**Subject**

**Algebra 2**

**Curriculum Guide**

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**LINDEN PUBLIC SCHOOLS**

**LINDEN, NEW JERSEY**

**DR. MARNIE HAZELTON**

 **SUPERINTENDENT**

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**DIRECTOR OF MATHEMATICS, VOCATIONAL, & TECHNICAL SUBJECTS**

**The Linden Board of Education adopted the Curriculum Guide on:**

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| **July 28, 2022**  |  | **Education Report #22** |
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| **Rationale** |

**EDUCATION EQUITY:** The Linden Public School District guarantees each student equal educational opportunity regardless of age, race, color, creed, religion, gender, language, affectional or sexual orientation, ancestry, national origin, marital or economic status. For Information, contact District Educational Equity Officer Kevin Thurston at **(**908) 486-2800 x 8307**.**

**NONDISCRIMATION:** The Linden Public School District does not discriminate against handicapped persons in admission or access to or treatment or employment in its programs, activities, and vocational opportunities. For information contact District Public 504 Officer Annabell Louis at (908) 486-2800 x 8025.

**Linden Public Schools Vision**

The Linden Public School District is committed to developing respect for diversity, excellence in education, and a commitment to service, in order to promote global citizenship and ensure personal success for all students

**Linden Public Schools Mission**

The mission of the Linden Public School District is to promote distinction through the infinite resource that is Linden’s diversity, combined with our profound commitment to instructional excellence, so that each and every student achieves their maximum potential in an engaging, inspiring, and challenging learning environment.

**Math Department Vision**

To equip students with the understanding and application of mathematical skills and processes to foster a drive for advanced mathematics and higher-level thinking.

**Math Department Mission Statement**

To develop a community of learners who construct and communicate meaning from the mathematical world around them. Students will experience mathematics that encourage them to think critically, discover and apply concepts to solve problems strategically. Students will be encouraged to solve equations with accuracy, efficiency, and flexibility. Furthermore, students will have a multitude of opportunities to apply mathematical tools and practice standards to solve real-world and multi-step problems.

**Math Department Goals**

* Provide opportunities for student to develop computation skills, conceptual understanding, and problem-solving skills
* Require students to explain, justify or prove their thinking through mathematical reasoning, modeling, and speaking

Course Description

This is a one-year course that reviews the skills from first-year Algebra and covers all the essential topics for a second- year course. Real numbers are examined with a focus on rational and irrational numbers. Applications involving logarithms and exponential equations are studied. Special emphasis is given to equation solving and its application to word problems.

Course Instructional Materials

* LPS Adopted Textbooks and Programs
	+ Pearson EnVision Geometry
	+ Pearson Realize (Computer Based program supplementing Envision)
* Khan Academy
* Edmentum Exact Path

Standards and NJDOE Mandates Guiding Instruction

* 1. New Jersey Student Learning Standards

 <https://www.state.nj.us/education/cccs/>

General Interdisciplinary Connections / Materials

Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, and liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other.

Diversity, Equity, and Inclusion

* Use students’ interests in conceptualized tasks
* Expose students to a diverse group of mathematicians
* Design assessments and assignments with a variety of response types
* Use systematic grading and participation methods
* Encourage students to embrace a growth mindset

**Algebra II/Algebra II Trig Honors Pacing Guide 2022-2023**

As part of the regular classroom activities and homework assignments be sure to include the following:

* Lesson objectives must include Essential Question to pinpoint learning outcomes
* Include “Digital Lesson Courseware” in your lessons
* MAP Testing will be completed three times throughout the school year; dates listed below
	+ 1. Pre – test scheduled below; Mid and Post Test dates pending
* Incorporate 4 Steps as defined in each lesson of the Teachers Edition:
	+ 1. Explore
		2. Understand and Apply
		3. Practice and Problem Solving
		4. Assess and Differentiate
* Be sure to utilize data results to drive instruction
* Daily DO NOW Activities
* Homework assignments must include a variety of levels from the Practice and Problem Solving Sections at the end of each lesson.
* Three benchmark exams and a final exam will be administered during exam week.

Please adhere to the following guide for Algebra II. The number of days per chapter is an estimate for your planning. When assigning exercises for homework, concentrate on the level appropriate for your students, tiering whenever possible according to the ability levels within the class.

**Throughout the year be sure to acknowledge the BIG IDEAS of each chapter and the Key Concepts of the sections with your students.**

