**Subject**

**AP Calculus A/B**

**Curriculum Guide**

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**LINDEN PUBLIC SCHOOLS**

**LINDEN, NEW JERSEY**

**DR. MARNIE HAZELTON**

 **SUPERINTENDENT**

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**DIRECTOR OF MATHEMATICS, VOCATIONAL, & TECHNICAL SUBJECTS**

**The Linden Board of Education adopted the Curriculum Guide on:**

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| **July 28, 2022**  |  | **Education Report #22** |
| **Date** |  | **Agenda Item** |
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| **Rationale** |

**EDUCATION EQUITY:** The Linden Public School District guarantees each student equal educational opportunity regardless of age, race, color, creed, religion, gender, language, affectional or sexual orientation, ancestry, national origin, marital or economic status. For Information, contact District Educational Equity Officer Kevin Thurston at **(**908) 486-2800 x 8307**.**

**NONDISCRIMATION:** The Linden Public School District does not discriminate against handicapped persons in admission or access to or treatment or employment in its programs, activities, and vocational opportunities. For information contact District Public 504 Officer Annabell Louis at (908) 486-2800 x 8025.

**Linden Public Schools Vision**

The Linden Public School District is committed to developing respect for diversity, excellence in education, and a commitment to service, in order to promote global citizenship and ensure personal success for all students

**Linden Public Schools Mission**

The mission of the Linden Public School District is to promote distinction through the infinite resource that is Linden’s diversity, combined with our profound commitment to instructional excellence, so that each and every student achieves their maximum potential in an engaging, inspiring, and challenging learning environment.

**Math Department Vision**

To equip students with the understanding and application of mathematical skills and processes to foster a drive for advanced mathematics and higher-level thinking.

**Math Department Mission Statement**

To develop a community of learners who construct and communicate meaning from the mathematical world around them. Students will experience mathematics that encourage them to think critically, discover and apply concepts to solve problems strategically. Students will be encouraged to solve equations with accuracy, efficiency, and flexibility. Furthermore, students will have a multitude of opportunities to apply mathematical tools and practice standards to solve real-world and multi-step problems.

**Math Department Goals**

* Provide opportunities for student to develop computation skills, conceptual understanding, and problem-solving skills
* Require students to explain, justify or prove their thinking through mathematical reasoning, modeling, and speaking

Course DescriptionAP Calculus AB is designed to be the equivalent of a first semester college calculus course devoted to topics in differential and integral calculus, focusing on students’ understanding of calculus concepts and provides experience with methods and applications. This course features a multi-representational approach to calculus, with concepts, results, and problems expressed graphically, numerically, analytically, and verbally. Exploring connections among these representations builds understanding of how calculus applies limits to develop important ideas, definitions, formulas, and theorems.

Course Instructional Materials

* LPS Adopted Textbooks and Programs
	+ Wiley: Calculus – AP Edition
	+ Cengage: Calculus – Early Transcendental Functions
	+ WebAssign (Computer Based program supplementing Cengage)
* Khan Academy
* Edmentum Exact Path

Standards and NJDOE Mandates Guiding Instruction

* 1. New Jersey Student Learning Standards

 <https://www.state.nj.us/education/cccs/>

Diversity, Equity, and Inclusion

* Use students’ interests in conceptualized tasks
* Expose students to a diverse group of mathematicians
* Design assessments and assignments with a variety of response types
* Use systematic grading and participation methods
* Encourage students to embrace a growth mindset

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| **Career Ready Practices** |
| CRP2.   Apply appropriate academic and technical skills.CRP4.   Communicate clearly and effectively and with reason.CRP6.   Demonstrate creativity and innovation.CRP8.   Utilize critical thinking to make sense of problems and persevere in solving them.CRP11.   Use technology to enhance productivity.CRP12.   Work productively in teams while using cultural global competence.**AP Calculus AB 2022-2023****Pacing Guide**

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| **Unit #0**Preparation for Calculus | 4 daysReview9/6-9/9 |  |
| Graphs and ModelsLinear ModelsRates of ChangeDomain & RangeFunctions & Their GraphsFitting Models to DataComposite FunctionsInverse FunctionsExponential FunctionsLogarithmic Functions | **Essential Questions:** What are the parts of each type of equation of a line?What is a function and why is it significant?What are the basic functions and what do their graphs look like?What are domain & range & how does one find them graphically and analytically?What are composite functions?How do you determine if functions are even or odd?What are the exponential & logarithmic rules?Can you use the trigonometric functions in problem solving situations?Can you graph the six basic trigonometric functions?How do you find an inverse of a function and why is it significant? |

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| **Unit #1**Limits & Continuity1.7, 1.8, 1.9, 4.7, Chapter 1 Review | 10 days9/12 – 9/23 | Big Idea 1 : Change (CHA) Big Idea 2 : Limits (LIM)Big Idea 3. Analysis of Functions (FUN)  |
| Lesson 1.7 : Introduction to Limits and ContinuityLesson 1.8 : Extending the Idea of LimitsLesson 1.9: Further Limit Calculations using Algebra Lesson 4.7: L’Hospitals Rule, Growth & Dominance | **Essential Questions:** What is continuity and what is its definition?What is the idea and definition of a limit?How do you calculate one-sided limits?How do calculate limits when asymptotes are present?How do you find the limits of quotients?How do calculate limits at infinity?What is the squeeze theorem and how do you apply it to functions? |

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| **Unit #2 :** Differentiation: Definition & Fundamental Properties2.1, 2.2, 2.3, 2.5, 2.5, 2.6, Chapter 2 review, 3.1, 3.2, 3.3 & Chapter 3 review | 18 days9/27 – 10/21 | Big Idea 1 : Change (CHA) Big Idea 2 : Limits (LIM)Big Idea 3. Analysis of Functions (FUN) |
| Lesson 2.1 : How Do We Measure Speed Lesson 2.2 : The Derivative at a Point Lesson 2.3 : The Derivative Function Lesson 2.4 : Interpretations of the Derivative Lesson 2.5 : The Second Derivative Lesson 2.6 : Differentiability Lesson 3.1 : Powers & PolynomialsLesson 3.2 : The Exponential FunctionLesson 3.3 : The Product & Quotient Rules | **Essential Questions:** A. How does the concept of the derivative show how a function is changing?B. How do you determine the rates of change at any instant by applying limits to knowledge about rates of change over intervals? C. How can you understand the behavior of the function using a derivative? D. How do you determine the rates of change at any instant by applying limits to knowledge about rates of change over intervals. E. When is it appropriate to apply derivative rules in order to simplify differentiation. |

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| **Unit #3**Differentiation: Composite, Implicit & Inverse Functions3.4, 3.5, 3.6, 3.7 & Chapter 3 review | 12 days10/24 – 11/7 | Big Idea 3. Analysis of Functions (FUN) |
| Lesson 3.4 : The Chain RuleLesson 3.5 : The Trigonometric FunctionsLesson 3.6 : The Chain Rule & Inverse FunctionsLesson 3.7 : Implicit Functions | **Essential Questions:** How do you derive a composite function?What are the derivatives of sine, cosine and tangent?What are the derivatives of ln *x* & a*x*.What is the procedures to derive an inverse general function or an inverse trigonometric function?What is an implicit function, and how is it derived? |

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| **Unit #4**Contextual Applications of Differentiation4.1, 4.2, 4.6 & Chapter 4 review | 14 days11/14 – 12/2 | Big Idea 1 : Change (CHA) Big Idea 2 : Limits (LIM) |
| Lesson 4.1 : Using First and Second Derivatives Lesson 4.2 : OptimizationLesson 4.6 : Rates and Related Rates | **Essential Questions:** What are the first and second derivatives of functions use to interpretate?How are the critical points of a function including maxima and minima of a function detected?How are the inflection points of a function including concavity of a function detected?How ate the upper and lower bounds of a function determined using optimization.How do you calculate rates of change using derivatives in various situations. |

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| **Unit #5**Analytical Applications of Differentiation3.10, 4.2, 4.3, 4.4 & Chapter 4 review |  14 days12/5 – 12/22 | Big Idea 3. Analysis of Functions (FUN) |
| Lesson 3.10 : Theorems About Differentiable FunctionsLesson 4.2 : OptimizationLesson 4.3 : Optimization and ModelingLesson 4.4 : Families of Functions and Modeling | **Essential Questions:** How is the average rate of change of a function on an interval related to the instantaneous rate of change at a point in that interval?Can you use mathematical modeling to translate a real life problem into a function with a known formula in order to determine global maxima or minima.What does changing the parameters of families of functions do to help solve modeling problems.  |

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| **Unit #6**Integration & Accumulation of Change5.1, 5.2, 5.3, 5.4 & Chapter 5 review6.1, 6.2 & Chapter 6 review7.1 | 20 days1/3 - 1/31 | Big Idea 1 : Change (CHA) Big Idea 2 : Limits (LIM)Big Idea 3. Analysis of Functions (FUN) |
| Lesson 5.1 : How Do We Measure Distance TraveledLesson 5.2 : The Definite IntegralLesson 5.3 : The Fundamental Theorem and InterpretationsLesson 5.4 : Theorems About Definite IntegralsLesson 6.1 : Antiderivatives Graphically & NumericallyLesson 6.2 : Constructing Antiderivatives AnalyticallyLesson 7.1 : Integration by Substitution | **Essential Questions:** How are the sums for any function *f* constructed using sigma notation?What is the definite interval and how is it used to determine the area under a curve? In what manner is the fundamental theorem of calculus used to determine how does one interpretate its results.What are the properties of definite intervals and why do its properties hold?How are the families of antiderivatives determined and how can you visualize them using slopes. What is the process used to compute the values of an antiderivative using definite integrals?How are antiderivatives constructed and solved analytically.How is integration by substitution completed using both the guess-check method and definite integration. |

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| **Unit #7**Differential Equations6.3, 11.1, 11.2, 11.5, 11.5, 11.6 & Chapter 11 review | 21 days2/1 – 3/2 | Big Idea 3. Analysis of Functions (FUN) |
| Lesson 6.3 : Differential Equations in MotionLesson 11.1 : What is a Differential EquationLesson 11.2 : Slope FieldsLesson 11.4 : Separation of VariablesLesson 11.5 : Growth & DecayLesson 11.6 : Applications of Modeling | **Essential Questions:** How can we pick one solution to the equation ?What is a differential equation?What are slope fields, and what are they used for?How does separating variables lead to solving differential equations?What are they types of growth and decay situations that calculus could be used to compute?Where do the theoretical functions come from that allow us to solve real world problems?  |

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| **Unit #8**Applications of Integration6.3, 6.4, 7.5, 8.1 & 8.2 | 20 days3/3 – 3/30 | Big Idea 1 : Change (CHA)  |
| Lesson 6.3 : Differential Equations & MotionLesson 6.4 : The Second Fundamental Theorem of CalculusLesson 7.5 : Numerical Methods for Definite IntegralsLesson 8.1 : Areas and VolumesLesson 8.2: Applications to Geometry | **Essential Questions:** What is the second fundamental theorem of calculus and how is it used to construct antiderivatives?How are the numerical methods for determining a definite integral including the midpoint and trapezoidal rules performed?What is Simpson’s rule and how does it ally in real life situations?How can areas and volumes of geometric figures be determined by the slicing method?Using applications of geometry, how can volumes of revolution and the volumes of regions of a know cross-section be determined? |

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| **Review for AP Exam** | 20 days4/3 – 5/5 | Big Idea 1 : Change (CHA) Big Idea 2 : Limits (LIM)Big Idea 3. Analysis of Functions (FUN) |
| AP College Board Review Questions onlinePast AP Exams | **Essential Questions:** What is on the AP test?What topics do I need to review?How are the questions set up and worded?What do I need to do to receive a high passing score? |

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| **AP Exam** | 1 days5/8 | Big Idea 1 : Change (CHA) Big Idea 2 : Limits (LIM)Big Idea 3. Analysis of Functions (FUN) |

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| **Post AP Test** | 19 days5/9 – 6/5 | Big Idea 1 : Change (CHA) Big Idea 2 : Limits (LIM)Big Idea 3. Analysis of Functions (FUN) |
| AP Calculus Video Project on one of the Units |  |

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| **Review for Final Exam** | 5 days6/7 – 6/13 | Big Idea 1 : Change (CHA) Big Idea 2 : Limits (LIM)Big Idea 3. Analysis of Functions (FUN) |
| LimitsDerivativesOptimization & Graphing IntegrationDifferential Equations Applications of Integration |  |

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| **Final Exam** | 1 days | Big Idea 1 : Change (CHA) Big Idea 2 : Limits (LIM)Big Idea 3. Analysis of Functions (FUN) |

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| **Modifications/Differentiation of Instruction** |
| Differentiation Strategies for Special Education Students* Remove unnecessary material, words, etc., that can distract from the content
* Use of off-grade level materials
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Time allowed
* Level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Varied homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Ability to work at their own pace
* Present ideas using auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment
* Differentiated checklists and rubrics, if available and appropriate

Differentiation Strategies for Gifted and Talented Students* Increase the level of complexity
* Decrease scaffolding
* Variety of finished products
* Allow for greater independence
* Learning stations, interest groups
* Varied texts and supplementary materials
* Use of technology
* Flexibility in assignments
* Varied questioning strategies
* Encourage research
* Strategy and flexible groups based on formative assessment or student choice
* Acceleration within a unit of study
* Exposure to more advanced or complex concepts, abstractions, and materials
* Encourage students to move through content areas at their own pace
* After mastery of a unit, provide students with more advanced learning activities, not more of the same activity
* Present information using a thematic, broad-based, and integrative content, rather than just single-subject areas

Differentiated Strategies for ELL Students* Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials, including visuals
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language.
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Allow students to work at their own pace
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Role play
* Provide graphic organizers, highlighted materials
* Strategy and flexible groups based on formative assessment

Differentiation Strategies for At Risk Students* Remove unnecessary materials, words, etc., that can distract from the content
* Provide appropriate scaffolding
* Limit the number of steps required for completion
* Gradually increase the level of independence required
* Tiered centers, assignments, lessons, or products
* Provide appropriate leveled reading materials
* Deliver the content in “chunks”
* Varied texts and supplementary materials
* Use technology, if available and appropriate
* Differentiate homework and products
* Varied questioning strategies
* Provide background knowledge
* Define key vocabulary, multiple-meaning words, and figurative language
* Use audio and visual supports, if available and appropriate
* Provide multiple learning opportunities to reinforce key concepts and vocabulary
* Meet with small groups to reteach idea/skill
* Provide cross-content application of concepts
* Presenting ideas through auditory, visual, kinesthetic, & tactile means
* Provide graphic organizers and/or highlighted materials
* Strategy and flexible groups based on formative assessment

**504 Plans**Students can qualify for 504 plans if they have physical or mental impairments that affect or limit any of their abilities to:* walk, breathe, eat, or sleep
* communicate, see, hear, or speak
* read, concentrate, think, or learn
* stand, bend, lift, or work

Examples of accommodations in 504 plans include:* preferential seating
* extended time on tests and assignments
* reduced homework or classwork
* verbal, visual, or technology aids
* modified textbooks or audio-video materials
* behavior management support
* adjusted class schedules or grading
* verbal testing
* excused lateness, absence, or missed classwork
* pre-approved nurse's office visits and accompaniment to visits
* occupational or physical therapy
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| **Modification Strategies** |
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| • Extended Time | **.** |
| • Frequent Breaks | **.** |
| • Highlighted Text | **.** |
| • Interactive Notebook | **.** |
| • Modified Test | **.** |
| • Oral Directions | **.** |
| • Peer Tutoring | **.** |
| • Preferential Seating | **.** |
| • Re-Direct | **.** |
| • Repeated Drill / Practice | **.** |
| • Shortened Assignments | **.** |
| • Teacher Notes | **.** |
| • Tutorials | **.** |
| • Use of Additional Reference Material | **.** |
| • Use of Audio Resources | **.** |

