | Apply trigonometry to general triangles  |

**Circles**

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| MA.G-C.A | Understand and apply theorems about circles  |
| MA.G-C.A.1 | Prove that all circles are similar.  |
| MA.G-C.A.2 | Identify and describe relationships among inscribed angles, radii, and chords.  |
| MA.G-C.A.3 | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.  |
| MA.G-C.A.4 | Construct a tangent line from a point outside a given circle to the circle.  |
| MA.G-C.B | Find arc lengths and areas of sectors of circles  |
| MA.G-C.B.5 | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.  |

**Expressing Geometric Properties with Equations**

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| MA.G-GPE.B.4 | Use coordinates to prove simple geometric theorems algebraically.  |
| MA.G-GPE.B.5 | Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).  |
| MA.G-GPE.B.6 | Find the point on a directed line segment between two given points that partitions the segment in a given ratio.  |
| MA.G-GPE.B.7 | Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.  |
| MA.G-GPE.A | Translate between the geometric description and the equation for a conic section  |
| MA.G-GPE.A.1 | Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.  |
| MA.G-GPE.A.2 | Derive the equation of a parabola given a focus and directrix.  |
| MA.G-GPE.A.3 | Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.  |
| MA.G-GPE.B | Use coordinates to prove simple geometric theorems algebraically  |

**Geometric Measurement and Dimension**

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| MA.G-GMD.B.4 | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.  |
| MA.G-GMD.A | Explain volume formulas and use them to solve problems  |
| MA.G-GMD.A.1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.  |
| MA.G-GMD.A.2 | Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures.  |
| MA.G-GMD.A.3 | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.  |
| MA.G-GMD.B | Visualize relationships between two-dimensional and three-dimensional objects  |

**Modeling with Geometry**

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| MA.G-MG.A | Apply geometric concepts in modeling situations  |
| MA.G-MG.A.1 | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).  |
| MA.G-MG.A.2 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).  |
| MA.G-MG.A.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).  |

**Career Ready Practices**CRP2.   Apply appropriate academic and technical skills.CRP4.   Communicate clearly and effectively and with reason.CRP6.   Demonstrate creativity and innovation.CRP8.   Utilize critical thinking to make sense of problems and persevere in solving them.CRP11.   Use technology to enhance productivity.CRP12.   Work productively in teams while using cultural global competence.**Essential Questions…** * How are three-dimensional figures and polygons related?
* How does the volume of a prism or cylinder relate to a cross section parallel to its base?
* How are the formulas for volume of a pyramid and volume of a cone alike?
* How does the volume of a sphere relate to the volumes of other solids?

**Enduring Understanding…*** The sum of the number of faces and vertices of a polyhedron equals two more than the number of edges.
* Many real-world situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly.
* Cones and pyramids with the same height and the same area at every cross section have equal volume.
* Apply Cavalieri’s Principle to show that the volume of a hemisphere is equal to the volume of a cylinder of equal diameter and height minus the volume of a cone of equal diameter and height.

**Students will know...*** Explain volume formulas and use them to solve problems.
* Visualize the relation between two-dimensional and three-dimensional objects.
* Apply geometric concepts in modeling situations. Focus on situations well modeled by trigonometric ratios for acute angles.
* Apply geometric concepts in modeling situations.

**Students will be able to...*** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri’s principle, and informal limit arguments.
* Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
* Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
* Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
* Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
* Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
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| **STAGE 2 – Evidence of Learning** |

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| **Formative Assessment Suggestions**

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| • 3- Minute Pause |
| • A-B-C Summaries |
| • Analogy Prompt |
| • Choral Response |
| • Debriefing |
| • Exit Card / Ticket |
| • Hand Signals |
| • Idea Spinner |
| • Index Card Summaries |
| • Inside-Outside Circle Discussion (Fishbowl) |
| • Journal Entry |
| • Misconception Check |
| • Observation |
| • One Minute Essay |
| • One Word Summary |
| • Portfolio Check |
| • Questions & Answers |
| • Quiz |
| • Self-Assessment |
| • Student Conference |
| • Think-Pair-Share |
| • Web or Concept Map |

**Authentic Assessment Suggestions**Through the following authentic assessments, students will develop traits regarding thinking and reasoning, settings, mathematical tools and attitudes and dispositions:* Performance Assessments
* Short Investigations
* Open Ended Response Questions
* Portfolios
* Self-Assessments