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| **Highlighted Items for Trig Honors Only** |
| **Marking Period 1** **September 6, 2022 to November 15, 2022** |
| Topic 1: Quadratic Functions and Factoring | Estimated Time: 16 Days | Standards: HSF.IF.B.4, HSF.IF.B.6, HSF.IF.C.7, HSF.IF.B.5, HSF.BF.B.3, HSF.BF.B.3, HSF.IF.C.7.B, HSF.LE.A.2, HSF.IF.A.3, HSF.IF.A.3, HSF.BF.A.1,HSF.BF.A.1.A, HSF.BF.A.2, HSF.LE.A.2, HAS.CED.A.1, HAS.REI.D.11, HAS.CED.A.3, HAS.REI.C.6, HAS.CED.A.2, HAS.CED.A.3, HAS.REI.C.6, HAS.CED.A.3, HAS.REI.C.6 |
| Big Ideas1. Key Features of Functions
2. Transformations of Functions
3. Piecewise – Defined Functions
4. Arithmetic Sequences and Series
5. Solving Equations and Inequalities by Graphing
6. Linear Systems
7. Solving Linear Systems Using Matrices
 | **Required:*** Topic Opener (Page 3) – Intro to Math Modeling in 3 Acts
* Topic Readiness Assessment (Page 2G in TE)
* envision STEM Project (Page 4)
* Mathematical Modeling in 3 Acts: Current Events (Page 55)
* Topic Review (Page 65) & Topic Assessment (Multiple Choice/Short Response)
* Topic Performance Assessment (Open Ended)
 |
| Topic 2: Quadratic Functions and Equations | Estimated Time: 17 Days | Standards: HAS.CED.A.2, HSF.IF.B.4, HSF.BF.B.3, HAS.CED.A.2, HSF.IF.B.4, HSS.ID.B.6, HSS.ID.B.6.A, HAS.SSE.A.2, HAS.SSE.B.3.A, HAS.APR.B.3, HSN.CN.A.1, HSN.CN.A.2, HSN.CN.A.3, HSN.CN.C.7, HAS.REI.B.4, HAS.REI.B.4.A, HAS.REI.B.4.B, HSN.CN.C.7, HAS.REI.B.4, HAS.REI.B.4.A, HAS.REI.B.4.B, HAS.REI.C.7, HAS.REI.D.11 |
| Big Ideas1. Vertex Form of a Quadratic Function
2. Standard Form of a Quadratic Function
3. Factored Form of a Quadratic Function
4. Complex Numbers and Operations
5. Completing the Square
6. The Quadratic Formula
7. Linear – Quadratic Systems
 | **Required:*** Topic Opener (Page 71) – Intro to Math Modeling in 3 Acts
* Topic Readiness Assessment (Page 70G in TE)
* envision STEM Project (Page 72)
* Mathematical Modeling in 3 Acts: Swift Kick (Page 102)
* Topic Review (Page 124) & Topic Assessment (Multiple Choice/Short Response)
* Topic Performance Assessment (Open Ended)
 |
| **Marking period 2** **November 16, 2022 to January 31, 2023** |
| Topic 3: Polynomial Functions | Estimated Time: 17 Days | Standards: HSF.IF.B.4, HSF.IF.B.6, HSF.IF.C.7.C, HAS.APR.A.1HSF.IF.C.P, HSF.BF.A.1.B, HAS.SSE.A.2,HAS.APR.C.4, HAS.APR.C.5, HAS.SSE.A.2, HAS.APR.B.2, HAS.APR.D.6, HAS.SSE.A.2, HAS.APR.B.3, HSF.IF.C.7.C, HAS.SSE.A.2, HAS.APR.A.1, HAS.APR.B.3, HSN.CN.C.8, HSN.CN.C.9, HAS.APR.B.2.HSA.APR.B.3, HSF.BF.B.3 |
| Big Ideas1. Graphing Polynomial Functions
2. Adding, Subtracting, and Multiplying Polynomials
3. Polynomial Identities
4. Dividing Polynomials
5. Zeros of Polynomial Functions
6. Theorems About Roots of Polynomial Equations
7. Transformations of Polynomial Functions
 | **Required:*** Topic Opener (Page 129) – Intro to Math Modeling in 3 Acts
* Topic Readiness Assessment (Page 128G in TE)
* envision STEM Project (Page 130)
* Mathematical Modeling in 3 Acts: What are the Rules? (Page 170)
* Topic Review (Page 187) & Topic Assessment (Multiple Choice/Short Response)
* Topic Performance Assessment (Open Ended)
 |
| Topic 4: Rational Functions  | Estimated Time: 13 Days | Standards: HAS.CED.A.2, HSF.IF.C.7.D, HSF.BF.B.3, HAS.APR.D.6, HAS.REI.D.11, HSF.IF.C.7.D, HAS.SSE.A.2, HAS.APR.D.6, HAS.APR.D.7, HAS.SSE.A.2, HAS.APR.D.7, HAS.CED.A.1, HAS.REI.A.1, HAS.REI.A.2  |
| Big Ideas1. Inverse Variation and the Reciprocal Function
2. Graphing Rational Functions
3. Multiplying and Dividing Rational Expressions
4. Adding and Subtracting Rational Expressions
5. Solving Rational Equations
 | **Required:*** Topic Opener (Page 191) – Intro to Math Modeling in 3 Acts
* Topic Readiness Assessment (Page 190G in TE)
* envision STEM Project (Page 192)
* Mathematical Modeling in 3 Acts: Real Cool Waters (Page 232)
* Topic Review (Page 233) & Topic Assessment (Multiple Choice/Short Response)
* Topic Performance Assessment (Open Ended)
 |
| Topic 5: Sequences and Series | Estimated Time: 12 Days | Standards: HSN.RN.A.1, HSN.RN.A.2, HAS.REI.A.1, HAS.SSE.A.1, HAS.SSE.A.2, HSF.IF.B.4, HSF.IF.C.7.B, HSF.BF.B.3, HSA.CED.A.4, HAS.REI.A.1, HAS.REI.A.2, HAS.REI.A.2, HAS.CED.A.1, HAS.CED.A.4, HSF.BF.B.4, HSF.BF.B.4.A, HSF.BF.B.4.C, HSF.BF.B.4.D |
| Big Ideas1. *n*th Roots, Radicals, and Rational Exponents
2. Properties of Exponents and Radicals
3. Graphing Radical Functions
4. Solving Radical Equations
5. Functions Operations
6. Inverse Relations and Functions
 | **Required:*** Topic Opener (Page 237) – Intro to Math Modeling in 3 Acts
* Topic Readiness Assessment (Page 236G in TE)
* envision STEM Project (Page 238)
* Mathematical Modeling in 3 Acts: The Snack Shack (Page 272)
* Topic Review (Page 290) & Topic Assessment (Multiple Choice/Short Response)
* Topic Performance Assessment (Open Ended)
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| **Marking Period 3****February 1, 2023 to April 5, 2023** |
| Topic 6: Exponential and Logarithmic Functions | 17 Days2/5 – 2/28 | Standards: HSF.IF.C.7.E, HSF.LE.A.2, HSF.LE.B.5, HSA.SSE.B.3.C, HSF.IF.C.8.B, HSF.LE.B.5, HSS.ID.B.6.A, HSF.LE.B.5, HSS.ID.B.6.A, HSF.BF.B.4.A, HSF.BF.B.5, HSF.LE.A.4, HSF.BF.B.3, HSF.BF.B.5, HSF.IF.B.6, HSF.IF.C.7.E, HSF.IF.C.9, HSA.SSE.A.2, HSA.REI.A.1, HSF.LE.A.4, HSA.SSE.A.2, HSA.CED.A.1, HSA.REI.A.1, HSF.LE.A.4, HSF.BF.A.2, HSF.LE.A.2, HSA.SSE.B.4, HSF.IF.A.3 |
| Big Ideas1. Key Features of Exponential Functions
2. Exponential Models
3. Logarithms
4. Logarithmic Functions
5. Properties of Logarithms
6. Exponential and Logarithmic Equations
 | **Required:*** Topic Opener (Page 295) – Intro to Math Modeling in 3 Acts
* Topic Readiness Assessment (Page 294G in TE)
* envision STEM Project (Page 296)
* Mathematical Modeling in 3 Acts: The Crazy Conditioning (Page 313)
* Topic Review (Page 349) & Topic Assessment (Multiple Choice/Short Response)
* Topic Performance Assessment (Open Ended)
 |
| Topic 7: Trigonometric Functions  | Estimated Time: 15 Days | Standards: HSG.SRT.C.6, HSG.SRT.C.7, HSG.SRT.C.8, HSF.TF.A.1, HSF.TF.A.2, HSF.TF.A.3, HSF.TF.C.8, HSF.IF.B.6, HSF.IF.C.9, HSF.BF.B.3, HSF.IF.B.4, HSF.BF.B.3, HSF.TF.B.5, HSF.IF.B.4, HSF.BF.B.3, HSF.TF.B.5 |
| Big Ideas1. Trigonometric Functions and Acute Angles
2. Angles and the Unit Circle
3. Trigonometric Functions and Real Numbers
4. Graphing Sine and Cosine Functions
5. Graphing Other Trigonometric Functions
6. Translating Trigonometric Functions
 | **Required:*** Topic Opener (Page 355) – Intro to Math Modeling in 3 Acts
* Topic Readiness Assessment (Page 354G in TE)
* envision STEM Project (Page 356)
* Mathematical Modeling in 3 Acts: What Note was That? (Page 392)
* Topic Review (Page 408) & Topic Assessment (Multiple Choice/Short Response)
* Topic Performance Assessment (Open Ended)
 |
| **Marking Period 4****April 17, 2023 to June 22, 2023** |
| Topic 12: Probability  | Estimated Time: 13 Days | Standards: HSS.CP.A.1, HSS.CP.A.2, HSS.CP.A.5, HSS.CP.B.7, HSS.CP.A.3, HSS.CP.A.4, HSS.CP.B.6, HSS.CP.B.8, HSG.SRT.C.8, HSS.CP.B.9, HSS.MD.A.1, HSS.MD.A.3, HSS.MD.A.4, HSS.CP.B.9, HSS.MD.A.2, HSS.MD.B.5, HSS.MD.B.5.A, HSS.MD.B.5.B, HSS.CP.B.9, HSS.MD.B.6, HSS.MD.B.67 |
| Big Ideas1. Probability Events
2. Conditional Probability
3. Permutations and Combinations
4. Probability Distributions
5. Expected Value
6. Probability and Decision Making
 | **Required:*** Topic Opener (Page 603) – Intro to Math Modeling in 3 Acts
* Topic Readiness Assessment (Page 602G in TE)
* envision STEM Project (Page 604)
* Mathematical Modeling in 3 Acts: Place Your Guess (Page 620)
* Topic Review (Page 650) & Topic Assessment (Multiple Choice/Short Response)
* Topic Performance Assessment (Open Ended)
 |
| Topic 9: Conic Sections | Estimated Time: 11 Days | Standards: HSG.GPE.A.2, HSA.SSE.B.3, HSA.SSE.A.2, HSG.GPE.A.1, HSA.REI.C.7, HSA.SSE.B.3, HSG.GPE.A.1, HSG.SSE.A.2, HSA.SSE.B.3, HSG.GPE.A.3, HSA.SSE.A.2, HSA.SSE.B.3 |
| Big Ideas1. Parabolas
2. Circles
3. Ellipses
4. Hyperbolas
 | **Required:*** Topic Opener (Page 461) – Intro to Math Modeling in 3 Acts
* Topic Readiness Assessment (Page 460G in TE)
* envision STEM Project (Page 462)
* Mathematical Modeling in 3 Acts: Watering the Lawn (Page 479)
* Topic Review (Page 498) & Topic Assessment (Multiple Choice/Short Response)
* Topic Performance Assessment (Open Ended)
 |
| Topic 8: Trigonometric Equations and Identities | Estimated Time: 13 Days | Standards: HSF.TF.B.6, HSF.TF.B.7, HSF.BF.B.4.D, HSG.SRT.D.10, HSG.SRT.D.11, HSF.TF.A.3, HSF.TF.A.4, HSF.TF.C.9, HSN.CN.A.3, HSN.CN.B.5, HSN.CN.B.6, HSN.CN.B.4, HSF.TF.C.9, HSN.CN.B.5 |
| Big Ideas1. Solving Trigonometric Equations Using Inverses
2. Law of Sines and Law of Cosines
3. Trigonometric Identities
4. The Complex Plane
5. Polar Form of Complex Numbers
 | **Required:*** Topic Opener (Page 413) – Intro to Math Modeling in 3 Acts
* Topic Readiness Assessment (Page 412G in TE)
* envision STEM Project (Page 414)
* Mathematical Modeling in 3 Acts: Ramp Up Your Design (Page 423)
* Topic Review (Page 457) & Topic Assessment (Multiple Choice/Short Response)
* Topic Performance Assessment (Open Ended)
 |
| Topic 11: Data Analysis and Statistics | Estimated Time: 15 Days(If Time Permits) | Standards: HSN. Q.A.2, HSS.IC.A.1, HSS.IC.A.1, HSS.IC.B.3, HSS.IC.B.6, HSS.ID.A.1, HSS.ID.A.2, HSS.IC.A.2, HSS.ID.A.4, HSS.IC.B.6, HSS.IC.B.4, HSS.IC.A.1, HSS.IC.A.2, HSS.IC.B.6, HSS.IC.A.1, HSS.IC.B.5, HSS.IC.B.6, HSS.IC.A.1, HSS.IC.A.2, HSS.IC.B.4  |
| Big Ideas1. Statistical Questions and Variables
2. Statistical Studies and Sampling Methods
3. Data Distributions
4. Normal Distributions
5. Margin of Error
6. Introduction to Hypothesis Testing
 | **Required:*** Topic Opener (Page 549) – Intro to Math Modeling in 3 Acts
* Topic Readiness Assessment (Page 548G in TE)
* envision STEM Project (Page 550)
* Mathematical Modeling in 3 Acts: Jar of Coins (Page 597)
* Topic Review (Page 598) & Topic Assessment (Multiple Choice/Short Response)
* Topic Performance Assessment (Open Ended)
 |
| Topic 10: Matrices | Estimated Time: 13 Days(If Time Permits) | Standards: HSN.VM.C.6, HSN.VM.C.7, HSN.VM.C.8, HSN.VM.C.12, HSN.VM.C.8, HSN.VM.C.9, HSN.VM.C.10, HSN.VM.A.1, HSN.VM.B.4, HSN.VM.B.5, HSN.VM.C.11, HSA.REI.C.9, HSN.VM.C.10, HSN.VM.C.12, HSA.REI.C.8, HSA.REI.C.9, HSA.CED.A.3, HSN.VM.C.6, HSA.REI.C.8 |
| Big Ideas1. Operations with Matrices
2. Matrix Multiplication
3. Vectors
4. Inverses and Determinants

Inverse Matrices and Systems of Equations | **Required:*** Topic Opener (Page 501) – Intro to Math Modeling in 3 Acts
* Topic Readiness Assessment (Page 500G in TE)
* envision STEM Project (Page 502)
* Mathematical Modeling in 3 Acts: The Big Burger (Page 544)
* Topic Review (Page 545) & Topic Assessment (Multiple Choice/Short Response)

Topic Performance Assessment (Open Ended) |

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| **Career Ready Practices** |
| CRP2.   Apply appropriate academic and technical skills.CRP4.   Communicate clearly and effectively and with reason.CRP6.   Demonstrate creativity and innovation.CRP8.   Utilize critical thinking to make sense of problems and persevere in solving them.CRP11.   Use technology to enhance productivity.CRP12.   Work productively in teams while using cultural global competence. |

**Unit 1 - Polynomials**

Content Area: **Mathematics**
Course(s): **Algebra II, Algebra II and Trigonometry**
Time Period: **First Marking Period**
Length: **8 Week**
Status: **Published**

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| **Unit Overview** |
|  Unit 1 standards will focus on the similarities of arithmetic with rational numbers and the arithmetic with rational expressions. |

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| **STAGE 1- DESIRED RESULTS** |
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| **Educational Standards** |
| The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:1. Make sense of problems and persevre in solving them.2. Reason abstractly and quantitatively.3. Construct viable arguments and critique the reasoning of others.4. Model with mathematics.5. Use appropriate tools strategically.6. Attend to precision.7. Look for and make use of structure.8. Look for and express regularity in repeated reasoning. |

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| **New Jersey Student Learning Standards- Mathematics** |
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| **Introduction- Algebra** |
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| **Expressions** |
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|  | An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.  |
|  | Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor.  |
|  | Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.  |
|  | A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.  |

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| **Equalities and Inequalities** |
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|  | An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.  |
|  | The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.  |
|  | An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.  |
|  | Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x² – 2 = 0 are real numbers, not rational numbers; and the solutions of x² + 2 = 0 are complex numbers, not real numbers.  |
|  | The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, 𝘈 = ((𝘣₁+𝘣₂)/2)𝘩, can be solved for 𝘩 using the same deductive process.  |
|  | Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.  |

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| **Connections to Functions and Modeling** |
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|  | Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.  |

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| **Seeing Structure in Expressions** |
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| MA.A-SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. For example, see 𝑥⁴ – 𝑦⁴ as (𝑥²)² – (𝑦²)², thus recognizing it as a difference of squares that can be factored as (𝑥² – 𝑦²)(𝑥² + 𝑦²).  |
| MA.A-SSE.B | Write expressions in equivalent forms to solve problems  |
| MA.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.  |
| MA.A-SSE.B.3a | Factor a quadratic expression to reveal the zeros of the function it defines.  |
| MA.A-SSE.B.3b | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.  |
| MA.A-SSE.A | Interpret the structure of expressions  |
| MA.A-SSE.B.3c | Use the properties of exponents to transform expressions for exponential functions.  |
| MA.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context.  |
| MA.A-SSE.A.1a | Interpret parts of an expression, such as terms, factors, and coefficients.  |
| MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.  |
| MA.A-SSE.A.1b | Interpret complicated expressions by viewing one or more of their parts as a single entity.  |

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| **Arithmetic with Polynomials and Rational Expressions** |
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| MA.A-APR.C.4 | Prove polynomial identities and use them to describe numerical relationships.  |
| MA.A-APR.A | Perform arithmetic operations on polynomials  |
| MA.A-APR.A.1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.  |
| MA.A-APR.B | Understand the relationship between zeros and factors of polynomials  |
| MA.A-APR.B.2 | Know and apply the Remainder Theorem: For a polynomial 𝑝(𝑥) and a number 𝑎, the remainder on division by 𝑥 – 𝑎 is 𝑝(𝑎), so 𝑝(𝑎) = 0 if and only if (𝑥 – 𝑎) is a factor of 𝑝(𝑥).  |
| MA.A-APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.  |
| MA.A-APR.C.5 | Know and apply the Binomial Theorem for the expansion of (𝑥 + 𝑦)ⁿ in powers of 𝑥 and 𝑦 for a positive integer 𝑛, where 𝑥 and 𝑦 are any numbers, with coefficients determined for example by Pascal’s Triangle.  |
| MA.A-APR.D | Rewrite rational expressions  |
| MA.A-APR.D.6 | Rewrite simple rational expressions in different forms; write 𝑎(𝑥)/𝑏(𝑥) in the form 𝑞(𝑥) + 𝑟(𝑥)/𝑏(𝑥), where 𝑎(𝑥), 𝑏(𝑥), 𝑞(𝑥), and 𝑟(𝑥) are polynomials with the degree of 𝑟(𝑥) less than the degree of 𝑏(𝑥), using inspection, long division, or, for the more complicated examples, a computer algebra system.  |
| MA.A-APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.  |
| MA.A-APR.C | Use polynomial identities to solve problems  |