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| **High Preparation Differentiation** |
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| • Alternative Assessments | **.** |
| • Choice Boards | **.** |
| • Games and Tournaments | **.** |
| • Group Investigations | **.** |
| • Guided Reading | **.** |
| • Independent Research / Project | **.** |
| • Interest Groups | **.** |
| • Learning Contracts | **.** |
| • Leveled Rubrics | **.** |
| • Literature Circles | **.** |
| • Menu Assignments | **.** |
| • Multiple Intelligence Options | **.** |
| • Multiple Texts | **.** |
| • Personal Agendas | **.** |
| • Project Based Learning (PBL) | **.** |
| • Stations / Centers | **.** |
| • Think-Tac-Toe | **.** |
| • Tiered Activities / Assignments | **.** |
| • Varying Graphic Organizers | **.** |

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| **Low Preparation Differentiation** |
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| • Choice of Book / Activity | **.** |
| • Cubing Activities | **.** |
| • Exploration by Interest (using interest inventories) | **.** |
| • Flexible Grouping | **.** |
| • Goal Setting With Student | **.** |
| • Homework Options | **.** |
| • Jigsaw | **.** |
| • Mini Workshops to Extend Skills | **.** |
| • Mini Workshops to Re-teach | **.** |
| • Open-ended Activities | **.** |
| • Think-Pair-Share by Interest | **.** |
| • Think-Pair-Share by Learning Style | **.** |
| • Think-Pair-Share by Learning Style | **.** |
| • Think-Pair-Share by Readiness | **.** |
| • Use of Collaboration | **.** |
| • Use of Reading Buddies | **.** |
| • Varied Journal Prompts | **.** |
| • Varied Product Choice | **.** |
| • Varied Supplemental Materials | **.** |
| • Work Alone / Together | **.** |

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| **Horizontal Integration- Interdisciplinary Connections** |
| Mathematical calculations occur at every step in Physics. The laws of motion, friction, expansion of solids, liquid pressure are explained using Mathematics. All the measurements in Physics need Mathematics. The coefficient of linear expansion of different metals, cubical expansion of liquids, expansion of gases and conversion of scales are a few to mention. New, exciting challenges in the Life Sciences can and are being met using mathematical modelling with a direct impact on improving people's quality of life in health, social and ecological issues. Knowledge of Mathematics is considered essential for a biologist for two reasons: firstly, biological study depends largely on its branches Bio-Physics and Bio-Chemistry. In Chemistry, all chemical combinations and their equations are governed by certain Mathematical laws. Also, Mathematics is the foundation of all Engineering Sciences, including IT. We know that Engineering Sciences deal with surveying, lending, construction, estimation, designing, measurement, calculation, drafting, drawing etc. Researchers in Economics, both theoretical and empirical, are using more mathematical tools in their research work and the growing importance of Econometrics. Mathematical terms like Relations, Functions, Continuity, etc., are very much used in Economics. Mathematics is used in almost all Social Science subjects. Mathematical knowledge is applied in History to know the dates, time, etc., of various historical events. In Geography to study the shape and size of earth, to measure area, height and distance, to study about latitude or longitude we need mathematical knowledge. To study the rivers, mountains, canals, population, climate, etc. all these studies need the tools of Mathematics in one way or other. |

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| **Vertical Integration- Discipline Mapping** |
| The standards in this unit were introduced in Geometry, Algebra II and Pre-Calculus. The AP Calculus A/B coursework focuses on preparing the students to be proficient in post high school coursework. |

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| **Unit # 1****8 Days****Limits & Continuity****(Weight on Exam 10-12%)** |
| **Overview** |

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| **STAGE 1****Desired Results** |

**Unit Overview**

You’ll start to explore how limits will allow you to solve problems involving change and to better understand mathematical reasoning about functions. Topics may include how limits help us to handle change at an instant, definition and properties of limits in various representations, definitions of continuity of a function at a point and over a domain, asymptotes and limits at infinity, & reasoning using the Squeeze theorem and the Intermediate Value Theorem. CR2 CR3 CR4 CR5 CR6

**Big Ideas**

Big Idea 1 : Change (CHA) CR22

Big Idea 2 : Limits (LIM) CR2

Big Idea 3. Analysis of Functions (FUN) CR2

**Essential Questions…** CR2 CR3 CR4 CR5 CR6

A. What is continuity and what is its definition?

B. What is the idea and definition of a limit?

C. How do you calculate one-sided limits?

D. How do calculate limits when asymptotes are present?

E. How do you find the limits of quotients?

F. How do calculate limits at infinity?

G. What is the squeeze theorem and how do you apply it to functions?

**Enduring Understanding…**

EU 1.1: The concept of a limit can be used to understand the behavior of function. CR22CR3 CR4

EU 1.2: Continuity is a key property of functions that is defined using limits. CR22CR4

CHA-2: Calculus allows us to generalize knowledge about motion to diverse problems involving change. CR22

­LIM-1 : Reasoning with definitions and properties can be used to justify claims about limits. CR22CR5

LIM-2: Reasoning with definitions, theorems, & properties can be used to justify claims about continuity. CR22CR4

FUN-1 : Existence theorems allow us to draw conclusions about a functions behavior on an interval without precisely locating that behavior. CR22CR4

**Students will be able to...** CR2 CR3 CR4 CR5 CR6

* Calculate average speed and instantaneous speed
* Define and calculate limits for function values and apply the properties of limits
* Find and interpret one-sided limits
* Apply the Squeeze Theorem to find certain limits indirectly
* Find and verify end-behavior models for various functions
* Calculate limits as x approaches infinity and identify vertical and horizontal tangents
* Identify intervals upon which a function is continuous and understand the meaning of continuity
* Remove removable discontinuities by extending or modifying a function
* Apply the Intermediate Value Theorem and the properties of algebraic combinations and composites of continuous functions
* Directly apply the definition of a slope of a curve in order to calculate slopes
* Find the equation of a tangent line and normal line to a curve at a given point
* Find the average rate of change of a function
* Determine and use sensitivity to approximate the change in a quantity
* Use a graphing calculator to solve equations
* Use a graphing calculator to graph functions and their tangent lines

**Students will know...** CR6

• The definitions for the following terms:

* 1. Average Speed
	2. Instantaneous Speed
	3. Definition of Limit
	4. Properties of Limits
	5. One Sided Limits
	6. Two Sided Limits
	7. Squeeze Theorem
	8. Finite Limits
	9. Infinite Limits
	10. End Behavior
	11. Continuity at a point
	12. Continuous Function
	13. Algebraic Combinations
	14. Composites
	15. Intermediate Value Theorem
	16. Average rate of change
	17. Tangent to a curve
	18. Slope of a curve
	19. Normal to a curve
	20. Sensitivity

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| **STAGE 2****Evidence of Learning** |

**Formative Activities, Tasks, or Projects:**

**Assessment Topic #1: Limits & Continuity** CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

* Use as an example of limits. Calculate limit using graphing utility.
* using graphing calculator to have a discussion of the properties of limits.
* Discuss the function to introduce limits approaching infinity.
* Discuss the three types of rational function end behavior.
* Use pencil and paper to trace a continuous function over any interval without lifting the pencil.
* Discuss continuity at an endpoint versus continuity at an interior point.
* Remind students that there can be points of discontinuity outside the domain of the function.
* Summarize the lesson by showing how to find the equation of the line.

Lesson Theme 1.7 : Introduction to Limits and Continuity (1.C)(1.E)(2.B)(3.B)(3.C)

Start with the concept of continuity on an interval. Begin graphically by using graphs of various functions: continuous, with a vertical asymptote, with jumps, etc. When limit notation is introduced, emphasize that the limit describes behavior of a function near a point, whereas continuity describes behavior both near and at the point. A function being continuous at a point versus a function having a limit at a point often become tangled together for students so be sure to concentrate on this difference. The Intermediate Value Theorem is best understood visually using a graph of a continuous function.

For most of the problems in this section, graphing will be a very useful tool. Be sure to include some problems that force students to practice other techniques (such as making a table or using basic algebra) for estimating limits or determining continuity, as well.

Day 1: Complete problems 1 – 9 odds, 13 – 17 odds, 23-33 odds

Day 2: Complete problems 37-39, 43-47 odds, 50-54 evens, 59, 61, 66, 69, 71

Lesson Theme 1.8 : Extending the Idea of Limits (1.C)(1.E)(3.B)

Begin with a function that has a jump, such as on [1*,*3) and on (3*,*5]. Ask why the limit does not exist as *𝑥* approaches 3, guiding them to the observation that as we approach *𝑥* from the left and from the right approaches two different values, 3 and 4, respectively. This naturally leads into left- and right-hand limits. Be sure to point out that the limit only exists when the left- and right-hand limits agree, if they exist. You might also ask if anything changes when we alter one of the domains to include 3. This should help reenforce the idea that limits are concerned with behavior near a point, not at a point.

Limits at infinity are introduced in this section, and the concept of *𝑥* “approaching" infinity can be confusing for students, as it is not a real number and they cannot graph the function at infinity. For this reason, it is important to be clear that we are considering *𝑥* values that can be as large as we want. A real-life situation of a quantity changing over time is a good way to introduce the idea of end behavior. For example, consider a hot cup of coffee brought into your office and left there for hours. If we consider the temperature of the coffee to be a function of time, what happens as the coffee sits for longer and longer? Note that students should soon realize that this function will eventually level off to approach the temperature of the room—*i.e.*, the temperature will not approach negative infinity as *𝑡* gets larger.

This section also includes properties of limits as they apply to sums, differences, products, etc. Give a few examples of these properties, making sure to give an example of the property for quotients. Something like:

This limit exits, but if what happens if we attempt to use the fourth property to find of the same function? Ask the students why property four does not work in this case. This topic will be algebraically addressed in the next section, although students can graph the function at this point. (Ask students to use radian mode and suggest a window and *.*)

This leads to a discussion of vertical asymptotes. Discuss what happens to the graph as and as You may also want to have students sketch graphs as in Problem 55. Then ask the students to sketch a graph such that

Give a few examples of how continuity applies to combinations of functions as well, including composite and inverse functions such as An important take-away from this section is for students recognize that complicated looking functions can often be broken down as combinations of functions they are familiar with.

Day 1: Complete problems 1, 3, 7-15 odds, 28-34 evens

Day 2: Complete problems 37-40, 43-55 odds, 59, 66

Lesson Theme 1.9: Further Limit Calculations using Algebra (1.C)(3.C)

We will summarize the previous techniques and describe further algebraic methods that can be used to calculate limits exactly.

Limits of continuous functions are straightforward to compute algebraically. The students look at some familiar continuous functions (polynomial, sine or cosine, etc) and evaluate the limit at a point.

Point out that the cancellation of the common factor is only valid because as we take the limit we are concerned about the behavior of the function *near*  but not at *.* (You can’t say this enough!)

In order to consider limits at infinity, refer students to the section on the end behavior of rational functions and the idea of *dominance*. As grows arbitrarily large, the function approaches 0.
Introduce the Squeeze Theorem by giving an example of a limit where the techniques students have learned so far cannot be used. Also give a visual example of how two functions “squeeze" between them, and have students graph the function. The visual aid of the graphs will help students understand what the Squeeze Theorem is saying.

Day 1: Complete problems 5-23 odds, 27-33 odds

Day 2: Complete problems 39-49 odds, 51-59 odds, 63, 65

Lesson Theme 4.7: L’Hospitals Rule, Growth & Dominance (3.C) (3.D)

Start with an example which demonstrates the case when is finite and , for example .

Approximate the limit graphically and numerically. Show that these functions satisfy the condition . Graph

and and the tangent line approximations for and on the same axes and give a graphical explanation of the rule in this case. Then use L’Hopital’s rule to evaluate the limit.

Remind students that the result of L’Hopital’s rule is a number and not a derivative function.

In another class activity, students will examine the graphs of rational functions in the form alongside the graphs of and . The objective is for students to discover why the hypothesis conditions for L’Hospital’s Rule are needed. Then the students summarize their findings in a shared online document. For homework, students are given a worksheet of limit problems, some of which involve the indeterminate forms and Students decide whether L’Hospital’s Rule can be applied and either explicitly confirm that hypothesis conditions are met or show that they are not.

Day 1: Complete problems 2, 4, 6, 13-37 odds, 48, 49

Lesson Theme: Chapter 1 Review

Complete problems 62, 64, 72, 74, 75, 79, 81, 84, 90, 93

AP Examination Preparation questions CR2 CR3 CR4 CR5 CR6 CR7 CR8

**Summative Activities, Tasks, or Projects:**

Do Now problems CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Unit 1 test CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Wiley course resources on Canvas CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Chapter 01 Additional problems

Example 1.7.3 Video: Using continuity to find zeros from a table of values

Example 1.7.4 Video: Finding a limit using a graph and algebra

Example 1.8.1 Video: Finding limits of a function involving absolute values

Example 1.8.4 Video: Describing the behavior of a function near a value of t

Example 1.9.4 Video: Finding a limit using substitution

Example 1.9.5 Video: Finding a limit at infinity

1.10 Optional Preview of the Formal Definition of a Limit

Example 4.7.2 Video: Using L'Hopital's rule to find a limit

Example 4.7.4 Video: Showing that one function dominates another function

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| **STAGE 3****Learning Plan** |

**Assessment Topics and Lesson Themes:**

**Assessment Topic #1: Limits & Continuity**  CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Lesson Theme 1.7 : Introduction to Limits and Continuity

Lesson Theme 1.8 : Extending the Idea of Limits

Lesson Theme 1.9: Further Limit Calculations using Algebra

Lesson Theme: Chapter 1 Review

Lesson Theme 4.7: L’Hospitals Rule, Growth & Dominance

**Unit Specific Accommodations and Modifications**

* Explain that some calculators may connect the two branches of a function such as suggesting the function is continuous for the domain even though there is a discontinuity at .
* Students may have difficulty dealing with the indeterminant form so use L’Hopital’s Rule to deal with evaluating these limits is covered in chapter 4.
* Students may have trouble with continuity at an interior point versus an endpoint.
* Students are prone to calculation errors when calculating the slope of a curve. Encourage students to find their own mistakes so they will be less likely to repeat these errors in the future.

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| **Additional Materials** |

**Digital Tools/Resources:**

TI-84 Plus Graphing Calculator provided to each student by the district

Wiley course resources on Canvas

http://apcentral.collegeboard.com

Edmentum Exact Path

**Additional Primary & Secondary Resources:**

Multiple Choice & Free-Response questions in preparation for the AP Calculus (AB) Examination

LPS Adopted Textbooks and Programs

* Wiley: Calculus – AP Edition CR1
* Cengage: Calculus – Early Transcendental Functions CR1

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| **Unit # 2****18 Days****Differentiation: Definition & Fundamental Properties****(Weight on Exam 10-12%)** |
| **Overview** |

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| **STAGE 1****Desired Results** |

**Unit Overview**

You’ll apply limits to define the derivative, become skillful at determining derivatives, and continue to develop mathematical reasoning skills. Topics include defining the derivative of a function at a point and as a function, connecting differentiability and continuity, determining derivatives for elementary functions & applying differentiation rules. CR2 CR3 CR4 CR5 CR6

**Big Ideas**

Big Idea 1 : Change (CHA) CR22

Big Idea 2 : Limits (LIM) CR2

Big Idea 3. Analysis of Functions (FUN) CR2

**Essential Questions…** CR3 CR4 CR5 CR6

A. How does the concept of the derivative show how a function is changing?

B. How do you determine the rates of change at any instant by applying limits to knowledge about rates of change over intervals?

C. How can you understand the behavior of the function using a derivative?

D. How do you determine the rates of change at any instant by applying limits to knowledge about rates of change over intervals.