**Benchmark Assessments**Edementum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3 – Learning Plan** |

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| **Instructional Map**

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| **Modifications/Differentiation of Instruction** |
| **Differentiation Strategies for Special Education Students*** Remove unnecessary material, words, etc., that can distract from the content
* Use of off-grade level materials
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Time allowed
* Level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Varied homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Ability to work at their own pace
* Present ideas using auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment
* Differentiated checklists and rubrics, if available and appropriate

**Differentiation Strategies for Gifted and Talented Students*** Increase the level of complexity
* Decrease scaffolding
* Variety of finished products
* Allow for greater independence
* Learning stations, interest groups
* Varied texts and supplementary materials
* Use of technology
* Flexibility in assignments
* Varied questioning strategies
* Encourage research
* Strategy and flexible groups based on formative assessment or student choice
* Acceleration within a unit of study
* Exposure to more advanced or complex concepts, abstractions, and materials
* Encourage students to move through content areas at their own pace
* After mastery of a unit, provide students with more advanced learning activities, not more of the same activity
* Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas.

**Differentiated Strategies for ELL Students*** Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials, including visuals
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Allow students to work at their own pace
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Role play
* Provide graphic organizers, highlighted materials
* Strategy and flexible groups based on formative assessment

**Differentiation Strategies for At Risk Students*** Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessments

**504 Plans**Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:* walk, breathe, eat, or sleep
* communicate, see, hear, or speak
* read, concentrate, think, or learn
* stand, bend, lift, or work

Examples of accommodations in 504 plans include:* preferential seating
* extended time on tests and assignments
* reduced homework or classwork
* verbal, visual, or technology aids
* modified textbooks or audio-video materials
* behavior management support
* adjusted class schedules or grading
* verbal testing
* excused lateness, absence, or missed classwork
* pre-approved nurse's office visits and accompaniment to visits
* occupational or physical therapy
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| **Modification Strategies** |
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| • Extended Time |  .  |
| • Frequent Breaks |  .  |
| • Highlighted Text |  .  |
| • Interactive Notebook |  .  |
| • Modified Test |  .  |
| • Oral Directions |  .  |
| • Peer Tutoring |  .  |
| • Preferential Seating |  .  |
| • Re-Direct |  .  |
| • Repeated Drill / Practice |  .  |
| • Shortened Assignments |  .  |
| • Teacher Notes |  .  |
| • Tutorials |  .  |
| • Use of Additional Reference Material |  .  |
| • Use of Audio Resources |  .  |

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| **High Preparation Differentiation** |
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| • Alternative Assessments |  .  |
| • Choice Boards |  .  |
| • Games and Tournaments |  .  |
| • Group Investigations |  .  |
| • Guided Reading |  .  |
| • Independent Research / Project |  .  |
| • Interest Groups |  .  |
| • Learning Contracts |  .  |
| • Leveled Rubrics |  .  |
| • Literature Circles |  .  |
| • Menu Assignments |  .  |
| • Multiple Intelligence Options |  .  |
| • Multiple Texts |  .  |
| • Personal Agendas |  .  |
| • Project Based Learning (PBL) |  .  |
| • Stations / Centers |  .  |
| • Think-Tac-Toe |  .  |
| • Tiered Activities / Assignments |  .  |
| • Varying Graphic Organizers |  .  |

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| **Low Preparation Differentiation** |
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| • Choice of Book / Activity |  .  |
| • Cubing Activities |  .  |
| • Exploration by Interest (using interest inventories) |  .  |
| • Flexible Grouping |  .  |
| • Goal Setting With Student |  .  |
| • Homework Options |  .  |
| • Jigsaw |  .  |
| • Mini Workshops to Extend Skills |  .  |
| • Mini Workshops to Re-teach |  .  |
| • Open-ended Activities |  .  |
| • Think-Pair-Share by Interest |  .  |
| • Think-Pair-Share by Learning Style |  .  |
| • Think-Pair-Share by Learning Style |  .  |
| • Think-Pair-Share by Readiness |  .  |
| • Use of Collaboration |  .  |
| • Use of Reading Buddies |  .  |
| • Varied Journal Prompts |  .  |
| • Varied Product Choice |  .  |
| • Varied Supplemental Materials |  .  |
| • Work Alone / Together |  .  |