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| **Creating Equations** |
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| MA.A-CED.A | Create equations that describe numbers or relationships  |
| MA.A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems.  |
| MA.A-CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  |
| MA.A-CED.A.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.  |
| MA.A-CED.A.4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.  |

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| **Reasoning with Equations and Inequalities** |
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| MA.A-REI.B | Solve equations and inequalities in one variable  |
| MA.A-REI.B.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.  |
| MA.A-REI.B.4 | Solve quadratic equations in one variable.  |
| MA.A-REI.C | Solve systems of equations  |
| MA.A-REI.C.5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.  |
| MA.A-REI.C.6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.  |
| MA.A-REI.C.7 | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.  |
| MA.A-REI.C.8 | Represent a system of linear equations as a single matrix equation in a vector variable.  |
| MA.A-REI.C.9 | Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).  |
| MA.A-REI.D | Represent and solve equations and inequalities graphically  |
| MA.A-REI.B.4a | Use the method of completing the square to transform any quadratic equation in 𝑥 into an equation of the form (𝑥 – 𝑝)² = 𝑞 that has the same solutions. Derive the quadratic formula from this form.  |
| MA.A-REI.D.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  |
| MA.A-REI.B.4b | Solve quadratic equations by inspection (e.g., for 𝑥² = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as 𝑎 ± 𝑏𝑖 for real numbers 𝑎 and 𝑏.  |
| MA.A-REI.A | Understand solving equations as a process of reasoning and explain the reasoning  |
| MA.A-REI.D.12 | Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.  |
| MA.A-REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.  |
| MA.A-REI.D.11 | Explain why the 𝑥-coordinates of the points where the graphs of the equations 𝑦 = 𝑓(𝑥) and 𝑦 = 𝑔(𝑥) intersect are the solutions of the equation𝑓(𝑥) = 𝑔(𝑥); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where 𝑓(𝑥) and/or 𝑔(𝑥) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.  |
| MA.A-REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.  |

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| **Essential Questions** |
| How might we classify polynomials? How can we use the polynomial operations of addition, subtraction, and multiplication in real life? How does finding greatest common factors in factoring polynomials? Why do we factor polynomials? How can we identify a difference of squares?  |

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| **Enduring Understanding** |
| Polynomials symbolize numeric relationships and can be manipulated in much the same way as real numbers.Polynomials functions are a class of curved relationships that fit many real life applications. |

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| **Students will know...** |
| 1. Perform arithmetic operations with complex numbers2. Use complex numbers in polynomial identities and equations3. Interpret the structure of expressions4. Write expressions in equivalent forms to solve problems5. Perform arithmetic operations on polynomials6. Understand the relationship between zeros and factors of polynomials. |

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| **Students will be able to...** |
| 1. Use properties of operations to add, subtract, and multiply complex numbers.2. Solve quadratic equations with real coefficients that have complex solutions.3. Interpret coefficients, terms, degree, powers (positive and negative), leading coefficients and monomials in polynomial and rational expressions in terms of context4. Restructure and perform arithmetic operations on polynomial/rational expressions and use them to solve problems (*i.e. calculate mortgage payments)*5. Use an appropriate factoring technique to factor expressions completely. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).*6. Explain the relationship between zeros and factors of polynomials and use zeros to construct a rough graph of the function defined by the polynomial |

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| **STAGE 2- EVIDENCE OF LEARNING** |
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| **Formative Assessment Suggestions** |
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| • 3- Minute Pause |  **.**  |
| • A-B-C Summaries |  **.**  |
| • Analogy Prompt |  **.**  |
| • Choral Response |  **.**  |
| • Debriefing |  **.**  |
| • Exit Card / Ticket |  **.**  |
| • Hand Signals |  **.**  |
| • Idea Spinner |  **.**  |
| • Index Card Summaries |  **.**  |
| • Inside-Outside Circle Discussion (Fishbowl) |  **.**  |
| • Journal Entry |  **.**  |
| • Misconception Check |  **.**  |
| • Observation |  **.**  |
| • One Minute Essay |  **.**  |
| • One Word Summary |  **.**  |
| • Portfolio Check |  **.**  |
| • Questions & Answers |  **.**  |
| • Quiz |  **.**  |
| • Self-Assessment |  **.**  |
| • Student Conference |  **.**  |
| • Think-Pair-Share |  **.**  |
| • Web or Concept Map |  **.**  |

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| **Authentic Assessments Suggestions** |
| Through the following authentic assessments, students will develop traits regarding thinking and reasoning, settings, mathematical tools and attitudes and dispositions: 1. Performance Assessments 2. Short Investigations 3. Open Ended Response Questions 4.  Portfolios 5.  Self-Assessments  |

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| **Benchmark Assessments** |
| Edmentum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3- LEARNING PLAN** |
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| **Instructional Map** |
| Pattern growth activity:Showing the change resulting from a linear or quadratic equation a group of images is presented that changes slightly from one image to another (either increasing or decreasing in size)Students create T-chart to represent the change and start to recognize that this change can also determine different types of functions. They also represent them graphically and as a function, and they begin making connections between them. * Collect class data, graph, and try to fit models or functions to the data
* Use software or graphing calculators to model transformations with functions to help students make connections between the various representations

• Use quadratics to help introduce polynomial identities. To help students perform arithmetic operations with complex numbers, have them treat i like they would any other variable once they understand what it means● Perform partner work to discuss similarities and differences in polynomials by reviewing the various properties and Euclidean Algorithm ● Use a sequence of diagrams to create a pattern of terms, then sum the areas to create a sequence. |

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| **Modifications/Differentiation of Instruction** |
| Differentiation Strategies for Special Education Students* Remove unnecessary material, words, etc., that can distract from the content
* Use of off-grade level materials
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Time allowed
* Level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Varied homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Ability to work at their own pace
* Present ideas using auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment
* Differentiated checklists and rubrics, if available and appropriate

Differentiation Strategies for Gifted and Talented Students* Increase the level of complexity
* Decrease scaffolding
* Variety of finished products
* Allow for greater independence
* Learning stations, interest groups
* Varied texts and supplementary materials
* Use of technology
* Flexibility in assignments
* Varied questioning strategies
* Encourage research
* Strategy and flexible groups based on formative assessment or student choice
* Acceleration within a unit of study
* Exposure to more advanced or complex concepts, abstractions, and materials
* Encourage students to move through content areas at their own pace
* After mastery of a unit, provide students with more advanced learning activities, not more of the same activity
* Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas

Differentiated Strategies for ELL Students* Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials, including visuals
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Allow students to work at their own pace
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Role play
* Provide graphic organizers, highlighted materials
* Strategy and flexible groups based on formative assessment

Differentiation Strategies for At Risk Students* Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment

**504 Plans**Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:* walk, breathe, eat, or sleep
* communicate, see, hear, or speak
* read, concentrate, think, or learn
* stand, bend, lift, or work

Examples of accommodations in 504 plans include:* preferential seating
* extended time on tests and assignments
* reduced homework or classwork
* verbal, visual, or technology aids
* modified textbooks or audio-video materials
* behavior management support
* adjusted class schedules or grading
* verbal testing
* excused lateness, absence, or missed classwork
* pre-approved nurse's office visits and accompaniment to visits
* occupational or physical therapy
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| **Modification Strategies** |
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| • Extended Time |  **.**  |
| • Frequent Breaks |  **.**  |
| • Highlighted Text |  **.**  |
| • Interactive Notebook |  **.**  |
| • Modified Test |  **.**  |
| • Oral Directions |  **.**  |
| • Peer Tutoring |  **.**  |
| • Preferential Seating |  **.**  |
| • Re-Direct |  **.**  |
| • Repeated Drill / Practice |  **.**  |
| • Shortened Assignments |  **.**  |
| • Teacher Notes |  **.**  |
| • Tutorials |  **.**  |
| • Use of Additional Reference Material |  **.**  |
| • Use of Audio Resources |  **.**  |

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| **High Preparation Differentiation** |
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| • Alternative Assessments |  **.**  |
| • Choice Boards |  **.**  |
| • Games and Tournaments |  **.**  |
| • Group Investigations |  **.**  |
| • Guided Reading |  **.**  |
| • Independent Research / Project |  **.**  |
| • Interest Groups |  **.**  |
| • Learning Contracts |  **.**  |
| • Leveled Rubrics |  **.**  |
| • Literature Circles |  **.**  |
| • Menu Assignments |  **.**  |
| • Multiple Intelligence Options |  **.**  |
| • Multiple Texts |  **.**  |
| • Personal Agendas |  **.**  |
| • Project Based Learning (PBL) |  **.**  |
| • Stations / Centers |  **.**  |
| • Think-Tac-Toe |  **.**  |
| • Tiered Activities / Assignments |  **.**  |
| • Varying Graphic Organizers |  **.**  |

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| **Low Preparation Differentiation** |
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| • Choice of Book / Activity |  **.**  |
| • Cubing Activities |  **.**  |
| • Exploration by Interest (using interest inventories) |  **.**  |
| • Flexible Grouping |  **.**  |
| • Goal Setting With Student |  **.**  |
| • Homework Options |  **.**  |
| • Jigsaw |  **.**  |
| • Mini Workshops to Extend Skills |  **.**  |
| • Mini Workshops to Re-teach |  **.**  |
| • Open-ended Activities |  **.**  |
| • Think-Pair-Share by Interest |  **.**  |
| • Think-Pair-Share by Learning Style |  **.**  |
| • Think-Pair-Share by Learning Style |  **.**  |
| • Think-Pair-Share by Readiness |  **.**  |
| • Use of Collaboration |  **.**  |
| • Use of Reading Buddies |  **.**  |
| • Varied Journal Prompts |  **.**  |
| • Varied Product Choice |  **.**  |
| • Varied Supplemental Materials |  **.**  |
| • Work Alone / Together |  **.**  |

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| **Horizontal Integration- Interdisciplinary Connections** |
| Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other. |

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| **Vertical Integration- Discipline Mapping** |
| The standards in this unit were introduced in Algebra 1 with a focus on numerical expressions and polynomial expressions in one variable, factoring quadratic and cubic polynomials, graphing quadratic functions and their transformations and interpret key features of graphs.  |

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| **Additional Materials** |
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| LPS Adopted Textbooks and Programs * Pearson EnVision Algebra II
* Pearson Realize (Computer Based program supplementing Envision)

Khan AcademyEdmentum Exact Path**Unit 2 - Polynomial, Rational and Radical Relationships**Content Area: **Mathematics**Course(s): **Algebra II, Algebra II and Trigonometry**Time Period: **First Marking Period** Length: **8 Week**Status: **Published**

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| **Unit Overview** |
| Unit 2 will extend student’s algebra knowledge of linear and exponential functions to include polynomial, rational, radical, and absolute value functions.  |

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| **STAGE 1- DESIRED RESULTS** |
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| **Educational Standards** |
| The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:1. Make sense of problems and persevre in solving them.2. Reason abstractly and quantitatively.3. Construct viable arguments and critique the reasoning of others.4. Model with mathematics.5. Use appropriate tools strategically.6. Attend to precision.7. Look for and make use of structure.8. Look for and express regularity in repeated reasoning. |