E. When is it appropriate to apply derivative rules in order to simplify differentiation.

**Enduring Understanding…**

How functions change gives key information about the function and related real-world phenomena.

EU 2.1: The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies. CR3

EU 2.2: A function’s derivative, which is itself a function, can be used to understand the behavior of the function. CR3

EU 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change. CR3

­CHA-2 : Derivatives allow us to determine the rates of change at any instant by applying limits to knowledge about rates of change over intervals. CR3

FUN-2 : Recognizing that a function’s derivative may also be a function allows us to develop knowledge about the related behaviors of both. CR3

FUN-3 : Recognizing opportunities to apply derivative rules can simplify differentiation. CR4

LIM-3: Reasoning with definitions theorems, and properties can be used to determine a limit. CR3

**Students will be able to...** CR2 CR3 CR4 CR5 CR6 CR7

• Measure speed using average and instantaneous velocity.

• Define and compute instantaneous velocity.

• Visualize velocity using the slope of a curve.

• Find the derivative at a point.

• Determine the average rate of change and instantaneous rate of change.

• Visualize the derivative using the slope of a curve or slope of a tangent.

• Compute a derivative algebraically.

• Find the derivative of a function graphically.

• Understand what the derivative of a function tells us graphically.

• Find the derivative of a function numerically.

• Find the derivative of a function using the constant function, linear function or power function formulas.

• Perform numerical differentiation on a graphing calculator.

• Interpret what the derivative means

• Understand the different notations the derivative could be in.

• Find the second derivative of a function.

• Interpret the second derivative as a rate of change.

• Understand what it means for a function to be differentiable.

• Understand what it means for a function to be continuous.

**Students will know...** CR2 CR3 CR4 CR5 CR6 CR7

• The definitions for the following terms:

a. derivative

b. one-sided derivatives

c. local linearity

d. Intermediate Value Theorem for derivatives

e. positive integer powers, multiples, sums and differences

f. products and quotients

g. second and higher order derivatives

h. instantaneous rate of change

i. derivative of the sine functions

j. derivative of the cosine function

k. derivatives of other basic trigonometric functions

• Relationships between the graphs of and

• Graphing the derivative from data

• How might fail to exist

• Why differentiability implies local linearity

• How to find numerical derivatives on a calculator

• Differentiability implies continuity

• Motion along a line

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| **STAGE 2****Evidence of Learning** |

**Formative Activities, Tasks, or Projects:**

* Present a graph to the students and show how the slopes of secant lines approach a limit corresponding to the slope of a

 tangent line.

* Discuss the different notations for derivatives.
* Have students discuss what it means for a function to be differentiable or nondifferentiable at a point.

 Give several examples to illustrate corners, cusps, vertical tangents and discontinuities.

* Start with differentiating linear functions and then have students differentiate all polynomial and rational functions.
* Have the students repeat the product and quotient rules out loud together in order for them to memorize the two.
* Discuss position, velocity and acceleration.

 Emphasize that velocity is the rate of change of position and acceleration is the rate of change of velocity.

* Use identities to show the rules for differentiating the other basic trigonometric functions.
* Use a graph of the sine function and examine the slopes at different points to have students come up with the derivative

 graph and for them to realize it is the graph of the cosine function.

**Assessment Topic #2: Key Concept : Differentiation: Definition & Fundamental Properties**

CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Lesson Theme 2.1 : How Do We Measure Speed (2.B)

Focus solely on measuring velocity in this class.

An example is that of something being thrown straight up in the air. The text uses a grapefruit and asks how fast it is moving at different times. When you plot your data (height above ground as a function of time), explain the graph very carefully. Some students will believe that you have drawn the trajectory of the grapefruit—i.e., its path through physical space. They’re (not unreasonably) assuming that the horizontal axis is spatial. Make sure that you convince them that the actual trajectory of the grapefruit is straight up and down, not arch-shaped. (Perhaps use a real grapefruit.) Mark units on the axes and ask if the units are realistic; get students to read from the graph how high the object went, how fast it was thrown, if the height and velocity fit their experience and are consistent with each other.

You should probably work with a table of data without using any formula. I

Compute various average velocities on the board, or have students work them out on their calculators, before getting to the idea of instantaneous velocity. Explain to the students why the slope of the line joining two points on your graph represents the average velocity of the grapefruit for some given interval of time. Emphasize that the secant line segment represents how the grapefruit would be moving if its velocity were constant over some interval, and that the slope represents this constant velocity.

When you begin to discuss instantaneous velocity, make the argument that looking at the function over smaller and smaller intervals reveals an increasingly linear curve, one that resembles more and more its own tangent line. Here, if you have the technology, it might be useful to have students zoom in on various curved functions on their calculators until they look linear.

Be sure to state explicitly that the difference quotient is just a disguised version of the slope formula they’ve always known*,* . Taking the limit is necessary only because the slope being measured is at a point on a curve and not of a straight line.

In a discussion section, it may be fun to spend some time discussing the paradox of the notion of instantaneous velocity.

Day 1: Complete problems 4, 5, 6, 8, 10, 12,13, 14, 15, 17, 19 & 21

Day 2: Complete problems 16, 18, 20, 22, 24, 26, 28, 29, 30 & 32

Lesson Theme 2.2 : The Derivative at a Point (1.E)

 Lots of examples from the text of numerical, graphical, and algebraic derivatives need to be completed. Begin by reviewing the difference quotient and the idea of rate of change of a function. When we define the derivative of at , make sure to emphasize that the definition applies at a point, so that the derivative is just a number. With technology, zoom in on a graph until it becomes straight; the slope of this line is the derivative. Calculate a few derivatives this way.

Make sure the students understand that the derivative is approximated by a difference quotient, and that it represents a slope.

Ask the students for the derivative of at numerically by calculating the difference quotient.

Compute derivatives of other functions numerically.

Hand out photocopies of a graph drawn on graph paper (for example, ) and ask the students for the value of the derivative at various points, or at what points the derivative is −1. Now combine the graphical and numerical approaches: draw the graph of on the whiteboard and ask what you can say about the derivative of at just from the graph. Then compute some difference quotients, with

Using graphing technology, zoom in on at for various values of. Have the students make a conjecture of what the derivative is at .

Finish with some simple algebraic examples. When going through these examples, and when assigning homework, be sure to remind students that derivatives should be found without short cuts, at least for the time being.

In this class, you should consider functions whose rates of change *aren’t* velocities. You should not assume that by the end of the class students completely understand the idea of the derivative.

Day 1: Complete problems 1, 3, 9, 10, 12, 13 & 14 - 18

Day 2: Complete problems 19, 21, 22, 23, 30, 36, 38, 42, 44 & 47

Lesson Theme 2.3 : The Derivative Function (1.D)(4.C)

 Hand out photocopies of a graph (for example, a generic cubic) and ask students to figure out the general shape of its derivative function by sketching small tangent lines, estimating their slope, and plotting them on the graph of . Each time a point is plotted, point out that the derivative graph is above the axis where the function is increasing, below it where the function is decreasing, and crosses it in between, where the function changes direction (from increasing to decreasing or vice-versa). Also point out that the slope is steepest (positively and negatively) where the concavity changes.

Sketch the graph of and its derivative on the same axes, without identifying the functions, and ask the students which function is the derivative of which. The way to see the answer is to observe that exactly one of these functions is always increasing, and exactly one of them is always positive. Then present the graph of a sine curve as the derivative of a mystery function and ask what it tells about the graph of the original function. Next try the same sort of thing with tables of values.

Cover the derivatives of constant and linear functions and then find the formula for the derivative of quickly; either algebraically, or by making a table of at different values of and looking for the pattern.

Finally talk the students through the power rule for *𝑛* an integer, using the binomial theorem, stating that the power rule works for all real .

Finish by making sure the students know the different notations used to represent derivatives, such as : .

Day 1: Complete problems 1 – 11 odds & 14 – 22 even

Day 2: Complete problems 28, 33, 35, 38, 40, 41, 43, 44-47, 49 & 50

Lesson Theme 2.4 : Interpretations of the Derivative (1.D) (3.E)

Open with the example: If is the fuel efficiency of a car going at *𝑣* miles per hour, what is the practical meaning of the statement ?

Go over some more of the examples, as most of the answers in this section require a written explanation. Upon answering correctly they will show concept of a rate of change using the notation is useful in this section.

Day 1: Complete problems 6-14 evens, 17 & 19

Day 2: Complete problems 26-34 evens, 35, 37, 38, 39 & 50

Lesson Theme 2.5 : The Second Derivative (1.E) (3.E)

Start by explaining why the sign of the second derivative gives the concavity of a function. Let students write out their own explanation of what the second derivative tells them about concavity. Mention that a straight line is neither concave up nor concave down and has second derivative zero.

Draw two possible position-versus-time graphs of an accelerating car: both increasing and starting from zero, but one concave up and one concave down and determine which is correct. Provide a table of velocity as a function of time, and have students find the average change in velocity over given intervals.

Complete problems 2, 4, 10, 12, 14, 15-25 odds, 26-29 & 38

Lesson Theme 2.6 : Differentiability (3.E)

Draw a graph of the function on the board and ask students whether it is continuous and/or differentiable at and why. Next, write the definition of differentiability (in terms of the limit of the difference quotient) on the board and note the graphical clues that indicate when a function fails to be differentiable at a point: (a) the function is not continuous at the point, (b) the graph has a sharp corner at that point, (c) the graph has a vertical tangent line at that point.

To check students’ understanding of the relationship between differentiability and continuity, use a graphical problem and make sure to ask students to explain their answers. Then use a piecewise-defined function for exploring differentiability and continuity.

Complete problems 1-7 odds, 8, 9, 11, 12, 14, 16 & 17

Lesson Theme: Chapter 2 Review

Complete problems 3-9 odds, 14-24 evens, 31, 33, 34, 37, 38, 41, 42 & 45

Lesson Theme 3.1 : Powers & Polynomials (1.E)

Whenever you compute a derivative from a formula by using the definition, draw a graph first and ask what the answer ought to look like before computing it. Since derivatives of power functions have been covered, we can show how to get the derivative of a polynomial as an application of the rules for sums and multiples, and then say that these rules work for any functions, not just power functions.

Day 1: Complete problems 5-55 multiples of 5, 57-71 odd

Day 2: Complete problems 73-81 odds, 83 ,84, 86, 87, 92-96

Lesson Theme 3.2 : The Exponential Function (1.E)

Start by drawing the graph of an exponential function and pointing out that its derivative shows much the same behavior as the function: it starts out small and gets larger and larger faster and faster.

The justification that the derivative is a constant times the original function is well worth the time and effort.

Practice many problems rewriting the rules if possible.

Day 1: Complete problems 1-27 odd, 40

Day 2: Complete problems 31, 33, 34, 36, 41-45

Lesson Theme 3.3 : The Product & Quotient Rules (1.C)(1.D)(1.E)

A large part of this class should be spent going over examples to make sure that the students become

proficient at using these rules. However, .

Start by reminding students of the differentiation rules for sums and differences:
 &

Then, ask them to predict a similar formula for .

To illustrate that derivatives of products do not work this way, show them the following two calculations:

 & 

Therefore, .

Give the formulas for the product rule and

quotient rule , then have the students have a lot of practice in order to use these rules correctly. Work through a few examples together as a class, and then let students work on problems together in small groups. Use the following examples for group work:

1. Find if

2. Find if *.*

3. Let be a twice differentiable function. If *,* find a formula for ’ and ’’ in terms of *.*

4. Find the derivative of each of the following functions. You may assume that *𝑎, 𝑏, 𝑐* and *𝑑* are constants. Make sure to first list the appropriate derivative rule that is needed to solve the problem (Quotient or product rule).

(a)

(b)

5. Suppose that , , , and ,

(a) Calculate *,* where .

(b) Calculate *,* where *.*
Then as a preliminary exercise for the product rule derive the product rule in a particular case:

 e.g.,  by writing down the local linearizations near *𝑥* = 0:

 &

Multiply them together to see

and the derivative of the right-hand side is 3 at . Therefore, the derivative of

should be approximately 3 at *𝑥* = 0. You can then use a numerical derivative program to calculate

that ,so this shows that .

Day 1: Complete problems 1-29 odds, 31, 32

Day 2: Complete problems 34-39, 40, 42, 43-53 odds, 84-87

Lesson Theme: Chapter 3 Review (1.C)

Upon reviewing for the test complete an activity in which students are presented with various functions described analytically and are asked to state which derivative rule or rules are needed to find their derivatives (Product rule, quotient rule, or power rule). They then will complete numerous examples of using these rules.

Complete 3-42 multiples of 3

AP Examination Preparation questions CR2 CR3 CR4 CR5 CR6 CR7 CR8

**Summative Activities, Tasks, or Projects:**

Do Now problems CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Unit 2 test CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Wiley course resources on Canvas CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Chapter 02 Additional problems

Example 2.1.1 Video: Finding average velocity from a table

Example 2.1.4 Video: Using algebra to find a limit of a difference quotient

Example 2.2.4 Video: Estimating graphically and numerically

Example 2.2.5 Video: Approximating an equation of a tangent line

Interactive Exploration 2.2: Visualizing the Derivative: Slope of Curve and Slope of Tangent

Example 2.3.2 Video: Sketching a graph of the derivative function

Example 2.3.3 Video: Finding a table of derivative values

Example 2.4.1 Video: A practical interpretation of

Example 2.5.3 Video: Interpreting the first and second derivative for a logistic curve

Example 2.5.5 Video: Describing movement and acceleration of a particle moving in a line

Example 2.6.2 Video: Investigating the differentiability of a function at a point

Example 2.6.3 Video: Determining if a piecewise defined function is continuous and differentiable

Interactive Exploration 2.6: Visualizing Differentiable Functions

Chapter 03 Additional problems

Example 3.1.4 Video: Using the power rule to differentiate functions

Example 3.1.7 Video: Finding and using a tangent line approximation

Interactive Exploration 3.1: Derivative of a Constant Times a Function

Example 3.2.1 Video: Differentiating an exponential function

Example 3.2.2 Video: Finding and interpreting a derivative of an exponential function

Example 3.3.1 Video: Using the product rule

Example 3.3.2 Video: Using the quotient rule

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| **STAGE 3****Learning Plan** |

**Assessment Topics and Lesson Themes:**

**Assessment Topic #2: Key Concept : Differentiation: Definition & Fundamental Properties**

CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Lesson Theme 2.1 : How Do We Measure Speed

Lesson Theme 2.2 : The Derivative at a Point

Lesson Theme 2.3 : The Derivative Function

Lesson Theme 2.4 : Interpretations of the Derivative

Lesson Theme 2.5 : The Second Derivative

Lesson Theme 2.6 : Differentiability

Lesson Theme: Chapter 2 Review

Lesson Theme 3.1 : Powers & Polynomials

Lesson Theme 3.2 : The Exponential Function

Lesson Theme 3.3 : The Product & Quotient Rules

Lesson Theme: Chapter 3 Review

**Unit Specific Accommodations and Modifications**

* Have students check their numerator in the difference quotient for errors with evaluating and simplifying.
* Stress that a piecewise function must be checked for continuity before checking the differentiability at a point.
* Have students repeat the quotient rule so that errors are not made with the order of the numerator.
* Discuss the difference between speed and velocity.
* In order to avoid mistakes when finding derivatives of trigonometric functions, make sure to review their reciprocals.
* Also review Pythagorean Theorem, angle sum identities and half angle identities.