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**Unit Specific Interdisciplinary Connections / Materials**With interdisciplinary instruction, the subject areas are woven together and explored through an overarching theme or concept. We use math to help us solve everyday problems in the kitchen, in the garden, and for many of us at our jobs.Brain research has shown that information in our brains is organized in schematic structures. These structures are made up of interconnected bits of information and serve as a framework for the knowledge we acquire. When a learner’s knowledge is connected, it is much more likely that they will apply the prior knowledge to a wide variety of new situations. They will acquire new information in a way that is more accessible and will be better able to relate it to previously acquired knowledge.Students learn about patterns in math, science, social studies, and even literature. Because of this, they are much more likely to “see” these patterns when they encounter new situations. Since patterns are not only studied in math, they are able to make the connection and gain the understanding that patterns can be found in many areas of their lives.  Interdisciplinary instruction allows students to understand the interconnectedness of the disciplines and makes learning more meaningful and relevant as fascinating connections are made across the subject areas.In Topic 7, the STEM Task will have students take a look into the first 3-dimensional printer. Students will use similarity to scale up the dimensions of a part of a rocket engine, then describe and draw steps for the production of the part. |

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| **Additional Materials** |

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| LPS Adopted Textbooks and Programs * Pearson EnVision Geometry
* Pearson Realize (Computer Based program supplementing Envision)

Khan AcademyEdmentum Exact Path  |

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| **Unit 5** |
| **Overview:**In unit 5, the following topics will be covered: ProbabilityTime Period: **Fourth Marking Period (if time permits)**Length: **3** **Weeks** |

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| **STAGE 1 – Desired Results** |

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| **Educational Standards**The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:1. Make sense of problems and persevere in solving them.2. Reason abstractly and quantitatively.3. Construct viable arguments and critique the reasoning of others.4. Model with mathematics.5. Use appropriate tools strategically.6. Attend to precision.7. Look for and make use of structure.8. Look for and express regularity in repeated reasoning.

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| **Introduction- Geometry** |
| Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations. Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion. |
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**Congruence**

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| MA.G-CO.A.1 | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.  |
| MA.G-CO.A.2 | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).  |
| MA.G-CO.A.3 | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.  |
| MA.G-CO.A.4 | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.  |
| MA.G-CO.A.5 | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.  |
| MA.G-CO.D.12 | Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).  |
| MA.G-CO.B | Understand congruence in terms of rigid motions  |
| MA.G-CO.D.13 | Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.  |
| MA.G-CO.B.6 | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.  |
| MA.G-CO.B.7 | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.  |
| MA.G-CO.B.8 | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.  |
| MA.G-CO.C | Prove geometric theorems  |
| MA.G-CO.C.9 | Prove theorems about lines and angles.  |
| MA.G-CO.C.10 | Prove theorems about triangles.  |
| MA.G-CO.C.11 | Prove theorems about parallelograms.  |
| MA.G-CO.D | Make geometric constructions  |
| MA.G-CO.A | Experiment with transformations in the plane  |

**Similarity, Right Triangles and Trigonometry**

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| MA.G-SRT.D.9 | Derive the formula 𝐴 = (1/2)𝑎𝑏 𝑠𝑖𝑛(𝐶) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.  |
| MA.G-SRT.D.10 | Prove the Laws of Sines and Cosines and use them to solve problems.  |
| MA.G-SRT.D.11 | Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).  |
| MA.G-SRT.A | Understand similarity in terms of similarity transformations  |
| MA.G-SRT.A.1 | Verify experimentally the properties of dilations given by a center and a scale factor:  |
| MA.G-SRT.A.1a | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.  |
| MA.G-SRT.A.1b | The dilation of a line segment is longer or shorter in the ratio given by the scale factor.  |
| MA.G-SRT.A.2 | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.  |
| MA.G-SRT.A.3 | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.  |
| MA.G-SRT.B | Prove theorems involving similarity  |
| MA.G-SRT.B.4 | Prove theorems about triangles.  |
| MA.G-SRT.B.5 | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.  |
| MA.G-SRT.C | Define trigonometric ratios and solve problems involving right triangles  |
| MA.G-SRT.C.6 | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.  |
| MA.G-SRT.C.7 | Explain and use the relationship between the sine and cosine of complementary angles.  |
| MA.G-SRT.C.8 | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.  |
| MA.G-SRT.D | Apply trigonometry to general triangles  |

**Circles**

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| MA.G-C.A | Understand and apply theorems about circles  |
| MA.G-C.A.1 | Prove that all circles are similar.  |
| MA.G-C.A.2 | Identify and describe relationships among inscribed angles, radii, and chords.  |
| MA.G-C.A.3 | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.  |
| MA.G-C.A.4 | Construct a tangent line from a point outside a given circle to the circle.  |
| MA.G-C.B | Find arc lengths and areas of sectors of circles  |
| MA.G-C.B.5 | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.  |