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| **New Jersey Student Learning Standards- Mathematics** |
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| **Introduction- Algebra** |
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| **Expressions** |
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|  | An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.  |
|  | Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor.  |
|  | Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.  |
|  | A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.  |

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| **Equalities and Inequalities** |
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|  | An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.  |
|  | The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.  |
|  | An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.  |
|  | Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x² – 2 = 0 are real numbers, not rational numbers; and the solutions of x² + 2 = 0 are complex numbers, not real numbers.  |
|  | The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, 𝘈 = ((𝘣₁+𝘣₂)/2)𝘩, can be solved for 𝘩 using the same deductive process.  |
|  | Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.  |

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| **Connections to Functions and Modeling** |
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|  | Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.  |

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| **Seeing Structure in Expressions** |
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| MA.A-SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. For example, see 𝑥⁴ – 𝑦⁴ as (𝑥²)² – (𝑦²)², thus recognizing it as a difference of squares that can be factored as (𝑥² – 𝑦²)(𝑥² + 𝑦²).  |
| MA.A-SSE.B | Write expressions in equivalent forms to solve problems  |
| MA.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.  |
| MA.A-SSE.B.3a | Factor a quadratic expression to reveal the zeros of the function it defines.  |
| MA.A-SSE.B.3b | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.  |
| MA.A-SSE.A | Interpret the structure of expressions  |
| MA.A-SSE.B.3c | Use the properties of exponents to transform expressions for exponential functions.  |
| MA.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context.  |
| MA.A-SSE.A.1a | Interpret parts of an expression, such as terms, factors, and coefficients.  |
| MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.  |
| MA.A-SSE.A.1b | Interpret complicated expressions by viewing one or more of their parts as a single entity.  |

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| **Arithmetic with Polynomials and Rational Expressions** |
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| MA.A-APR.C.4 | Prove polynomial identities and use them to describe numerical relationships.  |
| MA.A-APR.A | Perform arithmetic operations on polynomials  |
| MA.A-APR.A.1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.  |
| MA.A-APR.B | Understand the relationship between zeros and factors of polynomials  |
| MA.A-APR.B.2 | Know and apply the Remainder Theorem: For a polynomial 𝑝(𝑥) and a number 𝑎, the remainder on division by 𝑥 – 𝑎 is 𝑝(𝑎), so 𝑝(𝑎) = 0 if and only if (𝑥 – 𝑎) is a factor of 𝑝(𝑥).  |
| MA.A-APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.  |
| MA.A-APR.C.5 | Know and apply the Binomial Theorem for the expansion of (𝑥 + 𝑦)ⁿ in powers of 𝑥 and 𝑦 for a positive integer 𝑛, where 𝑥 and 𝑦 are any numbers, with coefficients determined for example by Pascal’s Triangle.  |
| MA.A-APR.D | Rewrite rational expressions  |
| MA.A-APR.D.6 | Rewrite simple rational expressions in different forms; write 𝑎(𝑥)/𝑏(𝑥) in the form 𝑞(𝑥) + 𝑟(𝑥)/𝑏(𝑥), where 𝑎(𝑥), 𝑏(𝑥), 𝑞(𝑥), and 𝑟(𝑥) are polynomials with the degree of 𝑟(𝑥) less than the degree of 𝑏(𝑥), using inspection, long division, or, for the more complicated examples, a computer algebra system.  |
| MA.A-APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.  |
| MA.A-APR.C | Use polynomial identities to solve problems  |

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| **Creating Equations** |
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| MA.A-CED.A | Create equations that describe numbers or relationships  |
| MA.A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems.  |
| MA.A-CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  |
| MA.A-CED.A.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.  |
| MA.A-CED.A.4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.  |

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| **Reasoning with Equations and Inequalities** |
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| MA.A-REI.B | Solve equations and inequalities in one variable  |
| MA.A-REI.B.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.  |
| MA.A-REI.B.4 | Solve quadratic equations in one variable.  |
| MA.A-REI.C | Solve systems of equations  |
| MA.A-REI.C.5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.  |
| MA.A-REI.C.6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.  |
| MA.A-REI.C.7 | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.  |
| MA.A-REI.C.8 | Represent a system of linear equations as a single matrix equation in a vector variable.  |
| MA.A-REI.C.9 | Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).  |
| MA.A-REI.D | Represent and solve equations and inequalities graphically  |
| MA.A-REI.B.4a | Use the method of completing the square to transform any quadratic equation in 𝑥 into an equation of the form (𝑥 – 𝑝)² = 𝑞 that has the same solutions. Derive the quadratic formula from this form.  |
| MA.A-REI.D.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  |
| MA.A-REI.B.4b | Solve quadratic equations by inspection (e.g., for 𝑥² = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as 𝑎 ± 𝑏𝑖 for real numbers 𝑎 and 𝑏.  |
| MA.A-REI.A | Understand solving equations as a process of reasoning and explain the reasoning  |
| MA.A-REI.D.12 | Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.  |
| MA.A-REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.  |
| MA.A-REI.D.11 | Explain why the 𝑥-coordinates of the points where the graphs of the equations 𝑦 = 𝑓(𝑥) and 𝑦 = 𝑔(𝑥) intersect are the solutions of the equation𝑓(𝑥) = 𝑔(𝑥); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where 𝑓(𝑥) and/or 𝑔(𝑥) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.  |
| MA.A-REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.  |

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| **Essential Questions** |
| How is factoring used to simplify a rational expression? How are reciprocals used to divide rational expressions? What makes a number a rational number? What are your essential rules when performing basic operations and simplification of square roots?  |

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| **Enduring Understanding** |
| Rational functions are not polynomials because their domains are not continuous. The behavior of rational functions provides an introduction to the mathematical concept of a limit. |

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| **Students will know...** |
| 1. Use polynomial identities to solve problems2. Rewrite rational expressions3. Create equations that describe numbers or relationships4. Understand solving equations as a process of reasoning and explain the reasoning5. Represent and solve equations and inequalities graphically6. Analyze functions using different representations |

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| **Students will be able to...** |
| 1. Rewrite simple rational expressions in different forms using inspection, long division, or, for the more complicated examples, a computer algebra system2. Create equations and inequalities in one variable and use them to solve problems using polynomial identities where appropriate (involving polynomial, rational and radical expressions).3. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.4. Solve simple rational and radical equations in one variable, explain the reasoning, and give examples showing how extraneous solutions may arise.5. Find approximate solutions for the intersections of functions and explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x) involving linear, polynomial, rational, absolute value, and exponential functions6. Analyze functions using different representations, graph functions expressed symbolically (by hand in simple cases and using technology for more complicated cases) and show key features of the graph. |

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| **STAGE 2- EVIDENCE OF LEARNING** |
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| **Formative Assessment Suggestions** |
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| • 3- Minute Pause |  **.**  |
| • A-B-C Summaries |  **.**  |
| • Analogy Prompt |  **.**  |
| • Choral Response |  **.**  |
| • Debriefing |  **.**  |
| • Exit Card / Ticket |  **.**  |
| • Hand Signals |  **.**  |
| • Idea Spinner |  **.**  |
| • Index Card Summaries |  **.**  |
| • Inside-Outside Circle Discussion (Fishbowl) |  **.**  |
| • Journal Entry |  **.**  |
| • Misconception Check |  **.**  |
| • Observation |  **.**  |
| • One Minute Essay |  **.**  |
| • One Word Summary |  **.**  |
| • Portfolio Check |  **.**  |
| • Questions & Answers |  **.**  |
| • Quiz |  **.**  |
| • Self-Assessment |  **.**  |
| • Student Conference |  **.**  |
| • Think-Pair-Share |  **.**  |
| • Web or Concept Map |  **.**  |

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| **Authentic Assessments Suggestions** |
| Through the following authentic assessments, students will develop traits regarding thinking and reasoning, settings, mathematical tools and attitudes and dispositions: 1. Performance Assessments 2. Short Investigations 3. Open Ended Response Questions 4.  Portfolios 5.  Self-Assessments |

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| **Benchmark Assessments** |
| Edmentum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3- LEARNING PLAN** |
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| **Instructional Map** |
| * Use real life examples to illustrate when to apply the various functions
* Use multiple representations (graphs, tables, equations) to help students make connections to other functions
* Use software or calculators to illustrate radical and rational functions to help students make connections between them as well as other functions
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| **Modifications/Differentiation of Instruction** |
| Differentiation Strategies for Special Education Students* Remove unnecessary material, words, etc., that can distract from the content
* Use of off-grade level materials
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Time allowed
* Level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Varied homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Ability to work at their own pace
* Present ideas using auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment
* Differentiated checklists and rubrics, if available and appropriate.

 Differentiation Strategies for Gifted and Talented Students* Increase the level of complexity
* Decrease scaffolding
* Variety of finished products
* Allow for greater independence
* Learning stations, interest groups
* Varied texts and supplementary materials
* Use of technology
* Flexibility in assignments
* Varied questioning strategies
* Encourage research
* Strategy and flexible groups based on formative assessment or student choice
* Acceleration within a unit of study
* Exposure to more advanced or complex concepts, abstractions, and materials
* Encourage students to move through content areas at their own pace
* After mastery of a unit, provide students with more advanced learning activities, not more of the same activity
* Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas

 Differentiated Strategies for ELL Students* Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials, including visuals
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Allow students to work at their own pace
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Role play
* Provide graphic organizers, highlighted materials
* Strategy and flexible groups based on formative assessment

 Differentiation Strategies for At Risk Students* Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment

 **504 Plans**Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:* walk, breathe, eat, or sleep
* communicate, see, hear, or speak
* read, concentrate, think, or learn
* stand, bend, lift, or work

 Examples of accommodations in 504 plans include:* preferential seating
* extended time on tests and assignments
* reduced homework or classwork
* verbal, visual, or technology aids
* modified textbooks or audio-video materials
* behavior management support
* adjusted class schedules or grading
* verbal testing
* excused lateness, absence, or missed classwork
* pre-approved nurse's office visits and accompaniment to visits
* occupational or physical therapy
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| **Modification Strategies** |
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| • Extended Time |  **.**  |
| • Frequent Breaks |  **.**  |
| • Highlighted Text |  **.**  |
| • Interactive Notebook |  **.**  |
| • Modified Test |  **.**  |
| • Oral Directions |  **.**  |
| • Peer Tutoring |  **.**  |
| • Preferential Seating |  **.**  |
| • Re-Direct |  **.**  |
| • Repeated Drill / Practice |  **.**  |
| • Shortened Assignments |  **.**  |
| • Teacher Notes |  **.**  |
| • Tutorials |  **.**  |
| • Use of Additional Reference Material |  **.**  |
| • Use of Audio Resources |  **.**  |

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| **High Preparation Differentiation** |
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| • Alternative Assessments |  **.**  |
| • Choice Boards |  **.**  |
| • Games and Tournaments |  **.**  |
| • Group Investigations |  **.**  |
| • Guided Reading |  **.**  |
| • Independent Research / Project |  **.**  |
| • Interest Groups |  **.**  |
| • Learning Contracts |  **.**  |
| • Leveled Rubrics |  **.**  |
| • Literature Circles |  **.**  |
| • Menu Assignments |  **.**  |
| • Multiple Intelligence Options |  **.**  |
| • Multiple Texts |  **.**  |
| • Personal Agendas |  **.**  |
| • Project Based Learning (PBL) |  **.**  |
| • Stations / Centers |  **.**  |
| • Think-Tac-Toe |  **.**  |
| • Tiered Activities / Assignments |  **.**  |
| • Varying Graphic Organizers |  **.**  |