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| **Additional Materials** |

**Digital Tools/Resources:**

TI-84 Plus Graphing Calculator provided to each student by the district

Wiley course resources on Canvas

http://apcentral.collegeboard.com

Khan Academy

Edmentum Exact Path

**Additional Primary & Secondary Resources:**

Multiple Choice & Free-Response questions in preparation for the AP Calculus (AB) Examination

LPS Adopted Textbooks and Programs

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* Cengage: Calculus – Early Transcendental Functions CR1

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| **Unit # 3****14 Days****Differentition: Composite, Implicit, & Inverse Functions****(Weight on Exam 9 – 13%)** |
| **Overview** |

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| **STAGE 1****Desired Results** |

**Unit Overview**

You’ll master using the chain rule, develop new differentiation techniques, and be introduced to higher-order derivatives. Topics include the chain rule for differentiating composite functions, implicit differentiation, differentiation of general and particular inverse functions & determining higher-order derivatives of functions. CR2 CR3 CR4 CR5 CR6

**Big Ideas**

Big Idea 3. Analysis of Functions (FUN) CR2

**Essential Questions…** CR2 CR3 CR4 CR5 CR6

A. How do you derive a composite function?

B. What are the derivatives of sine, cosine and tangent?

C. What are the derivatives of & .

D. What is the procedures to derive an inverse general function or an inverse trigonometric function?

E. What is an implicit function, and how is it derived?

**Enduring Understanding…**

Students will be able to understand the relationship between functions and their derivatives as well as apply basic rules to find derivatives of functions.

EU 2.1: The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies. CR3

EU 2.2: A function’s derivative, which is itself a function, can be used to understand the behavior of the function. CR3

EU 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change. CR3

­FUN-3 : Recognizing opportunities to apply derivative rules can simplify differentiation.CR4

**Students will be able to...**

• Differentiate composite functions using the Chain Rule

• Find derivatives using implicit differentiation

• Find derivatives using the Power Rule for Rational Powers of

• Calculate derivatives of functions involving the inverse trigonometric functions

• Calculate derivatives of exponential and logarithmic functions

**Students will know...**

• The derivatives of the following:

* 1. inverse functions
	2. composite functions
	3. arcsine
	4. arccosine
	5. arctangent
	6. arccotangent
	7. arcsecant
	8. arccosecant
	9.

• “Outside-inside” rule

• Repeated use of the Chain Rule

• Power Chain Rule

• Implicitly defined functions

• Lenses, tangents and normal Lines

• Derivatives of Higher Order

• Rational powers of differentiable functions

• Power Rule of arbitrary real powers

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| **STAGE 2****Evidence of Learning** |

**Formative Activities, Tasks, or Projects:**

* Use the “outside-inside” rule in order for students to use the Chain Rule automatic.
* Use a graphing utility to start a discussion about graphs of the form in order to understand the concept of implicit differentiation.
* Introduce derivatives of inverse functions by discussing the relationship between the slopes of linear functions and their inverses.
* Review the properties of logarithms before learning the derivatives of logs and exponentials and using log differentiation.

**Assessment Topic #3: Key Concept : Differentition: Composite, Implicit, & Inverse Functions** CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Lesson Theme 3.4 : The Chain Rule (1.C)(1.E)(2.C)(2.D)(2.E)

Start by recalling some of the types of functions that students can already differentiate using previous differentiation rules: power functions, exponential functions, and sums, differences, products and quotients of power and exponential functions. Point out that one type of operation that enables us to build a vast variety of functions is function composition. The Chain Rule is therefore a major tool, since it allows us to take the derivative of function compositions. As a simple illustration of the Chain Rule, we will start with the function and discuss the problem of finding *.* We will show them that if you let (the “inside" function), then the original function becomes *.* We will point out that and are easy to compute

using previous differentiation rules, and then show them the Chain Rule formula : which is easy to remember if we imagine canceling the two }}*𝑑𝑢*ε terms on the right hand side.

Therefore, our answer becomes  *.*

Next, we will go through a similar calculation by finding if .

Now is a good time to come up with a verbal description of what is happening when we apply the Chain Rule.

To illustrate this, go back to the first example in which we found that  *.* Emphasize that we actually calculated the derivative of where is the “inside" function that we called *,* and is the “outside" function. In words, the result of the calculation was the derivative of the outside function, evaluated at the inside function, times the derivative of the inside function, which illustrates the following alternate formula for the Chain Rule: .

Carefully work through several examples together as a class and try to give students plenty of time to attempt problems in groups. Below is a list of problems that will serve as good class examples or exercises:

1. For each of the following functions, identify which are composite functions, then apply the chain rule to differentiate them manually.

(a)

(b)

(c)

(d)

(e)

2. Find if .

3. Suppose that and are differentiable functions with the values given in the table below:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| 2 | 7 | 4 | 3 | 6 |
| 4 | 2 | −1 | 5 | 8 |

Find if

Alternatively, investigate how to take the derivative of the composition of two functions considering the following problem: Let , , and . Using the fact that and , ask students to make a conjecture about Construct a table to approximate numerically by taking for etc*.*

Finally have the students graph both the original equation and the derivative to show a representation of the chain rule numerically, analytically & graphically on their graphing calculator.

Day 1: Complete problems 1-35 odds

Day 2: Complete problems 37-57 odds, 58, 59, 63-73 odds

Day 3: Complete problems 75, 77, 78, 79-83 odds, 84-87

Lesson Theme 3.5 : The Trigonometric Functions (1.E)

Graphically show that the derivative formulas for sine, cosine, and tangent are reasonable. Start the class by drawing a graph of and asking what its derivative ought to look like: positive where is increasing, negative where is decreasing, repeating with the same period as . This produces a cosine-like graph. Alternatively, graph and then superimpose the graph of .

Compute the derivative of using the quotient rule. Note that this derivative is given as since we did not introduce the secant. Then do lots of examples involving the chain rule, as well as some with the product and quotient rules.

Next, on their graphing calculator, graph the function and its derivative for and using the numerical differentiation routine on a calculator. Now consider the function. The product rule and the trigonometric identity can be used to derive the formula .

Graph the function along with its derivative for and .

Next consider the function for and . From the graph, it is apparent that the function changes ever faster as *𝑥* increases.

Answer the questions. What about its derivative? The faster that the function changes, the larger the size of the derivative, so the derivative is an oscillatory function whose amplitude grows with *𝑥*.

The class can be wrapped up by graphing and observing how closely it matches the numerical derivative.

Use a graphing calculator to explore how changing periods or amplitudes of the trigonometric functions effect the graphs of said trigonometric functions. Also, how the trigonometric functions derivatives relate to the original trigonometric functions.

Day 1: Complete problems 3-31 odds

Day 2: Complete problems 33 - 59 odds

Day 3: Complete problems 60-71

Lesson Theme 3.6 : The Chain Rule & Inverse Functions (1.C) (1.E)

The derivative will be obtained from the chain rule. It is worthwhile to do some of the calculations in more than one way. For example, if , then can be found using the chain rule, but it can also be found using the product rule: & . Also &

In the derivation of ), use is made of the identity to simplify the expression .

Day 1: Complete problems 1-21 odds, 42-45

Day 2: Complete problems 23-41 odds, 47-51, 63

Day 3: Complete problems 53-61 odds, 65-72

Lesson Theme 3.7 : Implicit Functions (1.E)

First define an implicit function, as many students do not know the difference between implicit and explicit functions. Show the class that many implicit functions cannot be solved for either variable in terms of the other. Then go over some examples in class; the main point to keep in mind is that we are only using the chain rule. Where possible, draw graphs of the functions and their derivatives and check that the derivatives make sense graphically. Use an example like . Convince students that they cannot solve for (although they can solve for ). Show them a graph of the function and admit that it isn’t the graph of a function in the usual sense, since it fails the vertical line test. Point out that the graph does have a tangent line and a slope at (almost) every point. Express that the purpose of implicit differentiation is to find this slope.

Emphasize that you are using the product rule and the chain rule, since is (implicitly) a function of . It can be helpful to write instead of *𝑦*. Clearly express the rules of deriving in terms of , grouping the pieces with together and factoring out , and then solving for .

Day 1: Complete problems 1-17 odds

Day 2: Complete problems 19-37 odds

Lesson Theme : Chapter 3 Review

Day 1: Complete problems 51-72, multiples of 3

Day 2: Complete problems 75-91 odds, 105-110, 111, 112

AP Examination Preparation questions CR2 CR3 CR4 CR5 CR6 CR7 CR8

**Summative Activities, Tasks, or Projects:**

Do Now problems CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Unit 3 test CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Wiley course resources on Canvas CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Chapter 03 Additional problems

Example 3.4.1 Video: Using chain rule in an applied problem

Example 3.4.6 Video: Using product and chain rules to differentiate a quotient

Example 3.5.5 Video: Differentiating functions involving

Example 3.5.6 Video: An applied problem using cosine and chain rule

Example 3.6.2 Video: Differentiating inverse trig functions

Example 3.6.3 Video: Finding derivatives and tangent lines of inverse functions

Example 3.7.1 Video: Finding a tangent line for a graph of an implicit equation

Example 3.7.2 Video: Finding points where a tangent line is horizontal or vertical

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| **STAGE 3****Learning Plan** |

**Assessment Topics and Lesson Themes:**

**Assessment Topic #3: Key Concept : Differentition: Composite, Implicit, & Inverse Functions** CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Lesson Theme 3.4 : The Chain Rule

Lesson Theme 3.5 : The Trigonometric Functions

Lesson Theme 3.6 : The Chain Rule & Inverse Functions

Lesson Theme 3.7 : Implicit Functions

Lesson Theme : Chapter 3 Review

**Unit Specific Accommodations and Modifications**

* When applying the outside-inside rule to differentiate a common mistake is to omit the “derivative of the inside”.
* Need to stress the importance of using the Product Rule of within an implicit differentiation problem.
* Point out that the Power Rule has only been approved for rational powers. Until now when they learn the Power Rule for arbitrary real powers.
* Students are required to memorize all of the derivatives for the trigonometric functions.

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| **Additional Materials** |

**Digital Tools/Resources:**

TI-84 Plus Graphing Calculator provided to each student by the district

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Khan Academy

Edmentum Exact Path

**Additional Primary & Secondary Resources:**

Multiple Choice & Free-Response questions in preparation for the AP Calculus (AB) Examination

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| **Unit # 4****14 Days****Contexual Applications of Differentiation****(Weight on Exam 10-15%)** |
| **Overview** |

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| **STAGE 1****Desired Results** |

**Unit Overview**

You’ll apply derivatives to set up and solve real-world problems involving instantaneous rates of change and use mathematical reasoning to determine limits of certain indeterminate forms. Topics include identifying relevant mathematical information in verbal representations of real-world problems involving rates of change, applying understandings of differentiation to problems involving motion, generalizing understandings of motion problems to other situations involving rates of change, solving related rates problems, local linearity and approximation, & L’Hospital’s rule. CR2 CR3 CR4 CR5 CR6

**Big Ideas**

Big Idea 1 : Change (CHA) CR22

Big Idea 2 : Limits (LIM) CR2

**Essential Questions…** CR2 CR3 CR4 CR5 CR6

A. What are the first and second derivatives of functions use to interpretate?

B. How are the critical points of a function including maxima and minima of a function detected?

C. How are the inflection points of a function including concavity of a function detected?

D. How ate the upper and lower bounds of a function determined using optimization.

E. How do you calculate rates of change using derivatives in various situations.

**Enduring Understanding…**

How functions change gives key information about the function and related real world phenomena.

EU 2.1: The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies. CR3

EU 2.2: A function’s derivative, which is itself a function, can be used to understand the behavior of the function. CR3

EU 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change. CR3

CHA-3 : Derivatives allow us to solve real world problems involving rates of change. .CR3 CR8

LIM-4: L’Hospitals rule allows us to determine the limits of some indeterminate forms. CR3

**Students will be able to...**

* Determine the local or global extreme values of a function
* Determine the applicability of the Extreme Value Theorem
* Apply the Mean Value Theorem
* Find the intervals on which a function is increasing or decreasing
* Use the first and second derivative sign tests to determine the local extreme values of a function
* Determine the concavity of a function and locate the points of inflection by analyzing the second derivative
* Use the graph of using information
* Solve application problems involving finding maximum or minimum values of functions
* Find linearization and use Newton’s method to approximate the zeros of a function
* Solve related rate problems
* Utilize a calculator to approximate zeros of a function
* Use a graphing calculator to compare the graphs of ’ &

**Students will know...**

* Definitions for the following terms:
1. absolute(global) extreme values
2. local(relative) extreme values
3. increasing and decreasing functions
4. first derivative test for local extrema
5. concavity
6. points of inflection
7. second derivative test for local extrema
8. learning about functions from derivatives
9. linear approximation
* Mean Value Theorem
* learning about functions from derivatives
* linear approximation
* Newton’s Method
* Related Rate Equations

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| **STAGE 2****Evidence of Learning** |

**Formative Activities, Tasks, or Projects:**

* Sketch an arbitrary function and discuss the local and global minima and maxima of the function.
* Emphasize the terminology so that students truly understand the language of calculus.
* Complete a simple application of the Mean Value Theorem in order to introduce the lesson.
* Discuss graphs of functions, first derivative and second derivatives for students to be able to understand the connection and be able to analyze the graphs.
* Use *Desmos* to have students analyze and match graphs of derivatives.
* Have students suggest situations where a maximum or minimum need to be found.
* Stress the six-step “Strategy for Solving Max-Min Problems”.
* Review the Chain Rule and implicit differentiation to help the students solve related rates problems.

**Assessment Topic #4: Key Concept : Contexual Applications of Differentiation** CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Lesson Theme 3.10 : Theorems About Differentiable Functions (3.C)(3.E)

Give students an assessment question that with several functions described analytically on specified domains. The question asks students to determine whether each satisfies the hypotheses of the Mean Value Theorem, Extreme Value Theorem, and the Intermediate Value Theorem, as well as to provide written reasons for their choices. (3.C, 3.E)

Day 1: Complete problems 1-9 odd,10-28 even

Lesson Theme 4.1 : Using First and Second Derivatives (1.E)(2.B)(2.C)(2.E)(3.C)(3.E)

Start the class with some elementary curve before going on to anything more difficult. Sketch a simple cubic equation, and , as well as some functions where you can see quickly and easily how the derivative is behaving, where there is a chance of seeing as a whole the relation between the derivative and the shape of the function.

Illustrate the power of the result that if on an interval, then is increasing there. Explaining this result, then write down on the board the two cubic equations and . Figure out what is always increasing using a graphing program or drawing the graphs, and then answer, how could we have known which it was going to be?

Answer: the derivatives are and , and the quadratic formula tells us that the first has no real roots, the second has two. So, the first function is always increasing, as its derivative is always positive.

Define critical points and use the first derivative test to check for local extrema on one of the functions we picked to begin the class. Notice that a local maximum or minimum can be at a critical point or an endpoint.

Using the same example, discuss concavity, the role of the second derivative, and to introduce and use the second derivative test. Define inflection points and ask students where they would expect to find an inflection point on your example function. Then use the second derivative to find the point(s). Spend some time on simple examples

A particularly good example is ; since it has a maximum of 1 at and is asymptotic to the *𝑥*-axis, it must have at least two inflection points. Ask if it is possible to have a function which approaches a horizontal asymptote from above and stays concave down. Despite all the calculations necessary, keep the discussion primarily focused on graphs.

Next, complete a group activity in which two small pieces of paper are prepared for each of several functions, one that displays the graph of the function and the other that shows a well-labeled sign chart for the first derivative of the function over a relevant interval. Each student gets a card, and students walk around the room, talking to their classmates until they find their match. Then each pair of students writes on the board a verbal description of the graph of the second derivative of the original function and the verbal description of the second derivative (graphical, verbal).

Also include an instructional activity in which students are given a graph of and the graph of . Based on information found in these graphs, students work individually to identify local extrema for and write justifications. Then students compare their work with a neighbor’s, explaining their reasoning to each other, and both make refinements. Finally, the class works together to develop a clear and complete statement of both the first derivative test and the second derivative test based on what they have concluded from these examples. As an assessment of whether the students have learned the necessary concepts, they are given an exam question that asks them to determine whether the second derivative test applies to a variety of functions at specific x-values.