**Expressing Geometric Properties with Equations**

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| MA.G-GPE.B.4 | Use coordinates to prove simple geometric theorems algebraically.  |
| MA.G-GPE.B.5 | Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).  |
| MA.G-GPE.B.6 | Find the point on a directed line segment between two given points that partitions the segment in a given ratio.  |
| MA.G-GPE.B.7 | Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.  |
| MA.G-GPE.A | Translate between the geometric description and the equation for a conic section  |
| MA.G-GPE.A.1 | Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.  |
| MA.G-GPE.A.2 | Derive the equation of a parabola given a focus and directrix.  |
| MA.G-GPE.A.3 | Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.  |
| MA.G-GPE.B | Use coordinates to prove simple geometric theorems algebraically  |

**Geometric Measurement and Dimension**

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| MA.G-GMD.B.4 | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.  |
| MA.G-GMD.A | Explain volume formulas and use them to solve problems  |
| MA.G-GMD.A.1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.  |
| MA.G-GMD.A.2 | Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures.  |
| MA.G-GMD.A.3 | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.  |
| MA.G-GMD.B | Visualize relationships between two-dimensional and three-dimensional objects  |

**Modeling with Geometry**

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| MA.G-MG.A | Apply geometric concepts in modeling situations  |
| MA.G-MG.A.1 | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).  |
| MA.G-MG.A.2 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).  |
| MA.G-MG.A.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).  |

**Modeling**

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| Designing the layout of the stalls in a school fair so as to raise as much money as possible.  |
| Analyzing stopping distance for a car.  |
| Modeling savings account balance, bacterial colony growth, or investment growth.  |
| Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.  |
| Analyzing risk in situations such as extreme sports, pandemics, and terrorism.  |
| Relating population statistics to individual predictions.  |
| Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).  |
| Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.  |
| A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.  |
| In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.  |
| Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.  |
| Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.  |

**Functions**

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| Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.  |
| In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car’s speed in miles per hour, v; the rule T(v) = 100/v expresses this relationship algebraically and defines a function whose name is T.  |
| The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.  |
| A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, “I’ll give you a state, you give me the capital city;” by an algebraic expression like f(x) = a + bx; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function’s properties.  |
| Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.  |
| A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.  |
| Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.  |

**Interpreting Functions**

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| MA.F-IF.C.8b | Use the properties of exponents to interpret expressions for exponential functions.  |
| MA.F-IF.A | Understand the concept of a function and use function notation  |
| MA.F-IF.A.1 | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If 𝑓 is a function and 𝑥 is an element of its domain, then 𝑓(𝑥) denotes the output of 𝑓 corresponding to the input 𝑥. The graph of 𝑓 is the graph of the equation𝑦 = 𝑓(𝑥).  |
| MA.F-IF.A.2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.  |
| MA.F-IF.C.9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).  |
| MA.F-IF.A.3 | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.  |
| MA.F-IF.B | Interpret functions that arise in applications in terms of the context  |
| MA.F-IF.B.4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.  |
| MA.F-IF.B.5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.  |
| MA.F-IF.B.6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.  |
| MA.F-IF.C | Analyze functions using different representations  |
| MA.F-IF.C.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.  |
| MA.F-IF.C.7a | Graph linear and quadratic functions and show intercepts, maxima, and minima.  |
| MA.F-IF.C.7b | Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.  |
| MA.F-IF.C.7c | Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.  |
| MA.F-IF.C.7d | Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.  |
| MA.F-IF.C.7e | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.  |
| MA.F-IF.C.8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.  |
| MA.F-IF.C.8a | Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.  |

**Building Functions**

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| MA.F-BF.B.4b | Verify by composition that one function is the inverse of another.  |
| MA.F-BF.B.4c | Read values of an inverse function from a graph or a table, given that the function has an inverse.  |
| MA.F-BF.B.4d | Produce an invertible function from a non-invertible function by restricting the domain.  |
| MA.F-BF.B.5 | Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.  |
| MA.F-BF.A.1 | Write a function that describes a relationship between two quantities.  |
| MA.F-BF.A.1a | Determine an explicit expression, a recursive process, or steps for calculation from a context.  |
| MA.F-BF.A.1b | Combine standard function types using arithmetic operations.  |
| MA.F-BF.A | Build a function that models a relationship between two quantities  |
| MA.F-BF.A.1c | Compose functions.  |
| MA.F-BF.A.2 | Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.  |
| MA.F-BF.B | Build new functions from existing functions  |
| MA.F-BF.B.3 | Identify the effect on the graph of replacing 𝑓(𝑥) by 𝑓(𝑥) + 𝑘, 𝑘𝑓(𝑥), 𝑓(𝑘𝑥), and 𝑓(𝑥 + 𝑘) for specific values of 𝑘 (both positive and negative); find the value of 𝑘 given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.  |
| MA.F-BF.B.4 | Find inverse functions.  |
| MA.F-BF.B.4a | Solve an equation of the form 𝑓(𝑥) = 𝑐 for a simple function 𝑓 that has an inverse and write an expression for the inverse.  |