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| **Low Preparation Differentiation** |
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| • Choice of Book / Activity |  **.**  |
| • Cubing Activities |  **.**  |
| • Exploration by Interest (using interest inventories) |  **.**  |
| • Flexible Grouping |  **.**  |
| • Goal Setting With Student |  **.**  |
| • Homework Options |  **.**  |
| • Jigsaw |  **.**  |
| • Mini Workshops to Extend Skills |  **.**  |
| • Mini Workshops to Re-teach |  **.**  |
| • Open-ended Activities |  **.**  |
| • Think-Pair-Share by Interest |  **.**  |
| • Think-Pair-Share by Learning Style |  **.**  |
| • Think-Pair-Share by Learning Style |  **.**  |
| • Think-Pair-Share by Readiness |  **.**  |
| • Use of Collaboration |  **.**  |
| • Use of Reading Buddies |  **.**  |
| • Varied Journal Prompts |  **.**  |
| • Varied Product Choice |  **.**  |
| • Varied Supplemental Materials |  **.**  |
| • Work Alone / Together |  **.**  |

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| **Horizontal Integration- Interdisciplinary Connections** |
| Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other. |

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| **Vertical Integration- Discipline Mapping** |
| The standards in this unit were introduced in Algebra 1 with a focus on numerical expressions and polynomial expressions in one variable, factoring quadratic and cubic polynomials, graphing quadratic functions and their transformations and interpret key features of graphs.  |

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| **Additional Materials** |
| LPS Adopted Textbooks and Programs * Pearson EnVision Algebra II
* Pearson Realize (Computer Based program supplementing Envision)

Khan AcademyEdmentum Exact Path |

**Unit 3 - Modeling with Functions**Content Area: **Mathematics**Course(s): **Algebra II, Algebra II and Trigonometry**Time Period: **Second Marking period**Length: **8 Week** Status: **Published**

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| **Unit Overview** |
|  Unit 3 explores the effects of transformations on graphs of functions. The work of this unit will include identifying an appropriate model for a given situation. The standards require development of models more complex than those of previous courses. |

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| **STAGE 1- DESIRED RESULTS** |
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| **Educational Standards** |
| The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:1. Make sense of problems and persevre in solving them.2. Reason abstractly and quantitatively.3. Construct viable arguments and critique the reasoning of others.4. Model with mathematics.5. Use appropriate tools strategically.6. Attend to precision.7. Look for and make use of structure.8. Look for and express regularity in repeated reasoning. |

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| **New Jersey Student Learning Standards- Mathematics** |
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| **Introduction- Algebra** |
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| **Expressions** |
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|  | An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.  |
|  | Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor.  |
|  | Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.  |
|  | A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.  |

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| **Equalities and Inequalities** |
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|  | An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.  |
|  | The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.  |
|  | An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.  |
|  | Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x² – 2 = 0 are real numbers, not rational numbers; and the solutions of x² + 2 = 0 are complex numbers, not real numbers.  |
|  | The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, 𝘈 = ((𝘣₁+𝘣₂)/2)𝘩, can be solved for 𝘩 using the same deductive process.  |
|  | Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.  |

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| **Connections to Functions and Modeling** |
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|  | Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.  |

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| **Seeing Structure in Expressions** |
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| MA.A-SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. For example, see 𝑥⁴ – 𝑦⁴ as (𝑥²)² – (𝑦²)², thus recognizing it as a difference of squares that can be factored as (𝑥² – 𝑦²)(𝑥² + 𝑦²).  |
| MA.A-SSE.B | Write expressions in equivalent forms to solve problems  |
| MA.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.  |
| MA.A-SSE.B.3a | Factor a quadratic expression to reveal the zeros of the function it defines.  |
| MA.A-SSE.B.3b | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.  |
| MA.A-SSE.A | Interpret the structure of expressions  |
| MA.A-SSE.B.3c | Use the properties of exponents to transform expressions for exponential functions.  |
| MA.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context.  |
| MA.A-SSE.A.1a | Interpret parts of an expression, such as terms, factors, and coefficients.  |
| MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.  |
| MA.A-SSE.A.1b | Interpret complicated expressions by viewing one or more of their parts as a single entity.  |

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| **Arithmetic with Polynomials and Rational Expressions** |
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| MA.A-APR.C.4 | Prove polynomial identities and use them to describe numerical relationships.  |
| MA.A-APR.A | Perform arithmetic operations on polynomials  |
| MA.A-APR.A.1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.  |
| MA.A-APR.B | Understand the relationship between zeros and factors of polynomials  |
| MA.A-APR.B.2 | Know and apply the Remainder Theorem: For a polynomial 𝑝(𝑥) and a number 𝑎, the remainder on division by 𝑥 – 𝑎 is 𝑝(𝑎), so 𝑝(𝑎) = 0 if and only if (𝑥 – 𝑎) is a factor of 𝑝(𝑥).  |
| MA.A-APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.  |
| MA.A-APR.C.5 | Know and apply the Binomial Theorem for the expansion of (𝑥 + 𝑦)ⁿ in powers of 𝑥 and 𝑦 for a positive integer 𝑛, where 𝑥 and 𝑦 are any numbers, with coefficients determined for example by Pascal’s Triangle.  |
| MA.A-APR.D | Rewrite rational expressions  |
| MA.A-APR.D.6 | Rewrite simple rational expressions in different forms; write 𝑎(𝑥)/𝑏(𝑥) in the form 𝑞(𝑥) + 𝑟(𝑥)/𝑏(𝑥), where 𝑎(𝑥), 𝑏(𝑥), 𝑞(𝑥), and 𝑟(𝑥) are polynomials with the degree of 𝑟(𝑥) less than the degree of 𝑏(𝑥), using inspection, long division, or, for the more complicated examples, a computer algebra system.  |
| MA.A-APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.  |
| MA.A-APR.C | Use polynomial identities to solve problems  |

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| **Creating Equations** |
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| MA.A-CED.A | Create equations that describe numbers or relationships  |
| MA.A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems.  |
| MA.A-CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  |
| MA.A-CED.A.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.  |
| MA.A-CED.A.4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.  |

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| **Reasoning with Equations and Inequalities** |
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| MA.A-REI.B | Solve equations and inequalities in one variable  |
| MA.A-REI.B.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.  |
| MA.A-REI.B.4 | Solve quadratic equations in one variable.  |
| MA.A-REI.C | Solve systems of equations  |
| MA.A-REI.C.5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.  |
| MA.A-REI.C.6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.  |
| MA.A-REI.C.7 | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.  |
| MA.A-REI.C.8 | Represent a system of linear equations as a single matrix equation in a vector variable.  |
| MA.A-REI.C.9 | Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).  |
| MA.A-REI.D | Represent and solve equations and inequalities graphically  |
| MA.A-REI.B.4a | Use the method of completing the square to transform any quadratic equation in 𝑥 into an equation of the form (𝑥 – 𝑝)² = 𝑞 that has the same solutions. Derive the quadratic formula from this form.  |
| MA.A-REI.D.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  |
| MA.A-REI.B.4b | Solve quadratic equations by inspection (e.g., for 𝑥² = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as 𝑎 ± 𝑏𝑖 for real numbers 𝑎 and 𝑏.  |
| MA.A-REI.A | Understand solving equations as a process of reasoning and explain the reasoning  |
| MA.A-REI.D.12 | Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.  |
| MA.A-REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.  |
| MA.A-REI.D.11 | Explain why the 𝑥-coordinates of the points where the graphs of the equations 𝑦 = 𝑓(𝑥) and 𝑦 = 𝑔(𝑥) intersect are the solutions of the equation𝑓(𝑥) = 𝑔(𝑥); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where 𝑓(𝑥) and/or 𝑔(𝑥) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.  |
| MA.A-REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.  |

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| **Essential Questions** |
| How do polynomial, rational and radical functions model real world problems? What real world interpretation does a limit have? How can one function “undo” another function? What real world situations can be modeled by a system of non-linear equations? How can parabola be used to model/explain the real world?  |

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| **Enduring Understanding** |
| Algebra is the study of a system of symbols and mathematical operations essential for applications in many and various fields.  |

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| **Students will know...** |
| 1. Interpret functions that arise in applications in terms of context.2. Analyze functions using different representations3. Build a function that models a relationship between two quantities4. Build new functions from existing functions5. Construct and compare linear, quadratic, and exponential models and solve problems |

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| **Students will be able to...** |
| 1. Graph functions expressed symbolically and show key features of the graph (including intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity) by hand in simple cases and using technology for more complicated cases.2. Relate the domain of a function to its graph and identify any constraints on the domain3. Estimate, calculate and interpret the average rate of change of a function presented symbolically, in a table, or graphically over a specified interval.4. Rewrite a function in different but equivalent forms to identify and explain different properties of the function5. Analyze and compare properties of two functions when each is represented in a different form (algebraically, graphically, numerically in tables, or by verbal descriptions).6. Construct a function that combines standard function types using arithmetic operations to model a relationship between two quantities7. Identify and illustrate (using technology) an explanation of the effects on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.8. Express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. |

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| **STAGE 2- EVIDENCE OF LEARNING** |
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| **Formative Assessment Suggestions** |
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| • 3- Minute Pause |  **.**  |
| • A-B-C Summaries |  **.**  |
| • Analogy Prompt |  **.**  |
| • Choral Response |  **.**  |
| • Debriefing |  **.**  |
| • Exit Card / Ticket |  **.**  |
| • Hand Signals |  **.**  |
| • Idea Spinner |  **.**  |
| • Index Card Summaries |  **.**  |
| • Inside-Outside Circle Discussion (Fishbowl) |  **.**  |
| • Journal Entry |  **.**  |
| • Misconception Check |  **.**  |
| • Observation |  **.**  |
| • One Minute Essay |  **.**  |
| • One Word Summary |  **.**  |
| • Portfolio Check |  **.**  |
| • Questions & Answers |  **.**  |
| • Quiz |  **.**  |
| • Self-Assessment |  **.**  |
| • Student Conference |  **.**  |
| • Think-Pair-Share |  **.**  |
| • Web or Concept Map |  **.**  |

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| **Authentic Assessments Suggestions** |
| Through the following authentic assessments, students will develop traits regarding thinking and reasoning, settings, mathematical tools and attitudes and dispositions: 1. Performance Assessments 2. Short Investigations 3. Open Ended Response Questions 4.  Portfolios 5.  Self-Assessments |

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| **Benchmark Assessments** |
|  Edmentum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3- LEARNING PLAN** |
| The standards in this unit were introduced in Algebra 1 with a focus on numerical expressions and polynomial expressions in one variable, factoring quadratic and cubic polynomials, graphing quadratic functions and their transformations and interpret key features of graphs.  |

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| **Instructional Map** |
| Create multiple representations of Activity (4 person match-up). Teacher will create several function examples that can be represented in four ways. All students receive a card with one of the following function representations • Table of Values• Graph/Plot• Pattern Growth image•Verbal Explanation of the aforementioned imageStudents then seek to find others that match their function. As a group, students present how they determined their group • Write and recognize a function and its properties in multiple yet equivalent forms (e.g. Convert from standard form to vertex form of a quadratic equation and note the similarities and differences in the graph, table, and equation)  |

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| **Modifications/Differentiation of Instruction** |
| Differentiation Strategies for Special Education Students* Remove unnecessary material, words, etc., that can distract from the content
* Use of off-grade level materials
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Time allowed
* Level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Varied homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Ability to work at their own pace
* Present ideas using auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment
* Differentiated checklists and rubrics, if available and appropriate