Day 1: Complete problems 1, 3, 4-19

Day 2: Complete problems 28-38 evens, 39-49 odds

Day 3: Complete problems 50-54

Lesson Theme 4.2 : Optimization (1.D) (1.E) (1.F) (2.A) (3.F)

This is a class where all students should try some examples with this graphical example being excellent.

One example that cannot be done by a template but requires some thought is the following:

Ex. Find the absolute maximum and minimum of on the interval .
Solution: For the given function the derivative is of .

Thus, the only critical value is . The interval we are investigating only has one endpoint . Checking the function values at the critical value and the endpoint we see that and. However, since we are investigating the behavior of the function on a half open interval, the extreme value theorem does not apply and we have to graph the function to draw any conclusions. Note that as . Thus, we can describe the graph of the function on the interval The graph of the function starts at the origin where the function attains its absolute minimum, it increases until *𝑥* reaches its critical value at . where attains its absolute maximum of. After passing the critical value, is monotonically decreasing and the graph approaches the *𝑥*-axis. Since is continuous and has no more critical values we know that the graph will not change directions again.

Since the emphasis is on problem solving, we should work the same problem in different ways in class.

Use a graphing calculator to compute the definite integral based on the constraints of the graph.

Day 1: Complete problems 1, 2, 5-19 odds, 20, 22, 24, 26

Day 2: Complete problems 28-33, 36-39

Lesson Theme 4.6 : Rates and Related Rates (1.D)(1.E)(3.E)(3.F)(3.G)(4.A)(4.C)

Start with a do now assignment in which students are given various real-world situations described in words that they must translate into equivalent differential equations. Each student then picks one of these situations and writes a sentence on how they translated the description into a differential equation; the teacher grades these sentences for the use of appropriate mathematical language.

Then students will try some real-life examples together with one example being the following.

Students should draw pictures, give variable names to quantities that vary with time, and to explicitly write down what they know and what they are trying to find before trying to solve the problem. Also, we should give hints along the way and leave time to go over the problems together as a class after students have had a chance to try them.

Ex. 1 : Warmup with a classroom activity where students are asked to find the cylinder of maximum volume that can be inscribed in a cone of given dimensions. They first decide on the appropriate procedure (optimization, related rates, etc.) and then use algebraic rules and procedures for differentiation to find the answer.

Ex. 2 : A spherical balloon of radius *𝑟* centimeters has a volume given by *.*

1. (a)  Find when *𝑟* = 1 and when *𝑟* = 2 and give a practical interpretation of your answers.
2. (b)  Suppose that the balloon is being inflated in such a way that centimeters after*𝑡* seconds.

How fast is the volume of the balloon increasing when *𝑟* = 1? when *𝑟* = 2?

1. (c)  Air is begin blown into the balloon at a constant rate of 50 cubic centimeters per second.

 How fast is the radius of the balloon increasing when and when ?

Solution

In part (a) of the above problem, encourage the students to think about the relevant units. The quantity is the rate of change of the volume of the balloon with respect to its radius. Therefore, the number indicates that when the radius of the balloon is 1 centimeter, the volume of the balloon increases by 12.566 cubic centimeters for each centimeter increase in the radius. The number has a similar interpretation.

Before going through the solution of part (b) to the above problem, ask the students what they are being asked to calculate. Is part (b) asking for or some other derivative? Hopefully, the key words “how fast" will indicate to students that this part of the problem is asking for  *,* the rate at which the volume of the balloon is increasing with respect to time. Calculating, we find that

 so that .

Now, when *𝑟* = 1 cm and *𝑟* = 2 cm, we find that seconds and *𝑡* = 1 second, respectively, so we see that the volume of the balloon is increasing at a rate of 25.133 cubic centimeters per second when *𝑟* = 1 cm and increasing at a rate of 100.531 cubic centimeters per second when *𝑟* = 2 cm. Once again, emphasize the importance of the units in the answers.

Finally, for part (c), begin by asking students what is given and what they are being asked to find. Emphasize that even though in part (c) we cannot find an explicit formula in terms of *𝑡,* the differentiation that we do proceeds in exactly the same way as in (b); we simply remember that *𝑉* and *𝑟* are unknown functions of *𝑡* and go from there: .

Many students find the idea of differentiating with respect to *𝑡* confusing because *𝑡* doesn’t appear in the original equation

; they may ask where the comes from in equation above. If this happens, remind them that *𝑉* and *𝑟* are functions of *𝑡* and point out the similarities between the two equations. It is easier for some students to grasp this calculation and recognize it as an application of the Chain Rule if the starting equation is first rewritten as and differentiated into the form *.* At any rate, once the derivative has been taken, most students are comfortable substituting in the given information to obtain centimeters per second when cm and centimeters per second when cm. Be sure to insist on the inclusion of the proper units with each answer!

As a follow-up to part (c) of the preceding problem, ask students why is smaller when r=2than when *.* Point out that this result occurs even though *,* the rate at which the balloon is being inflated, is the same in both calculations. Blowing up a balloon in front of the class and letting students observe for themselves that the radius increases at a slower rate as the balloon gets larger makes an excellent demonstration.

Afterwards talk with the students how they came up with their answer& what was the reason they drew their conclusion. Also discuss what the formulas mean mathematically and show how they could check their answers to make sure they are accurate and appropriate.

Complete a homework activity in which students are assigned different released free-response questions involving related rates in different contexts. After answering the questions for their problem, they compare their problem and solution in groups of three the following day and write a summary of what was the same and what was different about the three problems.

Day 1: Complete problems 1-5, 7, 9-15

Day 2: Complete problems 16-26

Day 3: Complete problems 27, 28, 29, 30, 33, 34, 35, 36, 42, 43, 46, 50

Lesson Theme : Chapter 4 Review

Day 1: Complete problems 1-9 odds, 13-23 odds,

Day 2: Complete problems 56, 58, 59, 68, 70, 71, 73, 81

AP Examination Preparation questions CR2 CR3 CR4 CR5 CR6 CR7 CR8

**Summative Activities, Tasks, or Projects:**

Do Now problems CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Unit 4 test CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Wiley course resources on Canvas CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Chapter 04 Additional problems

Example 4.1.2 Video: Finding local maxima and minima analytically

Example 4.1.5 Video: Finding local extrema and inflection points to graph a function

Interactive Exploration 4.1: Studying a function using its first and second derivatives

Example 4.2.2 Video: Finding the global maximum and global minimum

Example 4.2.5 Video: A practical application of finding a global maximum

Interactive Exploration 4.2: Extrema

Example 4.6.3 Video: Finding the rate of a melting snowball

Example 4.6.6 Video: Finding how fast a camera rotates to view an airplane

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| **STAGE 3****Learning Plan** |

**Assessment Topics and Lesson Themes:**

**Assessment Topic #4: Key Concept : Contexual Applications of Differentiation** CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Lesson Theme 3.10 : Theorems About Differentiable Functions

Lesson Theme 4.1 : Using First and Second Derivatives

Lesson Theme 4.2 : Optimization

Lesson Theme 4.6 : Rates and Related Rates

Lesson Theme : Chapter 4 Review

**Unit Specific Accommodations and Modifications**

* Students should be exposed to examples of critical points that are not local extreme values so that the assumption that a critical point is always an extreme value is not made.
* Make sure students understand that does not guarantee that f has a local extremum atas well as does not guarantee a point of inflection.
* Students may overlook endpoints as possible candidates for optimal values.
* For Newton’s Method, students may stop after finding one zero or may not choose appropriate values for the initial guess x1.
* Emphasize that evaluation is the final step in the six-step strategy for Solving Related Rates Problems.

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| **Additional Materials** |

**Digital Tools/Resources:**

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**Additional Primary & Secondary Resources:**

Multiple Choice & Free-Response questions in preparation for the AP Calculus (AB) Examination

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| **Unit # 5****14 Days****Analytical Applications of Differentiation****(Weight on Exam 15-18%)** |
| **Overview** |

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| **STAGE 1****Desired Results** |

**Unit Overview**

After exploring relationships among the graphs of a function and its derivatives, you'll learn to apply calculus to solve optimization problems. Topics include the Mean Value Theorem and Extreme Value Theorem, derivatives and properties of functions, how to use the first derivative test, second derivative test, and candidates test, sketching graphs of functions and their derivatives, how to solve optimization problems& behaviors of Implicit relations. CR2 CR3 CR4 CR5 CR6

**Big Ideas**

Big Idea 3. Analysis of Functions (FUN) CR2

**Essential Questions…** CR2 CR3 CR4 CR5 CR6

A. How is the average rate of change of a function on an interval related to the instantaneous rate of change at a point in that interval?

B. Can you use mathematical modeling to translate a real- life problem into a function with a known formula in order to determine global maxima or minima.

C. What does changing the parameters of families of functions do to help solve modeling problems.

**Enduring Understanding…**

How functions change gives key information about the function and related real-world phenomena.

EU 2.1: The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies. CR3

EU 2.2: A function’s derivative, which is itself a function, can be used to understand the behavior of the function. CR3

EU 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change. CR3

FUN-1 : Existence theorems allow us to draw conclusions about a functions behavior on an interval without precisely locating that behavior. CR22CR4

FUN-4: A function’s derivative can be used to understand some behaviors of the function. CR2

**Students will be able to...**

* Apply the Mean Value Theorem
* Understand the increasing function theorem
* Apply the constant function theorem
* Find global extrema (maximum and minimums)
* Determine the upper and lower bounds of a function
* Optimize and model a graph from a word problem
* Create bell-shaped curves
* Determine how motion under gravity operates
* Solve exponential models using limits
* Use families of functions and modeling to complete a logistic model
* Use a graphing calculator to check the graphs of functions they created

**Students will know...**

• The definitions for the following terms:

 a. local extrema

 b. global extrema

 c. the racetrack principle

 d. global maximum

 e. global minimum

 f. upper and lower bounds

 g. optimization

 h. modeling

• Mean Value Theorem

• Extreme Value Theorem

• Global maximum and minimums on intervals

• How to find upper & lower bounds

• Optimization & modelling process

• The bell-shaped curve

• Modeling with families of functions

• Exponential model with a limit

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| **STAGE 2****Evidence of Learning** |

**Formative Activities, Tasks, or Projects:**

* Sketch an arbitrary function and discuss the local and global minima and maxima of the function.
* Emphasize the terminology so that students truly understand the language of calculus.
* Complete a simple application of the Mean Value Theorem in order to introduce the lesson.
* Discuss graphs of functions, first derivative and second derivatives for students to be able to understand the connection and be able to analyze the graphs.
* Use *Desmos* to have students analyze and match graphs of derivatives.
* Have students suggest situations where a maximum or minimum need to be found.
* Stress the six-step “Strategy for Solving Maximum/Minimum Problems”.
* Review the Chain Rule and implicit differentiation to help the students solve related rates problems.

**Assessment Topic #5: Key Concept : Analytical Applications of Differentiation** CR1 CR2 CR3 CR4 CR7 CR8

Lesson Theme 3.10 : Theorems About Differentiable Functions (2.D) (2.E) (3.D) (3.E)

For the Mean Value Theorem the picture that illustrates its statement will be stressed.

Complete an activity where the students are asked to decide whether the fact that a car covers a certain distance at an average velocity of 60 mph necessarily implies that at some point in time the car’s velocity was *exactly* 60 mph.

To justify the two hypotheses of the Mean Value Theorem, they will need to use :

 and

Have the students check if the hypotheses of the theorem are satisfied.

The Mean Value Theorem relates a local property (derivative at a point) to a global property (average rate of change) of a function which satisfies the requirements of the theorem. The Racetrack Principle will also be illustrated with pictures. Its proof is a direct application of the Increasing Function Theorem.

Students will have to justify their answers and explain the mathematical solution in context afterwards.

Day 1: Complete problems 6-11, 30, 31, 33-37

Lesson Theme 4.2 : Optimization (1.D) (1.E) (1.F) (2.A) (3.F)

Covered in previous lesson.

Day 1: Review problems from previous lesson

Lesson Theme 4.3 : Optimization and Modeling (1.D) (1.E) (1.F) (2.A) (3.F)
The focus will solely be on specific examples such as how to select optimal dimensions to minimize metal for a can of given volume. Go over where to begin this sort of question and remind them that the quantity to be optimized should be given as a function of quantities they can vary. Decide what the relevant variables are & discuss the importance of constraints here. Bring in a real soda can and conject why it is taller than predicted. Next try a problem on subsection on gasoline consumption

Encourage students to write down as much information as they can about their efforts, even if they can’t give complete solutions.

Base a project in the class on this topic.

Day 1: Complete problems 1-15, 17

Day 2: Complete problems 18, 19, 21-23, 24, 29, 31

Day 3: Complete problems 33, 34, 35, 39, 41, 45, 47, 52

Lesson Theme 4.4 : Families of Functions and Modeling (1.E) (2.D) (3.E)

When looking at families of functions, point out to students that they are already familiar with several important families including linear functions and trigonometric functions. Some of the effects that various parameters have on the graph of a general function (e.g., represents a horizontal shift of while represents a vertical shift). Emphasize that with calculus, families of functions can be studied in greater detail than previously possible.

Use a graphing calculator or computer program which allows you to enter parameterized families and then vary the parameters to see the movements. Break the class into groups and have each group investigate and report back on various families.

Possible examples to use in class are:

2.

3.

Day 1: Complete problems 1, 4, 5, 6, 7, 9, 10, 12, 17, 21

Day 2: Complete problems 22, 23, 25, 26, 27, 29, 31, 32, 33, 36, 37

Day 3: Complete problems 38, 39, 40, 41, 43, 44, 45, 46, 47, 49

Lesson Theme: Chapter 4 Review

Day 1: Complete problems 27, 28, 34, 38, 40, 42, 43

Day 2: Complete problems 48, 49, 50, 56

AP Examination Preparation questions CR2 CR3 CR4 CR5 CR6 CR7 CR8

**Summative Activities, Tasks, or Projects:**

Do Now problems CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Unit 5 test CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Wiley course resources on Canvas CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Example 3.10.1 Video: Using the Racetrack principle to prove an inequality

Chapter 04 Additional problems

Example 4.3.3 Video: Finding a rectangle of maximum area

Example 4.3.4 Video: Finding the maximum volume of a box given a fixed surface area

Example 4.4.2 Video: Finding critical points and inflection points

Example 4.4.5 Video: A two parameter family of functions

Example 4.5.4 Video: Finding maximum profit from cost and revenue graphs

Example 4.5.5 Video: Finding maximum and minimum profit from equations

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| **STAGE 3****Learning Plan** |

**Assessment Topics and Lesson Themes:**

**Assessment Topic #5: Key Concept : Analytical Applications of Differentiation** CR1 CR2 CR3 CR4 CR7 CR8

Lesson Theme 3.10 : Theorems About Differentiable Functions

Lesson Theme 4.2 : Optimization

Lesson Theme 4.3 : Optimization and Modeling

Lesson Theme 4.4 : Families of Functions and Modeling

Lesson Theme : Chapter 4 Review

**Unit Specific Accommodations and Modifications**

* Students should be exposed to examples of critical points that are not local extreme values so that the assumption that a critical point is always an extreme value is not made.
* Make sure students understand that *f’(c*)=0 does not guarantee that f has a local extremum at *(c,f(c))* as well as *f’’(x)*=0 does not guarantee a point of inflection.
* Students may overlook endpoints as possible candidates for optimal values.
* For Newton’s Method, students may stop after finding one zero or may not choose appropriate values for the initial guess x1.
* Emphasize that evaluation is the final step in the six-step strategy for Solving Related Rates Problems.