**Linear, Quadratic, and Exponential Models**

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| MA.F-LE.A | Construct and compare linear and exponential models and solve problems  |
| MA.F-LE.A.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions.  |
| MA.F-LE.A.1a | Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.  |
| MA.F-LE.A.1b | Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.  |
| MA.F-LE.A.1c | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.  |
| MA.F-LE.A.2 | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).  |
| MA.F-LE.A.3 | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.  |
| MA.F-LE.A.4 | Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to 𝑎𝑏 to the 𝑐𝘵 power = 𝑑 where 𝑎, 𝑐, and 𝑑 are numbers and the base 𝑏 is 2, 10, or 𝑒; evaluate the logarithm using technology.  |
| MA.F-LE.B | Interpret expressions for functions in terms of the situation they model  |
| MA.F-LE.B.5 | Interpret the parameters in a linear or exponential function in terms of a context.  |

**Trigonometric Functions**

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| MA.F-TF.A.3 | Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π – 𝑥, π + 𝑥, and 2π – 𝑥 in terms of their values for 𝑥, where 𝑥 is any real number.  |
| MA.F-TF.A.4 | Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.  |
| MA.F-TF.B | Model periodic phenomena with trigonometric functions  |
| MA.F-TF.B.5 | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.  |
| MA.F-TF.B.6 | Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.  |
| MA.F-TF.B.7 | Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.  |
| MA.F-TF.C | Prove and apply trigonometric identities  |
| MA.F-TF.C.8 | Prove the Pythagorean identity 𝑠𝑖𝑛²(θ) + 𝑐𝑜𝑠²(θ) = 1 and use it to find 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) given 𝑠𝑖𝑛(θ), 𝑐𝑜𝑠(θ), or 𝑡𝑎𝑛(θ) and the quadrant of the angle.  |
| MA.F-TF.C.9 | Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.  |
| MA.F-TF.A | Extend the domain of trigonometric functions using the unit circle  |
| MA.F-TF.A.1 | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.  |

**Algebra**

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| An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.  |
| Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor.  |
| Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.  |
| A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.  |
| An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.  |
| The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.  |
| An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.  |
| Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x² – 2 = 0 are real numbers, not rational numbers; and the solutions of x² + 2 = 0 are complex numbers, not real numbers.  |
| Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.  |
| The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, 𝘈 = ((𝘣₁+𝘣₂)/2)𝘩, can be solved for 𝘩 using the same deductive process.  |
| Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.  |

**Seeing Structures in Expressions**

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| MA.A-SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. For example, see 𝑥⁴ – 𝑦⁴ as (𝑥²)² – (𝑦²)², thus recognizing it as a difference of squares that can be factored as (𝑥² – 𝑦²)(𝑥² + 𝑦²).  |
| MA.A-SSE.B | Write expressions in equivalent forms to solve problems  |
| MA.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.  |
| MA.A-SSE.B.3a | Factor a quadratic expression to reveal the zeros of the function it defines.  |
| MA.A-SSE.B.3b | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.  |
| MA.A-SSE.A | Interpret the structure of expressions  |
| MA.A-SSE.B.3c | Use the properties of exponents to transform expressions for exponential functions.  |
| MA.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context.  |
| MA.A-SSE.A.1a | Interpret parts of an expression, such as terms, factors, and coefficients.  |
| MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.  |
| MA.A-SSE.A.1b | Interpret complicated expressions by viewing one or more of their parts as a single entity.  |