 Differentiation Strategies for Gifted and Talented Students* Increase the level of complexity
* Decrease scaffolding
* Variety of finished products
* Allow for greater independence
* Learning stations, interest groups
* Varied texts and supplementary materials
* Use of technology
* Flexibility in assignments
* Varied questioning strategies
* Encourage research
* Strategy and flexible groups based on formative assessment or student choice
* Acceleration within a unit of study
* Exposure to more advanced or complex concepts, abstractions, and materials
* Encourage students to move through content areas at their own pace
* After mastery of a unit, provide students with more advanced learning activities, not more of the same activity
* Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas

Differentiated Strategies for ELL Students* Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials, including visuals
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Allow students to work at their own pace
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Role play
* Provide graphic organizers, highlighted materials
* Strategy and flexible groups based on formative assessments

Differentiation Strategies for At Risk Students* Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment

**504 Plans**Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:* walk, breathe, eat, or sleep
* communicate, see, hear, or speak
* read, concentrate, think, or learn
* stand, bend, lift, or work

Examples of accommodations in 504 plans include:* preferential seating
* extended time on tests and assignments
* reduced homework or classwork
* verbal, visual, or technology aids
* modified textbooks or audio-video materials
* behavior management support
* adjusted class schedules or grading
* verbal testing
* excused lateness, absence, or missed classwork
* pre-approved nurse's office visits and accompaniment to visits
* occupational or physical therapy
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| **Modification Strategies** |
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| • Extended Time |  **.**  |
| • Frequent Breaks |  **.**  |
| • Highlighted Text |  **.**  |
| • Interactive Notebook |  **.**  |
| • Modified Test |  **.**  |
| • Oral Directions |  **.**  |
| • Peer Tutoring |  **.**  |
| • Preferential Seating |  **.**  |
| • Re-Direct |  **.**  |
| • Repeated Drill / Practice |  **.**  |
| • Shortened Assignments |  **.**  |
| • Teacher Notes |  **.**  |
| • Tutorials |  **.**  |
| • Use of Additional Reference Material |  **.**  |
| • Use of Audio Resources |  **.**  |

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| **High Preparation Differentiation** |
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| • Alternative Assessments |  **.**  |
| • Choice Boards |  **.**  |
| • Games and Tournaments |  **.**  |
| • Group Investigations |  **.**  |
| • Guided Reading |  **.**  |
| • Independent Research / Project |  **.**  |
| • Interest Groups |  **.**  |
| • Learning Contracts |  **.**  |
| • Leveled Rubrics |  **.**  |
| • Literature Circles |  **.**  |
| • Menu Assignments |  **.**  |
| • Multiple Intelligence Options |  **.**  |
| • Multiple Texts |  **.**  |
| • Personal Agendas |  **.**  |
| • Project Based Learning (PBL) |  **.**  |
| • Stations / Centers |  **.**  |
| • Think-Tac-Toe |  **.**  |
| • Tiered Activities / Assignments |  **.**  |
| • Varying Graphic Organizers |  **.**  |

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| **Low Preparation Differentiation** |
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| • Choice of Book / Activity |  **.**  |
| • Cubing Activities |  **.**  |
| • Exploration by Interest (using interest inventories) |  **.**  |
| • Flexible Grouping |  **.**  |
| • Goal Setting With Student |  **.**  |
| • Homework Options |  **.**  |
| • Jigsaw |  **.**  |
| • Mini Workshops to Extend Skills |  **.**  |
| • Mini Workshops to Re-teach |  **.**  |
| • Open-ended Activities |  **.**  |
| • Think-Pair-Share by Interest |  **.**  |
| • Think-Pair-Share by Learning Style |  **.**  |
| • Think-Pair-Share by Learning Style |  **.**  |
| • Think-Pair-Share by Readiness |  **.**  |
| • Use of Collaboration |  **.**  |
| • Use of Reading Buddies |  **.**  |
| • Varied Journal Prompts |  **.**  |
| • Varied Product Choice |  **.**  |
| • Varied Supplemental Materials |  **.**  |
| • Work Alone / Together |  **.**  |

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| **Horizontal Integration- Interdisciplinary Connections** |
| Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other. |

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| **Vertical Integration- Discipline Mapping** |
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| **Additional Materials** |
| LPS Adopted Textbooks and Programs * Pearson EnVision Algebra II
* Pearson Realize (Computer Based program supplementing Envision)

Khan AcademyEdmentum Exact Path |

**Unit 4: Trigonometric Functions**Content Area: **Mathematics** Course(s): **Algebra II, Algebra II and Trigonometry**Time Period: **Third Marking Period**Length: **8 Weeks**Status: **Published**

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| **Unit Overview** |
|  Unit 4 builds on the students’ previous knowledge of functions, trigonometric ratios and circles in geometry to extend trigon ometry to model periodic phenomena. |

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| **STAGE 1- DESIRED RESULTS** |
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| **Educational Standards** |
| The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:1. Make sense of problems and persevre in solving them.2. Reason abstractly and quantitatively.3. Construct viable arguments and critique the reasoning of others.4. Model with mathematics.5. Use appropriate tools strategically.6. Attend to precision.7. Look for and make use of structure.8. Look for and express regularity in repeated reasoning. |

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| **New Jersey Student Learning Standards- Mathematics** |
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| **Introduction- Algebra** |
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| **Expressions** |
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|  | An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.  |
|  | Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor.  |
|  | Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.  |
|  | A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.  |

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| **Equalities and Inequalities** |
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|  | An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.  |
|  | The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.  |
|  | An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.  |
|  | Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x² – 2 = 0 are real numbers, not rational numbers; and the solutions of x² + 2 = 0 are complex numbers, not real numbers.  |
|  | The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, 𝘈 = ((𝘣₁+𝘣₂)/2)𝘩, can be solved for 𝘩 using the same deductive process.  |
|  | Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.  |

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| **Connections to Functions and Modeling** |
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|  | Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.  |

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| **Seeing Structure in Expressions** |
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| MA.A-SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. For example, see 𝑥⁴ – 𝑦⁴ as (𝑥²)² – (𝑦²)², thus recognizing it as a difference of squares that can be factored as (𝑥² – 𝑦²)(𝑥² + 𝑦²).  |
| MA.A-SSE.B | Write expressions in equivalent forms to solve problems  |
| MA.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.  |
| MA.A-SSE.B.3a | Factor a quadratic expression to reveal the zeros of the function it defines.  |
| MA.A-SSE.B.3b | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.  |
| MA.A-SSE.A | Interpret the structure of expressions  |
| MA.A-SSE.B.3c | Use the properties of exponents to transform expressions for exponential functions.  |
| MA.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context.  |
| MA.A-SSE.A.1a | Interpret parts of an expression, such as terms, factors, and coefficients.  |
| MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.  |
| MA.A-SSE.A.1b | Interpret complicated expressions by viewing one or more of their parts as a single entity.  |

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| **Arithmetic with Polynomials and Rational Expressions** |
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| MA.A-APR.C.4 | Prove polynomial identities and use them to describe numerical relationships.  |
| MA.A-APR.A | Perform arithmetic operations on polynomials  |
| MA.A-APR.A.1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.  |
| MA.A-APR.B | Understand the relationship between zeros and factors of polynomials  |
| MA.A-APR.B.2 | Know and apply the Remainder Theorem: For a polynomial 𝑝(𝑥) and a number 𝑎, the remainder on division by 𝑥 – 𝑎 is 𝑝(𝑎), so 𝑝(𝑎) = 0 if and only if (𝑥 – 𝑎) is a factor of 𝑝(𝑥).  |
| MA.A-APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.  |
| MA.A-APR.C.5 | Know and apply the Binomial Theorem for the expansion of (𝑥 + 𝑦)ⁿ in powers of 𝑥 and 𝑦 for a positive integer 𝑛, where 𝑥 and 𝑦 are any numbers, with coefficients determined for example by Pascal’s Triangle.  |
| MA.A-APR.D | Rewrite rational expressions  |
| MA.A-APR.D.6 | Rewrite simple rational expressions in different forms; write 𝑎(𝑥)/𝑏(𝑥) in the form 𝑞(𝑥) + 𝑟(𝑥)/𝑏(𝑥), where 𝑎(𝑥), 𝑏(𝑥), 𝑞(𝑥), and 𝑟(𝑥) are polynomials with the degree of 𝑟(𝑥) less than the degree of 𝑏(𝑥), using inspection, long division, or, for the more complicated examples, a computer algebra system.  |
| MA.A-APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.  |
| MA.A-APR.C | Use polynomial identities to solve problems  |

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| **Creating Equations** |
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| MA.A-CED.A | Create equations that describe numbers or relationships  |
| MA.A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems.  |
| MA.A-CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  |
| MA.A-CED.A.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.  |
| MA.A-CED.A.4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.  |

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| **Reasoning with Equations and Inequalities** |
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| MA.A-REI.B | Solve equations and inequalities in one variable  |
| MA.A-REI.B.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.  |
| MA.A-REI.B.4 | Solve quadratic equations in one variable.  |
| MA.A-REI.C | Solve systems of equations  |
| MA.A-REI.C.5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.  |
| MA.A-REI.C.6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.  |
| MA.A-REI.C.7 | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.  |
| MA.A-REI.C.8 | Represent a system of linear equations as a single matrix equation in a vector variable.  |
| MA.A-REI.C.9 | Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).  |
| MA.A-REI.D | Represent and solve equations and inequalities graphically  |
| MA.A-REI.B.4a | Use the method of completing the square to transform any quadratic equation in 𝑥 into an equation of the form (𝑥 – 𝑝)² = 𝑞 that has the same solutions. Derive the quadratic formula from this form.  |
| MA.A-REI.D.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  |
| MA.A-REI.B.4b | Solve quadratic equations by inspection (e.g., for 𝑥² = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as 𝑎 ± 𝑏𝑖 for real numbers 𝑎 and 𝑏.  |
| MA.A-REI.A | Understand solving equations as a process of reasoning and explain the reasoning  |
| MA.A-REI.D.12 | Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.  |
| MA.A-REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.  |
| MA.A-REI.D.11 | Explain why the 𝑥-coordinates of the points where the graphs of the equations 𝑦 = 𝑓(𝑥) and 𝑦 = 𝑔(𝑥) intersect are the solutions of the equation𝑓(𝑥) = 𝑔(𝑥); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where 𝑓(𝑥) and/or 𝑔(𝑥) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.  |
| MA.A-REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.  |

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| **Essential Questions** |
| * How are trigonometric functions used in right triangles?
* What is radian measure?
* How can you evaluate trigonometric functions of any angle?
* How are inverse trigonometric functions used?
* When can the laws of sines be used to solve a triangle?
* In which cases can the law of cosines be used the solve a triangle?
* How do the graphs of  and  compare to the graph of ?
* How can you verify that a trigonometric equation is an identity?
* How can you write the general solution of a trigonometric equation?
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| **Enduring Understanding** |
| * Proving identities requires the use of the rules of arithmetic and algebra to produce equivalent expressions.
* Evaluating trigonometric functions requires the use of arithmetic and algebraic rules and geometric analysis to understand degree and radian measures.
* Trigonometric functions and features such as amplitude, frequency, and midline can be used to model periodic phenomena
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| **Students will know...** |
| 1. Build new functions from existing functions2. Extend the domain of trigonometric functions using the unit circle.3. Model periodic phenomena with trigonometric functions (Alg 2/Trig)4. Prove and apply trigonometric identities5. Analyze functions using different representations (Alg 2/Trig) |

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| **Students will be able to...** |
| 1. Uses the radian measure of an angle to find the length of the arc in the unit circle subtended by the angle and find the measure of the angle given the length of the arc.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers (interpreted as radian measures of angles traversed counterclockwise around the unit circle) and use the Pythagorean identity (sinA)2 + (cosA)2 = 1 to find sinA, cosA, or tanA, given sinA, cosA, or tanA, and the quadrant of the angle3. Construct trigonometric functions that model periodic phenomena with specified amplitude, frequency, and midline.4. Determine the inverse function for a simple function that has an inverse and write an expression for it.5. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (by hand in simple cases and using technology for more complicated cases). |