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| **Additional Materials** |

**Digital Tools/Resources:**

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Khan Academy

Edmentum Exact Path

**Additional Primary & Secondary Resources:**

Multiple Choice & Free-Response questions in preparation for the AP Calculus (AB) Examination

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* Cengage: Calculus – Early Transcendental Functions CR1

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| **Unit # 6****14 Days****Integration and Accumulation of Change****(Weight on Exam 17-20%)** |
| **Overview** |

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| **STAGE 1****Desired Results** |

**Unit Overview**

You’ll learn to apply limits to define definite integrals and how the Fundamental Theorem connects integration and differentiation. You’ll apply properties of integrals and practice useful integration techniques. Topics include using definite integrals to determine accumulated change over an interval, approximating integrals using Riemann Sums, accumulation functions, the Fundamental Theorem of Calculus, and definite integrals, antiderivatives and indefinite integrals & properties of integrals and integration techniques. CR2 CR3 CR4 CR5 CR6

**Big Ideas**

Big Idea 1 : Change (CHA) CR22

Big Idea 2 : Limits (LIM) CR2

Big Idea 3. Analysis of Functions (FUN) CR2

**Essential Questions…** CR2 CR3 CR4 CR5 CR6

A. How are the sums for any function *f* constructed using sigma notation?

B. What is the definite interval and how is it used to determine the area under a curve?

C. In what manner is the fundamental theorem of calculus used to determine how does one interpretate its results.

D. What are the properties of definite intervals and why do its properties hold?

E. How are the families of antiderivatives determined and how can you visualize them using slopes.

F. What is the process used to compute the values of an antiderivative using definite integrals?

G. How are antiderivatives constructed and solved analytically.

H. How is integration by substitution completed using both the guess-check method and definite integration.

**Enduring Understanding…**

How functions change gives key information about the function and related real world phenomena.

EU 3.1: Antidifferentiation is the inverse process of differentiation. CR3

EU 3.2: The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies. CR3

EU 3.3: The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration. CR4

EU 3.4: The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation. CR4

EU 3.5: Antidifferentiation is an underlying concept involved in solving separable differential equations. Solving separable differential equations involves determining a function or relation given its rate of change. CR3

CHA-4 : definite integrals allow us to solve problems involving the accumulation of change over an interval. CR3

LIM-5 : Definite integrals can be approximated using geometric and numerical methods. CR3 CR4

LIM-6 : The use of limits allows us to show that areas of unbounded regions may be finite. CR5

FUN-5: The Fundamental Theorem of Calculus connects differentiation and integration. CR3 CR4

FUN-6: Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration. CR4

**Students will be able to...**

* Approximate the area under a graph of a nonnegative continuous function by using rectangular approximation methods
* Interpret the area under a graph as a net accumulation rate of change
* Express the area under a curve using a definite integral and as a limit of Riemann sums
* Compute the area under a curve using a numerical integration procedure
* Apply rules for definite integrals and find the average value of a function over a closed interval
* Apply the Fundamental Theorem of Calculus.
* Understand the relationship between the derivative and definite integral as expressed in both parts of the Fundamental Theorem of Calculus
* Approximate the definite integral by using the Trapezoidal Rule and by using Simpson’s Rule, and estimate the error in using the Trapezoidal and Simpson’s Rule

**Students will know...**

• The definitions for the following terms:

* 1. rectangular approximation method
	2. volume of a sphere
	3. Riemann Sums
	4. notation of integration
	5. constant functions
	6. discontinuous integrable functions
	7. average value of a function
	8. Mean Value Theorem for definite integrals
	9. Fundamental Theorem: antiderivative part
	10. Fundamental Theorem: evaluation part
	11. trapezoidal approximations
	12. error analysis

• Accumulation problems as area

• Definite integral and area

• Definite integral as an accumulation function

• Integrals on a calculator

• Properties of definite integrals

• Connecting differential and integral calculus

• Graphing the function

• Area connection

• Analyzing antiderivatives graphically

• Analyzing antiderivatives with a graphing calculator

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| **STAGE 2****Evidence of Learning** |

**Formative Activities, Tasks, or Projects:**

* Have students graph a curve and shade the desired interval to visualize the area being sought.

Use either *Desmos* or the graphing calculators.

* Use a RAM program to help students understand what happens as *n* increases and converge to a limit.
* Review sigma notation with students.
* Introduce Riemann sums with nonnegative functions and then introduce negative functions.
* Emphasize the LRAM, RRAM and MRAM are examples of Riemann sums.
* Begin teaching the FTC with letting and .
* Introduction of the Trapezoidal Rule by having students discuss the relative accuracy of LRAM, RRAM and MRAM.

Have the students think of ways to increase the accuracy without increasing the number of intervals used

**Assessment Topic #6: Key Concept : Integration and Accumulation of Change** CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Lesson Theme 5.1 : How Do We Measure Distance Traveled (4.B)

Go over the first example in the Wiley Calculus text. Start with a table of velocities instead of graphs at first. Have students give lower and upper bounds for distance traveled, using what we call left and right hand sums. Demonstrate that more data (intermediate velocities) will make the lower and upper bounds closer together. Have the students answer themselves while getting them to give the initial distance estimates.

After students have given upper and lower estimates, show how they are represented on a graph. Prove that the difference between the left and right sums is making it clear that the left and right sums are converging to a common limit.

Start with monotonic increasing (or decreasing) functions.

Day 1: Complete problems 3, 9, 13, 15, 19, 22, 24, 31, 32, 33, 35, 38, 39

Lesson Theme 5.2 : The Definite Integral (1.D) (1.F)(2.B)(2.C)(2.D)

Introduce notation to the class and complete a couple warm up problems using it without using integration.

Write out the formulas you obtained during the last class for left- and right-hand sums and help students figure out how to express these using the notation. Make sure they understand what the subscript on the variable and means in the expression . Compute a Riemann sum by hand: for example, compute by making a table of values of the function for adding up all values but the last one, multiply by ; similarly for the right sum. For a good estimate, take the average of the left and right sums. Make to follow up with an example where , such as with . Emphasize that the definite integral is a number (the above examples will help).

Continue with more simple examples like .

The area interpretation of the integral follows directly from the idea of summing thinner and thinner rectangles so stress that the definite integral is a number and that area is only one of many interpretations of the integral.

Finish lesson by having students work on an assignment where they are given a function written analytically representing a population in terms of time and asked to graph the function, then to construct a table of values and use the table to estimate the area of the region bounded by the function, the t-axis, t = 2, and t = 6 using midpoint Riemann sums with four subintervals of equal length. Then ask them to discuss how the Riemann sum is connected to the context of the problem.

Day 1: Complete problems 2, 3, 8, 9, 11, 15, 17, 18, 19, 20

Day 2: Complete problems 22, 25, 28, 29, 31, 32, 33, 34, 35, 36

Lesson Theme 5.3 : The Fundamental Theorem and Interpretations (1.D) (2.D) (3.D)

Start the section with a discussion of units and notation for the definite integral that corresponds to the Leibniz notation thatwas introduced for the derivative. It will give meaning to the quantity in the integral which is often confusing to students.

To introduce the Fundamental Theorem of calculus, revisit the example from Section 5.1 that the change in position is the integral of velocity. Then revisit one of the examples we did in that section and work it again with the new definition and notation of the integral. This leads naturally to the Fundamental Theorem of Calculus in the context of change of position as the integral of velocity. From here it is a small step to generalize from velocity to rate of change of a quantity and from change of position to the total change of the quantity and we get that the integral of a rate of change gives total change.

Next, drive the point home that the theorem involves a function *and* its derivative. To illustrate the theorem, write down a function , give the students the formula for its derivative, compute a definite integral of the derivative using Riemann sums, and then compute

To become more comfortable with the Fundamental Theorem, use the area interpretation of the definite integral in several applications. A similar problem to the ones in this section, which usually generates an excellent class discussion, follows.

Example: Below is the graph of the rate *𝑟* (in arrivals per hour) at which patrons arrive at the theater in order to get rush seats for the evening performance. The first people arrive at 8 am and the ticket windows open at 9 am. Suppose that once the windows open, people can be served at an (average) rate of 200 per hour.

Use the graph to find or provide an estimate of:



(a) The length of the line at 9 am when the windows open.

(b) The length of the line at 10 am.
(c) The length of the line at 11 am.
(d) The rate at which the line is growing in length at 10 am.

(e) The time at which the length of the line is maximum.

(f) The length of time a person who arrives at 9 am has to stand in line.

(g) The time at which the line disappears.

Day 1: Complete problems 1-4, 16, 17 ,19, 20

Day 2: Complete problems 21-32, 38-42

Lesson Theme 5.4 : Theorems About Definite Integrals (3.D) (4.C)

This section gives and justifies properties of the definite integral. Many of the justifications are graphical and reinforce the area interpretation of the definite integral. We will choose several discuss in class, for example:

1.

2. If is even then

3. If is odd then

4. If for , then

In each case use pictures to illustrate the property.

Average value is a nice application of the integral that helps visualize the integral as area. The average value of a function is the height of a rectangle with width and whose area is equal to the area under the graph of from *𝑎* to *𝑏*. Another good example that makes sense to students is temperature. Start with a table of temperature readings (high temperature in Linden for each month over a year) and compute the average. Draw a graph with discrete data points and lead the discussion toward how to compute the true average temperature over the entire year by adding more and more temperature readings and finally integrating the temperature function.

Day 1: Complete problems 1-9 odds, 21-26

Day 2: Complete problems 28, 30, 32, 33, 34, 35, 36, 39-43, 49-52

Lesson Theme : Chapter 5 Review (1.C) (1.E)

When reviewing, give problems that require students to identify the appropriate integration techniques to apply to each of several definite integrals, compute the antiderivatives manually, and then check their results using their graphing calculator. In pairs, then have them discuss any of their results that do not match the results when using the technology.

Day 1: Complete problems 2, 3, 5, 6, 7, 10, 14, 15, 17, 18, 22, 23, 26, 27, 28, 32-36

Day 2: Complete problems 37, 38, 41, 46, 49, 50, 52, 53, 54, 55, 57, 58

Lesson Theme 6.1 : Antiderivatives Graphically & Numerically (1.E) (4.C)

Start with a graph of (no formula) and ask students for a possible sketch of . (It’s probably best to choose .) Start with a problem where is piecewise linear, for example a line segment from (0*,* 0) to (1*,* 2) followed by a line segment from (1*,* 2) to (2*,* 0). Have students to draw two antiderivatives and , where and After discussing the students’ answers, have them complete a table of values of for using the Fundamental Theorem of Calculus, and graph the antiderivative more carefully this time. Discuss the concavity of the graph.

Make sure the class remembers that
 on an interval ⇒is increasing there

 on an interval ⇒ is decreasing there.

Thus, if , the graph of will climb until crosses the *𝑥*-axis. To find out how far it climbs, we can estimate the area under the curve using the Fundamental Theorem.

Day 1: Complete problems 1-4, 9-11, 14, 15, 17, 20, 24, 28, 30, 33

Lesson Theme 6.2 : Constructing Antiderivatives Analytically (1.E) (3.D)

Emphasize the distinction between the definite integral (a number computed by limits of sums) and the indefinite integral (the general antiderivative, which happens to be useful in computing definite integrals). Because the notations are so similar this will have to be repeated over & over.

Explain the sum and constant multiple rules as reversals of the corresponding rules for derivatives, then set up the theme of reversing rules of differentiation, with the warning that it won’t be so easy for the chain and product rules.

Spend time using the sum and constant multiple rules to find antiderivatives, e.g.

Examples:

1.

2.

3.

Include some examples where the constant is in the denominator, like the following.

4.

Reminded students that in this case the integrand is just . 

Make sure students can do definite integrals using the Fundamental Theorem like:

5.

6.

Also include examples where the answer can be expressed both symbolically and numerically, e.g.

7.

Last, complete an example that requires thinking about whether the answers mechanically obtained make sense.

8. Determine which is bigger using students’ knowledge of the graphs of the integrands; then calculate the integrals to confirm their choice. or, . Afterwards ask students to guess the order of magnitude of and then calculate the answer.

Day 1: Complete problems 2-40 evens

Day 2: Complete problems 43-75 odds

Day 3: Complete problems 76, 77-93 odds

Lesson Theme : Chapter 6 Review

Day 1: Complete problems 1, 3-45 multiples of 3

Day 2: Complete problems 46, 47, 48, 49, 50, 53, 54

Lesson Theme 7.1 : Integration by Substitution (1.E)
The basic idea behind the substitution method is to find a way of reversing the chain rule.
Start with an easy example such as , introducing it by writing as showing how the obvious guess, , does not work because of the chain rule, and then point out that if the from the chain rule had been there in the first place you could have done the integral. Next attempt use the guess-and-check method to solve , and .Next, introduce the *𝑤*-substitution, & go back and do the same examples you did before. Remind students that this method grew out of looking for the end products of the chain rule; therefore, you are looking for an inside function whose derivative is somewhere outside, and when you have found it, you want to put *𝑤* equal to the inside function. This makes the tricky ones such as easier, since once you have decided to put the rest is mechanical. The substitution does not work if you try Students should also see an example where the outside function and the inside

are hard to recognize like:

1. where we let

2. where .

Encourage your students not to rely too much on the mechanics of substitution. The following examples all can be done without formal substitution, if you point out the patterns to students.:

3.

4.

5.

Always emphasize the reversal of differentiation; they are familiar with a constant coming out the front when they differentiate; so, when it is absent, you first have to divide by the constant.

An interesting example is:

6. with the answer being or (using ) with the two antiderivatives differing only by a constant.
Next, calculate a definite integrals:

7.

8.

in two ways: by evaluating the indefinite integral first (using a substitution) and then by changing the limits of integration when you make the substitution.

Choose other examples, similar to the first one you do, where a substitution does not work, and point out that it must be done numerically.

9.

10. and

 Sometimes it can be helpful to substitute for a complex part of the integrand and see what happens.

Another way of showing the usefulness of the substitution method in transforming integrals is to demonstrate the formula for the area of a circle . Start by expressing the area as an integral .Next suggest that students try the substitution *.*

This has the effect of transforming the integral and giving .

Because we know that our expression for must reduce to ., we know that we must have .
Demonstrate this geometrically, numerically and by using the substitution

Day 1: Complete problems 1-43 odds

Day 2: Complete problems 51-81 odds, 82

Day 3: Complete problems 85-95 odds, 111-121 odds, 135, 136, 137

Day 4: Complete problems 123-135 odds, 147-156

AP Examination Preparation questions CR2 CR3 CR4 CR5 CR6 CR7 CR8

**Summative Activities, Tasks, or Projects:**

Do Now problems CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Unit 6 test CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Wiley course resources on Canvas CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Chapter 05 Additional problems

Example 5.1.2 Video: Finding how often a velocity should be measured

Example 5.1.3 Video: Finding distance as area under a velocity graph

Interactive Exploration 5.1: Riemann Sums

Example 5.2.2 Video: Finding an exact value and an approximation of an integral

Example 5.2.3 Video: Relating a definite integral to area

Example 5.3.2 Video: Using the Fundamental Theorem of Calculus to approximate an increase in population

Example 5.3.3 Video: Using velocity graphs for two cars to determine which car is ahead.

Example 5.4.4 Video: Finding values of integrals for even and odd functions

Example 5.4.6 Video: Interpreting a definite integral

Interactive Exploration 5.4: Sums and Constant Multiples of the Integrand

Chapter 06 Additional problems

Example 6.1.2 Video: Graphing an Antiderivative

Example 6.1.4 Video: Using a graph of f' and the fundamental Theorem to graph f

Interactive Exploration 6.1: Visualizing Antiderivatives Using Slopes 1

Interactive Exploration 6.1: Visualizing Antiderivatives Using Slopes 2

Example 6.2.2 Video: Finding an Antiderivative Analytically

Example 6.2.4 Video: Evaluate a definite integral exactly

Interactive Exploration 6.2: Computing Values of an Antiderivative Using Definite Integrals

Interactive Exploration 6.2: What is the Most General Antiderivative of f?