**Arithmetic with Polynomials and Rational Functions**

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| MA.A-APR.C.4 | Prove polynomial identities and use them to describe numerical relationships.  |
| MA.A-APR.A | Perform arithmetic operations on polynomials  |
| MA.A-APR.A.1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.  |
| MA.A-APR.B | Understand the relationship between zeros and factors of polynomials  |
| MA.A-APR.B.2 | Know and apply the Remainder Theorem: For a polynomial 𝑝(𝑥) and a number 𝑎, the remainder on division by 𝑥 – 𝑎 is 𝑝(𝑎), so 𝑝(𝑎) = 0 if and only if (𝑥 – 𝑎) is a factor of 𝑝(𝑥).  |
| MA.A-APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.  |
| MA.A-APR.C.5 | Know and apply the Binomial Theorem for the expansion of (𝑥 + 𝑦)ⁿ in powers of 𝑥 and 𝑦 for a positive integer 𝑛, where 𝑥 and 𝑦 are any numbers, with coefficients determined for example by Pascal’s Triangle.  |
| MA.A-APR.D | Rewrite rational expressions  |
| MA.A-APR.D.6 | Rewrite simple rational expressions in different forms; write 𝑎(𝑥)/𝑏(𝑥) in the form 𝑞(𝑥) + 𝑟(𝑥)/𝑏(𝑥), where 𝑎(𝑥), 𝑏(𝑥), 𝑞(𝑥), and 𝑟(𝑥) are polynomials with the degree of 𝑟(𝑥) less than the degree of 𝑏(𝑥), using inspection, long division, or, for the more complicated examples, a computer algebra system.  |
| MA.A-APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.  |
| MA.A-APR.C | Use polynomial identities to solve problems  |

**Creating Equations**

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| MA.A-CED.A | Create equations that describe numbers or relationships  |
| MA.A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems.  |
| MA.A-CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  |
| MA.A-CED.A.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.  |
| MA.A-CED.A.4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.  |

**Reasoning with Equations and Inequalities**

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| MA.A-REI.B | Solve equations and inequalities in one variable  |
| MA.A-REI.B.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.  |
| MA.A-REI.B.4 | Solve quadratic equations in one variable.  |
| MA.A-REI.C | Solve systems of equations  |
| MA.A-REI.C.5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.  |
| MA.A-REI.C.6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.  |
| MA.A-REI.C.7 | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.  |
| MA.A-REI.C.8 | Represent a system of linear equations as a single matrix equation in a vector variable.  |
| MA.A-REI.C.9 | Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).  |
| MA.A-REI.D | Represent and solve equations and inequalities graphically  |
| MA.A-REI.B.4a | Use the method of completing the square to transform any quadratic equation in 𝑥 into an equation of the form (𝑥 – 𝑝)² = 𝑞 that has the same solutions. Derive the quadratic formula from this form.  |
| MA.A-REI.D.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  |
| MA.A-REI.B.4b | Solve quadratic equations by inspection (e.g., for 𝑥² = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as 𝑎 ± 𝑏𝑖 for real numbers 𝑎 and 𝑏.  |
| MA.A-REI.A | Understand solving equations as a process of reasoning and explain the reasoning  |
| MA.A-REI.D.12 | Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.  |
| MA.A-REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.  |
| MA.A-REI.D.11 | Explain why the 𝑥-coordinates of the points where the graphs of the equations 𝑦 = 𝑓(𝑥) and 𝑦 = 𝑔(𝑥) intersect are the solutions of the equation𝑓(𝑥) = 𝑔(𝑥); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where 𝑓(𝑥) and/or 𝑔(𝑥) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.  |
| MA.A-REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.  |

**Career Ready Practices**CRP2.   Apply appropriate academic and technical skills.CRP4.   Communicate clearly and effectively and with reason.CRP6.   Demonstrate creativity and innovation.CRP8.   Utilize critical thinking to make sense of problems and persevere in solving them.CRP11.   Use technology to enhance productivity.CRP12.   Work productively in teams while using cultural global competence.**Essential Questions…** * When are different types of graphs appropriate?
* What are outliers and how can they affect data?
* How can you make predictions?
* How can you find regression model?
* What is the difference between Z and T scores?

**Enduring Understanding…*** Analysis of different ways to organize and analyze data

**Students will know...*** Scatter plots.
* Stema and leaf plots
* Box and whisker
* Five number summary
* Lines of best fit
* Histograms
* Standard deviation
* Z scores
* T scores