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| **STAGE 2- EVIDENCE OF LEARNING** |
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| **Formative Assessment Suggestions** |
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| • 3- Minute Pause |  **.**  |
| • A-B-C Summaries |  **.**  |
| • Analogy Prompt |  **.**  |
| • Choral Response |  **.**  |
| • Debriefing |  **.**  |
| • Exit Card / Ticket |  **.**  |
| • Hand Signals |  **.**  |
| • Idea Spinner |  **.**  |
| • Index Card Summaries |  **.**  |
| • Inside-Outside Circle Discussion (Fishbowl) |  **.**  |
| • Journal Entry |  **.**  |
| • Misconception Check |  **.**  |
| • Observation |  **.**  |
| • One Minute Essay |  **.**  |
| • One Word Summary |  **.**  |
| • Portfolio Check |  **.**  |
| • Questions & Answers |  **.**  |
| • Quiz |  **.**  |
| • Self-Assessment |  **.**  |
| • Student Conference |  **.**  |
| • Think-Pair-Share |  **.**  |
| • Web or Concept Map |  **.**  |

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| **Authentic Assessments Suggestions** |
| Through the following authentic assessments, students will develop traits regarding thinking and reasoning, settings, mathematical tools and attitudes and dispositions: 1. Performance Assessments 2. Short Investigations 3. Open Ended Response Questions 4.  Portfolios5.  Self-Assessments |

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| **Benchmark Assessments** |
| Edmentum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3- LEARNING PLAN** |
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| **Instructional Map** |
| * Have students create a unit circle to discover it's properties inculding radian measures.
* Use software that models tides and periodicity, real world examples such as ferris wheels videos showing motion with graphing simultaneously (several websites offer this visual)
* Connect the trig identities to the Pythagorean Theorem, cutting out right triangles and fitting them to the unit circle, working hands on with the triangles
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| **Modifications/Differentiation of Instruction** |
| Differentiation Strategies for Special Education Students* Remove unnecessary material, words, etc., that can distract from the content
* Use of off-grade level materials
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Time allowed
* Level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Varied homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Ability to work at their own pace
* Present ideas using auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment
* Differentiated checklists and rubrics, if available and appropriate

Differentiation Strategies for Gifted and Talented Students* Increase the level of complexity
* Decrease scaffolding
* Variety of finished products
* Allow for greater independence
* Learning stations, interest groups
* Varied texts and supplementary materials
* Use of technology
* Flexibility in assignments
* Varied questioning strategies
* Encourage research
* Strategy and flexible groups based on formative assessment or student choice
* Acceleration within a unit of study
* Exposure to more advanced or complex concepts, abstractions, and materials
* Encourage students to move through content areas at their own pace
* After mastery of a unit, provide students with more advanced learning activities, not more of the same activity
* Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas

 Differentiated Strategies for ELL Students* Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials, including visuals
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Allow students to work at their own pace
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Role play
* Provide graphic organizers, highlighted materials
* Strategy and flexible groups based on formative assessment

 Differentiation Strategies for At Risk Students* Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment

 **504 Plans**Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:* walk, breathe, eat, or sleep
* communicate, see, hear, or speak
* read, concentrate, think, or learn
* stand, bend, lift, or work

 Examples of accommodations in 504 plans include:* preferential seating
* extended time on tests and assignments
* reduced homework or classwork
* verbal, visual, or technology aids
* modified textbooks or audio-video materials
* behavior management support
* adjusted class schedules or grading
* verbal testing
* excused lateness, absence, or missed classwork
* pre-approved nurse's office visits and accompaniment to visits
* occupational or physical therapy
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| **Modification Strategies** |
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| • Extended Time |  **.**  |
| • Frequent Breaks |  **.**  |
| • Highlighted Text |  **.**  |
| • Interactive Notebook |  **.**  |
| • Modified Test |  **.**  |
| • Oral Directions |  **.**  |
| • Peer Tutoring |  **.**  |
| • Preferential Seating |  **.**  |
| • Re-Direct |  **.**  |
| • Repeated Drill / Practice |  **.**  |
| • Shortened Assignments |  **.**  |
| • Teacher Notes |  **.**  |
| • Tutorials |  **.**  |
| • Use of Additional Reference Material |  **.**  |
| • Use of Audio Resources |  **.**  |

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| **High Preparation Differentiation** |
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| • Alternative Assessments |  **.**  |
| • Choice Boards |  **.**  |
| • Games and Tournaments |  **.**  |
| • Group Investigations |  **.**  |
| • Guided Reading |  **.**  |
| • Independent Research / Project |  **.**  |
| • Interest Groups |  **.**  |
| • Learning Contracts |  **.**  |
| • Leveled Rubrics |  **.**  |
| • Literature Circles |  **.**  |
| • Menu Assignments |  **.**  |
| • Multiple Intelligence Options |  **.**  |
| • Multiple Texts |  **.**  |
| • Personal Agendas |  **.**  |
| • Project Based Learning (PBL) |  **.**  |
| • Stations / Centers |  **.**  |
| • Think-Tac-Toe |  **.**  |
| • Tiered Activities / Assignments |  **.**  |
| • Varying Graphic Organizers |  **.**  |

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| **Low Preparation Differentiation** |
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| • Choice of Book / Activity |  **.**  |
| • Cubing Activities |  **.**  |
| • Exploration by Interest (using interest inventories) |  **.**  |
| • Flexible Grouping |  **.**  |
| • Goal Setting With Student |  **.**  |
| • Homework Options |  **.**  |
| • Jigsaw |  **.**  |
| • Mini Workshops to Extend Skills |  **.**  |
| • Mini Workshops to Re-teach |  **.**  |
| • Open-ended Activities |  **.**  |
| • Think-Pair-Share by Interest |  **.**  |
| • Think-Pair-Share by Learning Style |  **.**  |
| • Think-Pair-Share by Learning Style |  **.**  |
| • Think-Pair-Share by Readiness |  **.**  |
| • Use of Collaboration |  **.**  |
| • Use of Reading Buddies |  **.**  |
| • Varied Journal Prompts |  **.**  |
| • Varied Product Choice |  **.**  |
| • Varied Supplemental Materials |  **.**  |
| • Work Alone / Together |  **.**  |

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| **Horizontal Integration- Interdisciplinary Connections** |
| Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other. |

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| **Vertical Integration- Discipline Mapping** |
| * Students have worked with measuring angles in degrees since 4th grade. In 7th grade students first study the formulas for area and circumference of a circle. In Geometry students are first exposed to radian measure of an angle as the constant of proportionality.
* In Geometry, students are required to use the Pythagorean Theorem to derive the equation of a circle given the center and the radius. Students defined trigonometric ratios and solved problems involving right triangles
* In Geometry, students define trigonometric ratios and solve problems involving right triangles
* In 8th grade, students study and prove the Pythagorean Theorem.  In Geometry, students are required to use the Pythagorean Theorem to derive the equation of a circle given the center and the radius.
* In Algebra 1, students use linear and quadratic regression models fitted to data.
* In Calculus students will be expected to use the radian measure of angles in the vast majority of problems.
* In Pre-Calculus and Calculus, students will work with angle measures of all real numbers.
* In future mathematics courses students will work with trigonometric functions that have been translated vertically and horizontally and stretched or shrunk either horizontally or vertically.
* In Calculus, situations may arise where an expression needs to be altered by making a substitution based upon trigonometric identities.
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| **Additional Materials** |
| LPS Adopted Textbooks and Programs * Pearson EnVision Algebra II
* Pearson Realize (Computer Based program supplementing Envision)

Khan AcademyEdmentum Exact Path**Unit 5: Inference and Conclusions from Data**Content Area: **Mathematics** Course(s): **Algebra II, Algebra II and Trigonometry**Time Period: **Fourth Marking Period**Length: **8 Week**Status: **Published**

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| **Unit Overview** |
| Unit 5 will relate the visual displays and summary statistics learned in prior courses to different types of data and to probability distributions. Samples, surveys, experiments and simulations will be used as methods to collect data.  |

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| **STAGE 1- DESIRED RESULTS** |
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| **Educational Standards** |
| The following goals, as outlined in the NJSLS, will provide a framework for preparation and instruction in mathematics. They make up the eight mathematical practice standards:1. Make sense of problems and persevre in solving them.2. Reason abstractly and quantitatively.3. Construct viable arguments and critique the reasoning of others.4. Model with mathematics.5. Use appropriate tools strategically.6. Attend to precision.7. Look for and make use of structure.8. Look for and express regularity in repeated reasoning. |

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| **New Jersey Student Learning Standards- Mathematics** |
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| **Introduction- Algebra** |
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| **Expressions** |
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|  | An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.  |
|  | Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor.  |
|  | Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.  |
|  | A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.  |

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| **Equalities and Inequalities** |
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|  | An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.  |
|  | The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.  |
|  | An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.  |
|  | Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x² – 2 = 0 are real numbers, not rational numbers; and the solutions of x² + 2 = 0 are complex numbers, not real numbers.  |
|  | The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, 𝘈 = ((𝘣₁+𝘣₂)/2)𝘩, can be solved for 𝘩 using the same deductive process.  |
|  | Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.  |

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| **Connections to Functions and Modeling** |
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|  | Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.  |

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| **Seeing Structure in Expressions** |
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| MA.A-SSE.A.2 | Use the structure of an expression to identify ways to rewrite it. For example, see 𝑥⁴ – 𝑦⁴ as (𝑥²)² – (𝑦²)², thus recognizing it as a difference of squares that can be factored as (𝑥² – 𝑦²)(𝑥² + 𝑦²).  |
| MA.A-SSE.B | Write expressions in equivalent forms to solve problems  |
| MA.A-SSE.B.3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.  |
| MA.A-SSE.B.3a | Factor a quadratic expression to reveal the zeros of the function it defines.  |
| MA.A-SSE.B.3b | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.  |
| MA.A-SSE.A | Interpret the structure of expressions  |
| MA.A-SSE.B.3c | Use the properties of exponents to transform expressions for exponential functions.  |
| MA.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context.  |
| MA.A-SSE.A.1a | Interpret parts of an expression, such as terms, factors, and coefficients.  |
| MA.A-SSE.B.4 | Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.  |
| MA.A-SSE.A.1b | Interpret complicated expressions by viewing one or more of their parts as a single entity.  |

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| **Arithmetic with Polynomials and Rational Expressions** |
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| MA.A-APR.C.4 | Prove polynomial identities and use them to describe numerical relationships.  |
| MA.A-APR.A | Perform arithmetic operations on polynomials  |
| MA.A-APR.A.1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.  |
| MA.A-APR.B | Understand the relationship between zeros and factors of polynomials  |
| MA.A-APR.B.2 | Know and apply the Remainder Theorem: For a polynomial 𝑝(𝑥) and a number 𝑎, the remainder on division by 𝑥 – 𝑎 is 𝑝(𝑎), so 𝑝(𝑎) = 0 if and only if (𝑥 – 𝑎) is a factor of 𝑝(𝑥).  |
| MA.A-APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.  |
| MA.A-APR.C.5 | Know and apply the Binomial Theorem for the expansion of (𝑥 + 𝑦)ⁿ in powers of 𝑥 and 𝑦 for a positive integer 𝑛, where 𝑥 and 𝑦 are any numbers, with coefficients determined for example by Pascal’s Triangle.  |
| MA.A-APR.D | Rewrite rational expressions  |
| MA.A-APR.D.6 | Rewrite simple rational expressions in different forms; write 𝑎(𝑥)/𝑏(𝑥) in the form 𝑞(𝑥) + 𝑟(𝑥)/𝑏(𝑥), where 𝑎(𝑥), 𝑏(𝑥), 𝑞(𝑥), and 𝑟(𝑥) are polynomials with the degree of 𝑟(𝑥) less than the degree of 𝑏(𝑥), using inspection, long division, or, for the more complicated examples, a computer algebra system.  |
| MA.A-APR.D.7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.  |
| MA.A-APR.C | Use polynomial identities to solve problems  |