Chapter 07 Additional problems

Example 7.1.2 Video: Finding an integral using trial and error

Example 7.1.10 Video: Evaluating a definite integral using substitution

Interactive Exploration 7.1: Integration by substitution

Example 7.5.1 Video: Comparing the left, right, and midpoint sums to the exact value of an integral

Example 7.5.2 Video: Comparing the trapezoid rule with the left, right, and midpoint rules

Interactive Exploration 7.5: Comparing Numerical Estimates for Definite Integrals

Example 7.7.1 Video: Using an end behavior argument to determine if an improper integral converges

Example 7.7.2 Video: Using inequalities to determine if an improper integral converges or diverges

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| **STAGE 3****Learning Plan** |

**Assessment Topics and Lesson Themes:**

**Assessment Topic #6: Key Concept : Integration and Accumulation of Change** CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Lesson Theme 5.1 : How Do We Measure Distance Traveled

Lesson Theme 5.2 : The Definite Integral

Lesson Theme 5.3 : The Fundamental Theorem and Interpretations

Lesson Theme 5.4 : Theorems About Definite Integrals

Lesson Theme : Chapter 5 Review

Lesson Theme 6.1 : Antiderivatives Graphically & Numerically

Lesson Theme 6.2 : Constructing Antiderivatives Analytically

Lesson Theme : Chapter 6 Review

Lesson Theme 7.1 : Integration by Substitution

**Unit Specific Accommodations and Modifications**

* Students may assume the MRAM estimate is the average of LRAM and RRAM estimates.

Use an example of a quadratic to show this is not the case.

* Stress the importance of not forgetting the *dx*.
* Algebraic mistakes are often made when finding antiderivatives. Have students get in the habit of differentiating their answer to verify that they found the correct antiderivative.
* Make sure students pay attention to functions that are both above the *x-*axis and below the *x-*axis and to be careful with positive and negative definite integrals.
* Students may assume that the Trapezoidal Rule is equivalent to MRAM. Use a function such as *f(x)=x2* to show how the two rules differ.

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| **Additional Materials** |

**Digital Tools/Resources:**

TI-84 Plus Graphing Calculator provided to each student by the district

Wiley course resources on Canvas

http://apcentral.collegeboard.com

Khan Academy

Edmentum Exact Path

**Additional Primary & Secondary Resources:**

Multiple Choice & Free-Response questions in preparation for the AP Calculus (AB) Examination

LPS Adopted Textbooks and Programs

* Wiley: Calculus – AP Edition CR1
* Cengage: Calculus – Early Transcendental Functions CR1

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| **Unit # 7****14 Days****Differential Equations****(Weight on Exam 6-12%)** |
| **Overview** |

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| **STAGE 1****Desired Results** |

**Unit Overview**

You’ll learn how to solve certain differential equations and apply that knowledge to deepen your understanding of exponential growth and decay. Topics include interpreting verbal descriptions of change as separable differential equations, sketching slope fields and families of solution curves, solving separable differential equations to find general and particular solutions & deriving and applying a model for exponential growth and decay. CR2 CR3 CR4 CR5 CR6

**Big Ideas**

Big Idea 3. Analysis of Functions (FUN) CR2

**Essential Questions…** CR2 CR3 CR4 CR5 CR6 CR88

A. How can we pick one solution to the equation ?

B. What is a differential equation?

C. What are slope fields, and what are they used for?

D. How does separating variables lead to solving differential equations?

E. What are they types of growth and decay situations that calculus could be used to compute?

F. Where do the theoretical functions come from that allow us to solve real world problems?

**Enduring Understanding…**

How functions change gives key information about the function and related real-world phenomena. CR88

EU 3.1: Antidifferentiation is the inverse process of differentiation. CR3

EU 3.2: The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies. CR3

EU 3.3: The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration. CR3

EU 3.4: The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation. CR3

EU 3.5: Antidifferentiation is an underlying concept involved in solving separable differential equations. Solving separable differential equations involves determining a function or relation given its rate of change. CR3

­FUN-7: Solving differential equations allows us to determine functions and develop models. CR3

**Students will be able to...**

* Solve initial value problems in the form , )
* Be able to construct slope fields using technology and interpret slope fields as visualizations of differential equations
* Use Euler’s Method for graphing a solution to an initial value problem
* Compute indefinite and definite integrals by the method of substitution
* Use integration by parts to evaluate indefinite and definite integrals
* Use tabular integration or the method of solving for the unknown integral in order to evaluate integrals that require repeated use of integration by parts
* Use integration by parts to integrate inverse trigonometric and logarithmic functions
* Solve separable differential equations
* Solve problems involving exponential growth and decay in a variety of applications
* Solve logistic differential equations using the technique of partial fractions and by the general formula.
* Solve problems involving exponential or logistic population growth

**Students will know...**

• The definitions for the following terms:

* 1. differential equations
	2. slope fields
	3. Euler’s Method
	4. indefinite integrals
	5. product rule in integral form
	6. law of exponential change
	7. separable differential equations
	8. continuously compounded interest
	9. modeling growth with other bases
	10. partial fractions
	11. the logistic differential equation

• Leibniz notation and antiderivatives

• Substitution in indefinite integrals

• Substitution in definite integrals

• Solving for the unknown integral

• Tabular integration

• Inverse trigonometric and logarithmic functions

• Logistic growth models

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| **STAGE 2****Evidence of Learning** |

**Formative Activities, Tasks, or Projects:**

* Discuss the relationship between a slope field and a differential equation.
* Discuss and stress to the students that the solutions to a differential equation given an initial condition is a function.
* Have students use *Desmos* to understand a slope field.
* As an introduction to integration using substitution, review the Chain Rule since *u*-substitution is a method for “reversing” the Chain Rule.
* Make sure to stress the basic steps for finding an antiderivative by substitution and practice with students in order for students to thoroughly understand the process.
* Review the double angle formulas need to integrate and
* To introduce integration by parts, use the Product Rule to derive the formula for integration by parts.
* Use LIPET to help the students determine what should be *u*.
* Discuss why tabular integration works and when it can be used to integrate.
* Before completing exponential growth and decay problems, review the algebraic rules for logarithms and exponents.
* Review the partial fraction decomposition method and use the “cover up” method where possible.
* Complete examples together using differential equations to model physical situations.

**Assessment Topic #7: Key Concept : Differential Equations** CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Lesson Theme 6.3 : Differential Equations in Motion (1.E) (4.B)

Start with a brief introduction to differential equations. Next emphasize the idea of a family of solution by choosing a particular solution using initial or other values. Focus on antidifferentiation as a process of “working backward.” Place emphasis on the fact that though the antiderivatives of a function form a family, when one additional condition is imposed, there is a unique antiderivative.

Use a situation is in describing an object thrown in the air to get this point across. Starting with its initial velocity, we need one additional constraint—its initial height—to find its position. Starting with its acceleration (that due to gravity) you need two additional pieces of information—the initial velocity and position.

Next give the class the following example of an object that is being dropped from a certain height to solve together.

An egg is dropped from the student union’s roof, which is 20 m high. Find the velocity, in meters per second, and the position, in meters, afterseconds.

At first it seems that we don’t have enough information until we remember that we know the ac celeration due to gravity, , is 9.81 m/sec2. Therefore, we know that . Thus, and . At first glance we obtain a family of solutions for both, velocity and position of the egg. Looking at the problem statement more carefully, we can write down two initial conditions. Since the student union is 20 m height, the initial height is given as , and since the egg is dropped and not thrown down the initial velocity is zero, thus . Using this information we can solve for and and obtain particular solutions and

As follow-up questions I will ask:

(a) When will the egg hit the ground?

(b) What is the velocity of the egg when it hits the ground?

Day 1: Complete problems 1, 2, 5-12

Day 2: Complete problems 13, 14, 16, 17, 19-27, 30, 31

Lesson Theme 11.1 : What is a Differential Equation (1.E) (2.C)(4.A)

Differential equations will be introduced by analogy with algebraic equations. Differential equations have functions as solutions rather than numbers. Algebraic equations model simple problems where the solution is a number; differential equations model more complex problems where the solution is described by a function.

We will start with a simple example that illustrates this modeling procedure, such as the following.

A yam is placed inside a 200◦C oven. The yam gets hotter at a rate proportional to the difference between its temperature and the oven’s temperature. When the yam is at 120◦C, it is getting hotter at a rate of 2◦ per minute. Write a differential equation that models the temperature, , of the yam as a function of time, *𝑡*. (Answer: . Show by substitution that

 is a solution to this differential equation for any constant . A discussion of the significance of the constant (it’s the initial temperature difference between the yam and the oven) will take place next, followed by solving for in the particular case when the initial temperature of the yam is 20◦. Discuss from an intuitive point of view why you would expect an arbitrary constant in the solution (the differential equation describes many different situations, with different initial temperature differences) and how you calculate the specific value of the constant given a specific initial value.

Point out that antidifferentiation is a particular case of solving a differential equation, namely and that there the arbitrary constant appears added to the solution, not multiplied.

Repeat that the solution to a differential equation is a *function*, and that unless specific conditions are given there are usually many solutions to a given differential equation.

Use examples such as the following:

1. Verify that is a solution to

2. Verify that is a solution to .

 At this stage you should also mention the concept of the order of a differential equation, and its relation to the number of arbitrary constants to expect in the solution.

Finish by solving a simple second-order differential equation such as to show that you get two arbitrary constants.

Make sure to use precise mathematical language to communicate the understanding of this concept.

Day 1: Complete problems 2-7, 8, 12, 13, 14-17, 20-23

Day 2: Complete problems 24, 27-31, 33-36

Lesson Theme 11.2 : Slope Fields (4.D)

Have students using a graphing calculator and the teacher use a computer program that draws slope fields and project them on a smartboard. Go over how to find slope fields on the calculator with the class by using an online graphing calculator and having iot projected on my smartboard as well via an overhead projector. On top of that use photocopied prepared slope fields from the Wiley AP Calculus manual that match overhead tprojections of various slope fields where we will trace solutions together as a class. Complete at least three examples by hand, just to make sure that the students see how a slope field is constructed. In particular, make sure that students remember what various slopes look like: a large positive slope, a small positive slope, a large negative slope, a slope of 1/2, 1 or 2. Explain that slope fields are like a set of sign posts, where there is one at each point, and wherever you are and it tells you in what direction to move. You move a little, and there’s the next sign post, etc.

Try drawing the solution:

1. from , to , and see how close to you get (then have the students to do this on work sheets).

Get students to come up and draw solutions directly on the whiteboard, over the projected slope field, and then ask them to critique each other’s efforts. Make sure that they understand that the solutions they draw on the slope field are the same as the solutions given in the previous class.

One good problem that can be done either in the way described above, or on worksheets that you hand out, is to give students the slope field of the yam equation from the previous class (or, if you gave them another example, give the slope field from that example). You can see a lot more about the general behavior of solutions from the slope field than from the specific solution. Illustrate how the general behavior of the solution depends on the initial conditions by getting the students to draw three solutions, one starting at the equilibrium solution one starting above it, and one starting below it. It is a good test of their understanding whether they will cross the equilibrium solution, or whether starting on it, they will stay on it. Ask them why a solution can never cross the equilibrium solution. (Once you are on the line *𝑇* = 200, you can’t leave it because the signposts don’t let you.) Also, point out how to read the arbitrary constant in the previously derived analytic solution from the graph of the solution on the slope field (it is the difference between 200 and the point where the solution crosses the *𝑦*-axis).

Since they feel like they are “connecting the dots,” students can easily think that the graphical solution is not as valid as Euler’s method or an analytic solution. It’s important to stress to students that this method is a viable one, and that it strengthens their geometric intuition about differential equations and their solutions. To illustrate this, give them a differential equation such as and ask them to describe the long-term behavior of the solution passing through (i.e. what happens to as Then draw the slope field and ask them the same question.

Day 1: Complete problems 1, 2, 3, 4, 5, 8, 9, 11, 17, 23

Day 2: Complete problems 10, 12, 13, 15, 18, 19-22, 24, 25

Lesson Theme 11.4 : Separation of Variables (1.E)

This is the only analytic technique students will be shown for solving first-order differential equations. Success with the method depends on being able to antidifferentiate both sides after the variables have been separated.

Consequently, this section provides a lot of practice with integration.

It is very important to maintain the graphical point of view while doing these problems because students often fail to make the connection between the analytic solutions and the ones pictured in the slope fields. Take the yam equation and solve it using separation of variables. Then graph the solutions for various different values of the constant and show that they look the same as the ones obtained from the slope field.

Day 1: Complete problems 1-29 odds

Day 2: Complete problems 2-30 evens

Lesson Theme 11.5 : Growth & Decay (1.E)(3.G)

While this section has an emphasis on modeling including how to go from a verbal statement to a differential equation. Use a graphing calculator to solve the equations. Start with a simple population or bank balance example. For instance, consider a population of rabbits that starts at 100 and grows at a continuous rate of 3% per year. How many rabbits are there after 10 years? Some students will know immediately that the answer is , but they may not know how to translate the percentage growth rate into a statement about derivatives; that is the point of going over this problem in detail. Explain how the continuous growth rate of 3% per year translates into the equation where is the population after years.

Ex. Since the growth rate is 3% per year, the percentage growth during a time interval of years is , so the actual change in population is therefore, You need to be careful about the distinction between continuous and annual percentage growth rates. Emphasize that it is not necessary to know what is to write down the differential equation for it; in fact, that’s the whole point. Then solve the differential equation by separation of variables () , and show how to go back and look at the problem to find the initial value ( when ). Then use the initial value to find the arbitrary constant (). Finally, use the solution to answer the question: , so there are 135 rabbits after 10 years. Since we are interested in modeling the real world here, it is worthwhile discussing why it does not make sense to quote to more decimal places ().

Day 1: Complete problems 1, 7, 20-22, 25, 27, 29, 33, 37, 39

Day 2: Complete problems 2, 9, 23, 24, 26, 28, 30, 34, 36, 38, 43

Lesson Theme 11.6 : Applications of Modeling (3.G) (3.F)

A compartmental analysis problem (such as the problem about salt concentration in a reservoir) is a good example to start with because although it is too complex to solve in one step, it is susceptible to a logical analysis. To help students set up differential equations, urge them to write the equations in words first.

Show how to recognize an equilibrium solution from a differential equation of the form (Set

Show how to recognize if an equilibrium is stable or unstable from a graph of the solutions.

Example: A person is in an unventilated room, 3 m long, 2 m wide, and 2.5 m high. The person’s rate of breathing depends linearly on the amount of carbon dioxide in the air. When the air has its usual amount of carbon dioxide, 0.04%, the person breathes at a rate of 0.015 m3 per minute, but when the carbon dioxide increases to 3.0%, the rate of breathing doubles. Expired air contains about 4.0 percentage points more carbon dioxide than the air breathed in. Write a differential equation for the concentration of carbon dioxide in the air at time Answer: Let be the concentration of carbon dioxide in the air in the room at time . The rate of breathing *𝑟* depends linearly on we have when, and when , giving

*.*  Now the only way the quantity of carbon dioxide in the room can increase is from the *extra* carbon dioxide in expired air. Since expired air contains 4% more carbon dioxide than the air breathed in:

Rate additional carbon dioxide is being added to room (in ) (Rate of respiration in )

If is the amount of carbon dioxide in the room at time , then putting the previous two equations together gives:

*.*

In addition, the concentration, , is given by so .
Thus, rewriting our differential equation in terms of , we get 100 volume of room

so .
This equation can be solved using separation of variables or a slope field.

Upon finishing this problem, have the students explain the meaning of the mathematical solution in context of how they solved it.