**Students will be able to...*** Construct plots of data.
* Construct stem and leaf plots.
* Construct box and whisker plots.
* Find the five-number summary by hand and with a graphing calculator.
* Understand the implications of each statistic.
* Understand the effects of outliers.
* Predict the effect of a new piece of data on the five numbers.
* Construct lines of best fit for a scatterplot.
* Use technology to find the least squares regression model.
* Construct histograms for two variable data by hand, and using technology.
* Construct cumulative frequency tables
* Display ability to find the five number summary from cumulative table
* Find standard deviation for a data set by hand.
* Find standard deviation using software
* Understand and interpret the advantages and limitations of the standard deviation as a statistical measure.
* Understand the different approaches to finding the Standard deviation
* Understand the genesis of the bell curve.
* Use Gauss' formula to normalize a data set.
* Construct normal distributions from the data set.
* Generalize and infer from a given normal curve
* Use the formula to find Z scores as a measure of the distance from the mean of a given set of data.
* Use Z scores to calculate the probability of an event occurring.
* Use a standard normal table to find Z scores and their corresponding probabilities
* Use a table to find necessary probabilities
* Find probabilities from a. Normal distribution with a graphing calculator
* Graph normal distributions using a graphing calculator
* Find inverse normal probabilities from a table.
* Find normal probabilities using calculator
* To find appropriate models for population with unknown SD.
* Find t scores for a given data set.
* Generate a confidence interval for a promotion with a known standard.
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| **STAGE 2 – Evidence of Learning** |

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| **Formative Assessment Suggestions**

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| • 3- Minute Pause |
| • A-B-C Summaries |
| • Analogy Prompt |
| • Choral Response |
| • Debriefing |
| • Exit Card / Ticket |
| • Hand Signals |
| • Idea Spinner |
| • Index Card Summaries |
| • Inside-Outside Circle Discussion (Fishbowl) |
| • Journal Entry |
| • Misconception Check |
| • Observation |
| • One Minute Essay |
| • One Word Summary |
| • Portfolio Check |
| • Questions & Answers |
| • Quiz |
| • Self-Assessment |
| • Student Conference |
| • Think-Pair-Share |
| • Web or Concept Map |

**Authentic Assessment Suggestions**Through the following authentic assessments, students will develop traits regarding thinking and reasoning, settings, mathematical tools and attitudes and dispositions:* Performance Assessments
* Short Investigations
* Open Ended Response Questions
* Portfolios
* Self-Assessments

**Benchmark Assessments*** Edementum Exact Path (BOY, MOY, EOY)
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| **STAGE 3 – Learning Plan** |

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| **Instructional Map**

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| **Modifications/Differentiation of Instruction** |
| **Differentiation Strategies for Special Education Students*** Remove unnecessary material, words, etc., that can distract from the content
* Use of off-grade level materials
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Time allowed
* Level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Varied homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Ability to work at their own pace
* Present ideas using auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment
* Differentiated checklists and rubrics, if available and appropriate

**Differentiation Strategies for Gifted and Talented Students*** Increase the level of complexity
* Decrease scaffolding
* Variety of finished products
* Allow for greater independence
* Learning stations, interest groups
* Varied texts and supplementary materials
* Use of technology
* Flexibility in assignments
* Varied questioning strategies
* Encourage research
* Strategy and flexible groups based on formative assessment or student choice
* Acceleration within a unit of study
* Exposure to more advanced or complex concepts, abstractions, and materials
* Encourage students to move through content areas at their own pace
* After mastery of a unit, provide students with more advanced learning activities, not more of the same activity
* Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas.

**Differentiated Strategies for ELL Students*** Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials, including visuals
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Allow students to work at their own pace
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Role play
* Provide graphic organizers, highlighted materials
* Strategy and flexible groups based on formative assessment

**Differentiation Strategies for At Risk Students*** Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessments

**504 Plans**Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:* walk, breathe, eat, or sleep
* communicate, see, hear, or speak
* read, concentrate, think, or learn
* stand, bend, lift, or work

Examples of accommodations in 504 plans include:* preferential seating
* extended time on tests and assignments
* reduced homework or classwork
* verbal, visual, or technology aids
* modified textbooks or audio-video materials
* behavior management support
* adjusted class schedules or grading
* verbal testing
* excused lateness, absence, or missed classwork
* pre-approved nurse's office visits and accompaniment to visits
* occupational or physical therapy
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| **Modification Strategies** |
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| • Extended Time |  .  |
| • Frequent Breaks |  .  |
| • Highlighted Text |  .  |
| • Interactive Notebook |  .  |
| • Modified Test |  .  |
| • Oral Directions |  .  |
| • Peer Tutoring |  .  |
| • Preferential Seating |  .  |
| • Re-Direct |  .  |
| • Repeated Drill / Practice |  .  |
| • Shortened Assignments |  .  |
| • Teacher Notes |  .  |
| • Tutorials |  .  |
| • Use of Additional Reference Material |  .  |
| • Use of Audio Resources |  .  |