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| **Creating Equations** |
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| MA.A-CED.A | Create equations that describe numbers or relationships  |
| MA.A-CED.A.1 | Create equations and inequalities in one variable and use them to solve problems.  |
| MA.A-CED.A.2 | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  |
| MA.A-CED.A.3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.  |
| MA.A-CED.A.4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.  |

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| **Reasoning with Equations and Inequalities** |
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| MA.A-REI.B | Solve equations and inequalities in one variable  |
| MA.A-REI.B.3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.  |
| MA.A-REI.B.4 | Solve quadratic equations in one variable.  |
| MA.A-REI.C | Solve systems of equations  |
| MA.A-REI.C.5 | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.  |
| MA.A-REI.C.6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.  |
| MA.A-REI.C.7 | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.  |
| MA.A-REI.C.8 | Represent a system of linear equations as a single matrix equation in a vector variable.  |
| MA.A-REI.C.9 | Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).  |
| MA.A-REI.D | Represent and solve equations and inequalities graphically  |
| MA.A-REI.B.4a | Use the method of completing the square to transform any quadratic equation in 𝑥 into an equation of the form (𝑥 – 𝑝)² = 𝑞 that has the same solutions. Derive the quadratic formula from this form.  |
| MA.A-REI.D.10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  |
| MA.A-REI.B.4b | Solve quadratic equations by inspection (e.g., for 𝑥² = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as 𝑎 ± 𝑏𝑖 for real numbers 𝑎 and 𝑏.  |
| MA.A-REI.A | Understand solving equations as a process of reasoning and explain the reasoning  |
| MA.A-REI.D.12 | Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.  |
| MA.A-REI.A.1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.  |
| MA.A-REI.D.11 | Explain why the 𝑥-coordinates of the points where the graphs of the equations 𝑦 = 𝑓(𝑥) and 𝑦 = 𝑔(𝑥) intersect are the solutions of the equation𝑓(𝑥) = 𝑔(𝑥); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where 𝑓(𝑥) and/or 𝑔(𝑥) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.  |
| MA.A-REI.A.2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.  |

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| **Essential Questions** |
| * How do you determine whether a set of data fits into a linear, quadratic or exponential pattern?
* What is the difference, in terms of constants and variables, between an exponential model and a power model?
* How do you compare functions represented in different ways?
* What is binomial distribution?
* How does experimental probability compare to theoretical probability?
* Where are the values in a normal distribution that rarely occur displayed on a normal curve?
* How can we determine a trigonometric equation for modeling sinusoidal data?
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| **Enduring Understanding** |
| * The way that data is collected, organized and displayed influences interpretation.
* The probability of an event’s occurrence can be predicted with varying degrees of confidence.
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| **Students will know...** |
| 1. Construct and compare linear, quadratic and exponential models and solve problems2. Summarize, represent, and interpret data on a single count or measurement variable3. Understand and evaluate random processes underlying statistical experiments4. Make inferences and justify conclusions from sample surveys, experiments, and observational studies5. Use probability to evaluate outcomes of decisions |

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| **Students will be able to...** |
| 1. Make inferences about population parameters based on a random sample from that population.2. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages; recognize that there may be different distributions that apply to the data sets for which normal distribution procedures are not appropriate. (Use calculators, spreadsheets, and tables to estimate areas under the normal curve.)3. Determine if the outcomes and properties of a specified model are consistent with results from a given data-generating process using simulation.4. Identify different methods and purposes for conducting sample surveys, experiments, and observational studies and explain how randomization relates to each. 5. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.6. Apply probabilities in making decisions that are fair and analyze strategies based on their probabilities and likelihoods of occurring. |

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| **STAGE 2- EVIDENCE OF LEARNING** |
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| **Formative Assessment Suggestions** |
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| • 3- Minute Pause |  **.**  |
| • A-B-C Summaries |  **.**  |
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| • Student Conference |  **.**  |
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| • Web or Concept Map |  **.**  |

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| **Authentic Assessments Suggestions** |
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| **Benchmark Assessments** |
| Edmentum Exact Path (BOY, MOY, EOY) |

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| **STAGE 3- LEARNING PLAN** |
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| **Instructional Map** |
| * Collect data based on certain criteria or something relevant to student/class.
* Graph and analyzing data using mean, standard deviation
* Find out which data sets apply to the statistics tools, if not why? (normal distribution, etc)
* Compare and Contrast the different graphical displays (box plot-- previously referred to as a box- whisker, stem-leaf, bar, pie, etc)
* Explore a variety of statistical experiments to arrive at statistical techniques (graphs, distribution)
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| **Modifications/Differentiation of Instruction** |
| Differentiation Strategies for Special Education Students* Remove unnecessary material, words, etc., that can distract from the content
* Use of off-grade level materials
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Time allowed
* Level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Varied homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Ability to work at their own pace
* Present ideas using auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment
* Differentiated checklists and rubrics, if available and appropriate

Differentiation Strategies for Gifted and Talented Students* Increase the level of complexity
* Decrease scaffolding
* Variety of finished products
* Allow for greater independence
* Learning stations, interest groups
* Varied texts and supplementary materials
* Use of technology
* Flexibility in assignments
* Varied questioning strategies
* Encourage research
* Strategy and flexible groups based on formative assessment or student choice
* Acceleration within a unit of study
* Exposure to more advanced or complex concepts, abstractions, and materials
* Encourage students to move through content areas at their own pace
* After mastery of a unit, provide students with more advanced learning activities, not more of the same activity
* Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas

Differentiated Strategies for ELL Students* Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials, including visuals
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Allow students to work at their own pace
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Role play
* Provide graphic organizers, highlighted materials
* Strategy and flexible groups based on formative assessments

Differentiation Strategies for At Risk Students* Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment

**504 Plans**Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:* walk, breathe, eat, or sleep
* communicate, see, hear, or speak
* read, concentrate, think, or learn
* stand, bend, lift, or work

Examples of accommodations in 504 plans include:* preferential seating
* extended time on tests and assignments
* reduced homework or classwork
* verbal, visual, or technology aids
* modified textbooks or audio-video materials
* behavior management support
* adjusted class schedules or grading
* verbal testing
* excused lateness, absence, or missed classwork
* pre-approved nurse's office visits and accompaniment to visits
* occupational or physical therapy
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| **Modification Strategies** |
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| • Extended Time |  **.**  |
| • Frequent Breaks |  **.**  |
| • Highlighted Text |  **.**  |
| • Interactive Notebook |  **.**  |
| • Modified Test |  **.**  |
| • Oral Directions |  **.**  |
| • Peer Tutoring |  **.**  |
| • Preferential Seating |  **.**  |
| • Re-Direct |  **.**  |
| • Repeated Drill / Practice |  **.**  |
| • Shortened Assignments |  **.**  |
| • Teacher Notes |  **.**  |
| • Tutorials |  **.**  |
| • Use of Additional Reference Material |  **.**  |
| • Use of Audio Resources |  **.**  |

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| **High Preparation Differentiation** |
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| • Alternative Assessments |  **.**  |
| • Choice Boards |  **.**  |
| • Games and Tournaments |  **.**  |
| • Group Investigations |  **.**  |
| • Guided Reading |  **.**  |
| • Independent Research / Project |  **.**  |
| • Interest Groups |  **.**  |
| • Learning Contracts |  **.**  |
| • Leveled Rubrics |  **.**  |
| • Literature Circles |  **.**  |
| • Menu Assignments |  **.**  |
| • Multiple Intelligence Options |  **.**  |
| • Multiple Texts |  **.**  |
| • Personal Agendas |  **.**  |
| • Project Based Learning (PBL) |  **.**  |
| • Stations / Centers |  **.**  |
| • Think-Tac-Toe |  **.**  |
| • Tiered Activities / Assignments |  **.**  |
| • Varying Graphic Organizers |  **.**  |

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| **Low Preparation Differentiation** |
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| • Choice of Book / Activity |  **.**  |
| • Cubing Activities |  **.**  |
| • Exploration by Interest (using interest inventories) |  **.**  |
| • Flexible Grouping |  **.**  |
| • Goal Setting With Student |  **.**  |
| • Homework Options |  **.**  |
| • Jigsaw |  **.**  |
| • Mini Workshops to Extend Skills |  **.**  |
| • Mini Workshops to Re-teach |  **.**  |
| • Open-ended Activities |  **.**  |
| • Think-Pair-Share by Interest |  **.**  |
| • Think-Pair-Share by Learning Style |  **.**  |
| • Think-Pair-Share by Learning Style |  **.**  |
| • Think-Pair-Share by Readiness |  **.**  |
| • Use of Collaboration |  **.**  |
| • Use of Reading Buddies |  **.**  |
| • Varied Journal Prompts |  **.**  |
| • Varied Product Choice |  **.**  |
| • Varied Supplemental Materials |  **.**  |
| • Work Alone / Together |  **.**  |

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| **Horizontal Integration- Interdisciplinary Connections** |
| Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other. |

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| **Vertical Integration- Discipline Mapping** |
| In previous curriculum, students understand: defining trigonometric functions within the unit circle; angles of rotation and radian measure; evaluating trigonometric functions; and transformations of the graphs of trigonometric functions. In future curriculum, students will study: statistics; shape, center, and spread; and confidence intervals and margins of error.  |

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| **Additional Materials** |
| LPS Adopted Textbooks and Programs * Pearson EnVision Algebra II
* Pearson Realize (Computer Based program supplementing Envision)

Khan AcademyEdmentum Exact Path**Interdisciplinary Connections & Standards**With interdisciplinary instruction, the subject areas are woven together and explored through an overarching theme or concept. We use math to help us solve everyday problems in the kitchen, in the garden, and for many of us at our jobs.Brain research has shown that information in our brains is organized in schematic structures. These structures are made up of interconnected bits of information and serve as a framework for the knowledge we acquire. When a learner’s knowledge is connected, it is much more likely that they will apply the prior knowledge to a wide variety of new situations. They will acquire new information in a way that is more accessible and will be better able to relate it to previously acquired knowledge.Students learn about patterns in math, science, social studies, and even literature. Because of this, they are much more likely to “see” these patterns when they encounter new situations. Since patterns are not only studied in math they are able to make the connection and gain the understanding that patterns can be found in many areas of their lives. Interdisciplinary instruction allows students to understand the interconnectedness of the disciplines and makes learning more meaningful and relevant as fascinating connections are made across the subject areas.**Science**HS-LS3-3 Apply concepts of statistics and probability to explain the variation and distribution of expressed traits in a population.HS-ESS3-2 Evaluate competing design solutions for developing, managing, and utilizing energy and mineral resources based on cost-benefit ratios.HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.**Language Arts**RL.9-10.1 Cite strong and thorough textual evidence and make relevant connections to support analysis of what the text says explicityas well as inferentially, including determining where the text leaves matters uncertain.RI.9-10.7 Analyze various perspectives as presented in different mediums, determining which details are emphasized in each account.RI.9-10.8 Describe and evaluate the argument and specific claims in a text, assessing whether the reasoning is valid and the evidence is relevant and sufficient; identity false statements and reasoning.**Social Studies**6.1.12.EconGE.16 Use quantitative data and other sources to assess the impact of international, global business organizations, and oversees competition on the United States economy and workforce.6.1.12.HistoryCA.2 Research multiple perspectives to explain the struggle to create an American identity. |

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