Day 1: Complete problems 1, 7-10, 12, 13, 15, 17, 18, 19, 26

Lesson Theme : Chapter 11 Review

Day 1: Complete problems 2- 14, 17

Day 2: Complete problems 18, 21, 22, 23, 25, 32, 35, 40 , 42

AP Examination Preparation questions CR2 CR3 CR4 CR5 CR6 CR7 CR8

**Summative Activities, Tasks, or Projects:**

Do Now problems CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Unit 7 test CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Wiley course resources on Canvas CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Example 6.3.2 Video: Finding a solution to an initial value problem

Example 6.3.3 Video: Finding velocity and position functions for a stone dropped from a building

Chapter 11 Additional problems

Example 11.1.1 Video: Verifying that a function is not a solution to a differential equation

Example 11.2.2 Video: Using a slope field to guess the form of solutions to a differential equation

Example 11.2.3 Video: Sketching solutions on a slope field and looking at long run behavior

Example 11.4.1 Video: Using separation of variables to solve a differential equation

Example 11.4.2 Video: Solving a differential equation and graphing solutions

Example 11.5.1 Video: Writing and solving a differential equation involving continuous compound interest

Example 11.5.4 Video: Finding an equilibrium solution and determining if it is stable

Example 11.6.1 Video: Writing and solving a differential equation for an application in economics

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| **STAGE 3****Learning Plan** |

**Assessment Topics and Lesson Themes:**

**Assessment Topic #7: Key Concept : Differential Equations** CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Lesson Theme 6.3 : Differential Equations in Motion

Lesson Theme 11.1 : What is a Differential Equation

Lesson Theme 11.2 : Slope Fields

Lesson Theme 11.4 : Separation of Variables

Lesson Theme 11.5 : Growth & Decay

Lesson Theme 11.6 : Applications of Modeling

Lesson Theme : Chapter 11 Review

**Unit Specific Accommodations and Modifications**

* Calculus Accelerated will require a formula sheet of integrals to be given as they are not required to memorize all of the derivatives for the transcendental functions.
* Students will compete integration by parts, tabular integration and integrating by partial fractions after the AP Test.
* Algebraic mistakes are common when evaluating antiderivatives. Students should check answers by differentiating.
* A very common mistake when integrating by using substitution is to insert the wrong constant multiplier.

Stress the mechanical nature of the process.

* Have students write the variable name by the bounds when changing bounds for definite integrals.
* Encourage students to use LIPET to help choose *u* for integration by parts.
* Watch out for sign errors when using integration by parts.
* Be careful with units from the given data at the beginning of exponential growth and decay problems.

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| **Additional Materials** |

**Digital Tools/Resources:**

TI-84 Plus Graphing Calculator provided to each student by the district

Wiley course resources on Canvas

http://apcentral.collegeboard.com

Khan Academy

Edmentum Exact Path

**Additional Primary & Secondary Resources:**

Multiple Choice & Free-Response questions in preparation for the AP Calculus (AB) Examination

LPS Adopted Textbooks and Programs

* Wiley: Calculus – AP Edition CR1
* Cengage: Calculus – Early Transcendental Functions CR1

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| **Unit # 8****14 Days****Applications of Integration** **(Weight on Exam 10-15%)** |
| **Overview** |

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| **STAGE 1****Desired Results** |

**Unit Overview**

You’ll make mathematical connections that will allow you to solve a wide range of problems involving net change over an interval of time and to find areas of regions or volumes of solids defined using functions. Topics include determining the average value of a function using definite integrals, modeling particle motion, solving accumulation problems, finding the area between curves & determining volume with cross-sections, the disc method, and the washer method. CR2 CR3 CR4 CR5 CR6

**Big Ideas**

Big Idea 1 : Change (CHA) CR22

**Essential Questions…** CR2 CR3 CR4 CR5 CR6 CR8

A. What is the second fundamental theorem of calculus and how is it used to construct antiderivatives?

B. How are the numerical methods for determining a definite integral including the midpoint and trapezoidal rules performed?

C. What is Simpson’s rule and how does it ally in real life situations?

D. How can areas and volumes of geometric figures be determined by the slicing method?

E. Using applications of geometry, how can volumes of revolution and the volumes of regions of a known cross-section be determined?

**Enduring Understanding…**

How functions change gives key information about the function and related real-world phenomena.

EU 3.1: Antidifferentiation is the inverse process of differentiation. CR3

EU 3.2: The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies. CR3

EU 3.3: The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration. CR3

EU 3.4: The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation. CR3

EU 3.5: Antidifferentiation is an underlying concept involved in solving separable differential equations. Solving separable differential equations involves determining a function or relation given its rate of change. CR3

CHA-4 : Definite integrals allow us to solve problems involving the accumulation of change over an interval. CR3

CHA-5 : Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval. CR3

CHA-6 : definite integrals allow us to solve problems involving the accumulation of change in length over an interval. CR3

**Students will be able to...**

* Apply the definite integral to problems involving motion
* Use definite the definite integral to solve problems involving accumulation
* Use the definite integral to calculate the areas of regions in a plane
* Use the definite (by slices or shells) to calculate the volume of solids
* Use the definite integral to calculate the length of curves in a plane

**Students will know...**

• The definitions for the following terms:

1. linear motion
2. position
3. velocity
4. acceleration
5. area between two curves
6. area enclosed by intersecting curves
7. boundaries with changing functions
8. integrating with respect to y
9. volume as an integral
10. circular cross sections
11. square cross sections
12. cylindrical shells
13. other cross sections
14. length of a smooth curve
15. vertical tangents, corners and cusps
16. midpoint rule
17. trapezoidal rule
18. Simpson’s rule

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| **STAGE 2****Evidence of Learning** |

**Formative Activities, Tasks, or Projects:**

* Review the concepts of position, velocity and acceleration
* Students need to understand that the integral of a rate gives the net change
* Utilize examples that take students beyond the velocity position relationship
* Students need to graph the region to decide on function dominance, limits and variable of integration
* Emphasis on volumes of rotations vs. solids
* Disk vs. washer method with shells as an alternate method.
* Use websites available for students to see the three-dimensional creation of the solids
* For length of a curve discuss the reasoning behind x versus y integration variable

**Assessment Topic #8: Key Concept : Applications of Integration** CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Lesson Theme 6.3 : Differential Equations & Motion (1.E) (4.B)

Revisit lesson 6.3 going over problems competed as a class previously.

Lesson Theme 6.4 : The Second Fundamental Theorem of Calculus (1.D) (1.E)

Many functions do not have elementary function antiderivatives so the construction theorem allows us to define and analyze an antiderivative function through the graphical justification and the “squeeze theorem.” Section 5.4 could be referred back to for review. Extend the examples on by using problems in the text, or use the error . . Finish by using the construction theorem on a function that does antidifferentiate easily to show that the answers agree.

Day 1: Complete problems 2, 4, 5, 7-17

Day 2: Complete problems 19-33 odds, 34-37

Lesson Theme 7.5 : Numerical Methods for Definite Integrals (1.D) (1.E)

Students should draw lots of graphs to explain the ideas as well as have a TI-84 plus to show how the different methods converge for increasing values of *𝑛*. This section can be made into lab where the students do their own calculations and report the results and conclusions. They will describe geometrically what each method is computing and how to tell whether the approximation is an overestimate or an underestimate by looking at whether the function is increasing or decreasing, concave up or concave down.

A small set of examples will be chosen and students will be guided through using the various approximation methods. This should be done for one problem where the exact answer is known, and another for which it is not known. Make sure to check all of the answers of numerical integration with a graphing calculator. Examples:

1.

2.

3.

Explain that you have found the value to a specified degree of accuracy (e.g. 3 decimal places) by watching the decimal places stabilize in the numerical approximations.

Teach students that the true value of an integral is bracketed by the left and right rules if the integrand is monotonic, and by the midpoint and trapezoid rules if the function does not change concavity. Since most integrals can be broken into integrals in which the integrand is monotonic or doesn’t change concavity, students can easily construct upper and lower estimates for their integrals, and hence bound the error in their approximations.

Using the method described, where the true value is bracketed above and below, allow students to focus on the integral they are approximating while estimating the error.

Next generate a discussion by drawing two curves, one gently rising the other rapidly rising and ask for which curve the left or right sum will give a better approximation for a given *𝑛*. It is easy to see that the errors are smaller in the gently rising curve. Then draw two curves which have small and large second derivatives and examine the behavior of the midpoint and trapezoidal rules. This is done in more detail in the next section, but it can be done here to firm up students’ understanding of the geometry.

Ensure students observe that doubling makes the error twice as small (or 4 or 16 times as small), but may have trouble understanding how this indicates that the error is proportional to (or or ), and how this in turn implies that increasing by a factor of 10 gives one (or 2 or 4) extra decimal places of accuracy.

Teach this class using technology emphasizing that the rules presented are approximate and won’t hold exactly in every given case. For examples in this section you should continue with whatever examples you used in the previous section. A good conclusion to draw is that Simpson’s rule gives reasonably accurate results in most cases for relatively small values of

Day 1: Complete problems 1-24 multiples of 3, 28-31, 32, 33, 34, 39

Lesson Theme 8.1 : Areas and Volumes (1.E)(2.B)(4.C)

Introduce the idea of finding areas and volumes by slicing with emphasis on the idea of slicing the region or solid into pieces whose area or volume is easy to find. Take the time to write down the sums that then approximate the area or volume.

Write the sum that gives an approximation, then point out that the limiting process gives the definition of a definite integral.

Have students find areas as a Riemann sum, then find the areas by evaluating the definite integrals. Have the students work some of the exercises in class having them draw a lot of pictures. Use handouts with pictures prepared ahead of time.

Cover Example 5 from the Wiley Calculus book in class (the volume of the Great Pyramid of Egypt) since it is revisited in the Section 8.5 where the total work required to build it is computed. It is easy to think of the Great Pyramid as being laid down in layers of square cross sections. Further, it will provide an example that cannot be done with “washers" or “shells."

Day 1: Complete problems 1-10, 28

Day 2: Complete problems 11-18, 30

Day 3: Complete problems 19, 22, 23, 26, 27, 29, 36, 37, 41

Lesson Theme 8.2: Applications to Geometry (1.D) (3.D)

Make sure to do each example by slicing, finding the volume of that slice, and summing while encouraging students to sketch pictures. A simpler version of the same idea is Problem 65 in the Wiley Calculus book. Ask the students why it is natural to have circumference measures for the tree rather than radius measures. Then derive the general formula for the volume of a right circular cone, and discuss a few variations; a skewed cone, a pyramid (cone with square cross-sections). Without going through the derivation, mention the general formula; all of these figures have volume equal to 1/3 the area of their base times their height; this is the generalization to three dimensions of the formula for the area of a triangle.

Cover arc length too, so that the integral can be easily calculated using the Fundamental Theorem. Any interesting curve can be chosen, such as a piece of a parabola or a cubic, or an arch of the sine function, and use numerical methods to evaluate the arc length integral.

Surface area can be assigned as a project.

Example: Viewing the earth from a spacecraft orbiting the earth and from the moon.

On Christmas Day, 1968, the Apollo 8 crew orbited the moon. In April, 1983, two members of the space shuttle Challenger performed an activity outside the Challenger at an altitude of 280 km above the surface of the earth.

What percentage of the Earth’s surface could the Apollo 8 team see? What percentage could the Challenger team see?

How far above the surface of the earth would you have to be to see 10% of the earth’s surface?

Use the facts: Earth is approximately a sphere of radius 6,380 km with the moon is about 376,000 km from the center of the earth.

This requires the student to know the formula for the area of a segment of a sphere. Students can check their formula by looking it up in a book. However, make sure they explain why it works. The ability to find an answer by research will stand them in good stead and should not be discouraged. The point of this project is not to teach the students the method for calculating areas of solids of revolution, but to train them in solving real world problems where the method is not laid out before them.

Day 1: Complete problems 1-4, 5-13 odds, 15-17, 29-32

Day 2: Complete problems 6-10 even, 14, 33-36, 39-49 odds

Day 3: Complete problems 40-48 evens, 50-58

AP Examination Preparation questions CR2 CR3 CR4 CR5 CR6 CR7 CR8

**Summative Activities, Tasks, or Projects:**

Do Now problems CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Unit 8 test CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Wiley course resources on Canvas CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Example 6.3.2 Video: Finding a solution to an initial value problem

Example 6.3.3 Video: Finding velocity and position functions for a stone dropped from a building

Example 6.4.1 Video: Constructing a table of values for Si(x)

Interactive Exploration 6.4: Second Fundamental Theorem of Calculus

Chapter 08 Additional problems

Example 8.1.2 Video: Using slices to set up a definite integral to find the area of a semicircle

Example 8.1.3 Video: Using slices to find a volume of a cone

Example 8.2.2 Video: Using slices to find the volume of a table leg

Example 8.2.4 Video: Finding volume of a solid of known cross-section

Interactive Exploration 8.2: Volumes of Solids of Revolution

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| **STAGE 3****Learning Plan** |

**Assessment Topics and Lesson Themes:**

**Assessment Topic #8: Key Concept : Applications of Integration** CR1 CR2 CR3 CR4 CR5 CR6 CR7 CR8

Lesson Theme 6.3 : Differential Equations & Motion

Lesson Theme 6.4 : The Second Fundamental Theorem of Calculus

Lesson Theme 7.5 : Numerical Methods for Definite Integrals

Lesson Theme 8.1 : Areas and Volumes

Lesson Theme 8.2: Applications to Geometry

**Unit Specific Accommodations and Modifications**

* Students will need clarification on the variable of integration, students should always graph the regions in question to give themselves a working diagram to help choose the correct variable.
* Stress the ability to recognize that using a geometry formula may be the easier option than a definite integral or in the case of a semicircle that the only way to find the area by hand.
* The focus of the chapter is to use the definite integral, so to that end students should be writing the definite integral necessary and then using the graphing calculator to evaluate.

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| **Additional Materials** |

**Digital Tools/Resources:**

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Khan Academy

Edmentum Exact Path

**Additional Primary & Secondary Resources:**

Multiple Choice & Free-Response questions in preparation for the AP Calculus (AB) Examination

LPS Adopted Textbooks and Programs

* Wiley: Calculus – AP Edition CR1
* Cengage: Calculus – Early Transcendental Functions CR1

**Interdisciplinary Connections & Standards**

With interdisciplinary instruction, the subject areas are woven together and explored through an overarching theme or concept. We use math to help us solve everyday problems in the kitchen, in the garden, and for many of us at our jobs.

Brain research has shown that information in our brains is organized in schematic structures. These structures are made up of interconnected bits of information and serve as a framework for the knowledge we acquire. When a learner’s knowledge is connected, it is much more likely that they will apply the prior knowledge to a wide variety of new situations. They will acquire new information in a way that is more accessible and will be better able to relate it to previously acquired knowledge.

Students learn about patterns in math, science, social studies, and even literature. Because of this, they are much more likely to “see” these patterns when they encounter new situations. Since patterns are not only studied in math they are able to make the connection and gain the understanding that patterns can be found in many areas of their lives. Interdisciplinary instruction allows students to understand the interconnectedness of the disciplines and makes learning more meaningful and relevant as fascinating connections are made across the subject areas.

**Science**

HS-PS3-1 Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other components(s) and energy flows in and out of the system.

HS-PS3-3 Design, build, and refine a device that works within given constraints to covert one form of energy into another form of energy.

HS-ETS1-2 Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

**Language Arts**

RL.11-12.1. Cite strong and thorough textual evidence and make relevant connections to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain.

RI.11-12.1. Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.

RI.11-12.2. Determine two or more central ideas of a text, and analyze their development and how they interact to provide a complex analysis; provide an objective summary of the text.

**Social Studies**

6.1.12.EconEM.2 Analyze how technological developments transformed the economy, created international markets, and affected the environment in New Jersey and the nation.

6.1.12.EconGE.16 Use quantitative data and other sources to assess the impact of international, global business organizations, and oversees competition on the United States economy and workforce.