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| **High Preparation Differentiation** |
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| • Alternative Assessments |  .  |
| • Choice Boards |  .  |
| • Games and Tournaments |  .  |
| • Group Investigations |  .  |
| • Guided Reading |  .  |
| • Independent Research / Project |  .  |
| • Interest Groups |  .  |
| • Learning Contracts |  .  |
| • Leveled Rubrics |  .  |
| • Literature Circles |  .  |
| • Menu Assignments |  .  |
| • Multiple Intelligence Options |  .  |
| • Multiple Texts |  .  |
| • Personal Agendas |  .  |
| • Project Based Learning (PBL) |  .  |
| • Stations / Centers |  .  |
| • Think-Tac-Toe |  .  |
| • Tiered Activities / Assignments |  .  |
| • Varying Graphic Organizers |  .  |

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| **Low Preparation Differentiation** |
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| • Choice of Book / Activity |  .  |
| • Cubing Activities |  .  |
| • Exploration by Interest (using interest inventories) |  .  |
| • Flexible Grouping |  .  |
| • Goal Setting With Student |  .  |
| • Homework Options |  .  |
| • Jigsaw |  .  |
| • Mini Workshops to Extend Skills |  .  |
| • Mini Workshops to Re-teach |  .  |
| • Open-ended Activities |  .  |
| • Think-Pair-Share by Interest |  .  |
| • Think-Pair-Share by Learning Style |  .  |
| • Think-Pair-Share by Learning Style |  .  |
| • Think-Pair-Share by Readiness |  .  |
| • Use of Collaboration |  .  |
| • Use of Reading Buddies |  .  |
| • Varied Journal Prompts |  .  |
| • Varied Product Choice |  .  |
| • Varied Supplemental Materials |  .  |
| • Work Alone / Together |  .  |

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**Unit Specific Interdisciplinary Connections / Materials**With interdisciplinary instruction, the subject areas are woven together and explored through an overarching theme or concept. We use math to help us solve everyday problems in the kitchen, in the garden, and for many of us at our jobs.Brain research has shown that information in our brains is organized in schematic structures. These structures are made up of interconnected bits of information and serve as a framework for the knowledge we acquire. When a learner’s knowledge is connected, it is much more likely that they will apply the prior knowledge to a wide variety of new situations. They will acquire new information in a way that is more accessible and will be better able to relate it to previously acquired knowledge.Students learn about patterns in math, science, social studies, and even literature. Because of this, they are much more likely to “see” these patterns when they encounter new situations. Since patterns are not only studied in math, they are able to make the connection and gain the understanding that patterns can be found in many areas of their lives.  Interdisciplinary instruction allows students to understand the interconnectedness of the disciplines and makes learning more meaningful and relevant as fascinating connections are made across the subject areas.For the final unit, students will work on two STEM Tasks:1. Measure a Distance
	1. Trigonometry is a powerful tool for measuring lengths and distances indirectly. Students will use trigonometry and indirect measurement to find the height of an object that is too tall to measure directly.
2. Design a Solar Collector
	1. Giant solar power plants are not the only place to see parabolic trough collectors – you might find a water purifier made from a single 6ft x 4ft mirror in a neighbor’s back yard. Students will analyze parabolas and design a solar collector and for use in your school or community.
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| **Additional Materials** |

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| LPS Adopted Textbooks and Programs * Pearson EnVision Geometry
* Pearson Realize (Computer Based program supplementing Envision)

Khan AcademyEdmentum Exact Path  |

**Interdisciplinary Connections/Standards**

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**Science**

HS-ESS3-2 Evaluate competing design solutions for developing, managing, and utilizing energy and mineral resources based on cost-benefit ratios.

HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

HS-ESS1-4 Use mathematical or computational representations to predict the motion of orbiting objects in the solar system.

**Language Arts**

RL.9-10.1 Cite strong and thorough textual evidence and make relevant connections to support analysis of what the text says explicityas well as inferentially, including determining where the text leaves matters uncertain.

RI.9-10.7 Analyze various perspectives as presented in different mediums, determining which details are emphasized in each account.

RI.9-10.8 Describe and evaluate the argument and specific claims in a text, assessing whether the reasoning is valid and the evidence is relevant and sufficient; identity false statements and reasoning.

**Social Studies**

6.1.12.CivicsPD.1 Use multiple sources to analyze the factors that led to an increase in the political rights and participatiuon in government.

6.1.12.HistoryCA.2 Research multiple perspectives to explain the struggle to create an American identity.